International Journal of Engineering & Technology, 7 (4) (2018) 2044-2050

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET
doi: 10.14419/ijet.v7i4.15298
Research paper

Effect of axial magnetic field tapering on whistler-pumped FEL amplifier in collective Raman regime operation

Ram Gopal[1]*, Pradip Kumar Jain[1], *Senior Member, IEEE*

[1]*Centre of Research in Microwave Tubes, Department of Electronics Engineering, Indian Institute of Technology (Banaras Hindu University), Varanasi-221005, India*
**Corresponding author E-mail: ramgopal.rs.eee@itbhu.ac.in*

Abstract

The dispersion relation of the FEL Amplifiers is sensitive to the linear tapered strong axial magnetic fields, electron cyclotron frequency and plasma frequency of electrons. For the synchronism of the pumped frequency, it should be closed to electron cyclotron frequency which is resonantly enhanced the wiggler wave number that produces the amplifier radiation for higher frequency from sub millimeter wave to optical ranges. The guiding of radiation signal into the waveguide and charge neutralization phenomenon, the beam density should be greater than the background plasma density with tapered strong axial magnetic field. It is quite considerable that radiation signal slowed down at much higher background plasma density comparable to the density of beams and enhanced the instability growth rate also. In Raman Regime operation, the growth rate decreases as increases with operation frequency of the amplifier, however, the growth rate is larger in this regime. It is noted that as increases with background plasma density, the beat wave frequency of the Ponderomotive waves is increases thus the mechanism of background plasma density can serve for tenability of the higher frequencies. The tapering of the strong guided magnetic field is a crucial role for enhancing the efficiency of the net transfer energy as well as reduction of interaction region along the axis. It is observed that, an efficiency of the transfer energy enhanced by while the reduction along the interaction region of about with the variation of tapering in a strong axial guided magnetic fields.

Keywords: *FEL Amplifier; Linear Tapered Axial Magnetic Field; Magnetic Wiggler and Whistler.*

1. Introduction

The free electron laser amplifier (FELA) is an attractive device to employment of an electrostatic and electromagnetic wiggler for the production of tunable and coherent high power radiated signal using with mildly relativistic electron beams (REBs) to the higher frequencies from sub millimetre wave to optical ranges. In the interaction region, the synchronism of the pumped frequency with strong magnetized plasma, it should be closed to electron cyclotron frequency which is resonantly enhanced the wiggler wave number that produces the amplifier radiation in slow whistler mode, however, below the electron cyclotron frequency, the frequency emission of radiation is limited [1] [2]. The operating radiation frequency of FEL Amplifier λ_1 scales with wiggler period λ_o and beam Lorentz factor or the relativistic gamma factor (γ_o) of the electron beam, $\gamma_o = 1 + E_b / mc^2$ as $\lambda_1 = \lambda_0 / 2\gamma_o^2$ where E_b is the beam kinetic energy, m is the rest mass of electrons and c is the light velocity in vacuum [3]. Typically not all, the science of the free electron lasers studied in many areas as classical physics, engineering physics, applied mathematics, material science, biology, medicine, life science, nuclear physics and engineering, electrical engineering, electronics engineering, mechanical engineering and many other curriculums. The free electron laser is the tunable and fascinating device which have distinguishing feature and their properties is determined by the relativistic electron beams (REBs) or a short pulse laser. It is based on radiation from "free" electrons or unbound electrons rather than electrons bound in atomic and molecular systems [4]-[8]. The HUMAN BODY is the best example of free electron lasers that produces radiated energy after gyrated foods (seed-signal) into the human body chamber interact with water as e-beams and finally dumped it through depressed collector.

Sharma et al. [9] examined the operation of the electromagnetic and electrostatic wiggler with whistler pumped free electron laser for the higher frequencies. They have also studied the feasibility of the FELs in the low-current Compton Regime (CR), however practically, in the Compton regime, the whistler wave does not exist as suitable wiggler for FEL Amplifier operation due to requirements of extremely high pump power density (10-15 Tesla for operating frequencies scaled upto 200-250 GHz) for reasonable growth rate, hence including the effect of finite space charged mode, Raman Regime operation plays an important role in whistler-pumped FEL only [10] [11]. In the FEL Amplifier, the nonlinear state of Raman Regime (RR), amplitude of the beat wave (i.e., free space charge wave) is growing with trapped electrons; however, the trapped electrons could not be decelerated with the beat wave continuously. Pant and Tripathi [12] have proposed a nonlocal theory and studied their operation in whistler mode. In whistler pumped free electron laser, they had also examined the operating possibilities using a strong axial guided magnetic field and a static magnetic wiggler.

First time, in 1984, S. H. Gold et al. experimentally demonstrated a high power FEL Amplifier (FELA) using relativistic electron beams (REBs) at $35GHz$ for $1.2dB/cm$ growth rates and experimental efficiency $>3\%$ with $50dB$ gain. They have also examined an effect of strong axial magnetic field tapering on both ends of the device to enhancement of the efficiency and power for the generation of $>75MW$ at $75GHz$ with 6% experimental efficiency to the device [13]-[15]. Orzechowski et al. [16] [17] examined the operation of a FEL Amplifier at $35GHz$ with $180MW$ peak output power and electron beams energy governed by $3.6MeV\ /850A$ and find out an extraction efficiency of 6% with operating bandwidth of approximately 10% to $1.4m$ wiggler lengths. An efficiency of FEL Amplifiers (FELA) was 35% for $8mm$ wavelength with tapered wiggler [18] and it is about 1% at $800nm$ wavelengths [19].

Freund et al. [20] have introduced a nonlinear theory and simulation techniques for high-power in collective regime free electron laser with an efficiency of 27% at $33.4GHz$ frequency using a uniform wiggler while tapered wiggler experiment achieved 35% efficiency for same frequency using an electron beams of $3.5MeV\ /850A$. In Darmstadt, FELs at electron beam energy of $31MeV$ with $7\mu m$ wavelengths was investigated by Khodyachykh and their groups for tapered undulator on the main parameters of FELs [21]. Chung et al. [22] have proposed nonlinear theory and numerical simulation techniques of a FEL Amplifier (FELA) with the axial guided magnetic field and the tapered magnetic wiggler in one dimensional to amplification of $14kW$ signal and radiation wavelengths $10.6\mu m$ to a $2GW$ power. Gardelle et al. [23] explored the effects of space charge and quality of electron beams to improve the efficiency of FEL Amplifiers. They have also examined the collective effect on super radiant free electron laser amplifiers for a given interaction region and 2.5% efficiency is estimated to $35MW$ at $4mm$ lengths [24] [25].

In this paper, we examined and study the tapering effects of an axial magnetic field on whistler-Pumped FEL Amplifier in Collective Raman Regime operation. The tapering of the strong axial guided magnetic field is a crucial role for enhancing the efficiency and output power of the net transfer energy as well as reduction of interaction region along the axis. Now an electrostatic perturbation, i.e., $\vec{E}_{pb} = -\nabla\Phi_{pb}, \Phi_{pb} \approx e^{-i(\omega t - kz)}$ is subjected for the consideration of the electron beam density n^{o}_{ob} and the beam velocity $v^{o}_{ob}\hat{z}$, in a uniform cold electron beams for the free space charge waves, here is the consideration only of (ω, k) and real part of the forces, where $\omega = \omega_1 - \omega_0 \approx \omega_1$ and $k = k_1 + k_0$. Apart from the Ponderomotive potential, the self-consistent of free space charge potential $\Phi \approx e^{-i(\omega t - kz)}$ is also experienced on electrons to the high relativistic beam current as $\vec{E}_b \geq 40KA$ [1]. Since the beam current is very high, the susceptibility (χ_b) is greater than unity to the medium, i.e., $(\chi_b >> 1)$, hence, therefore the self-consistent of free space charge potential (Φ) is considered comparatively as Ponderomotive potential (Φ_{pb}) ,i.e., $(\Phi >> \Phi_{pb})$. In Eq. (27), the one with the lower sign has $\omega\partial\varepsilon / \partial\omega_1 < 0$, i.e., mode of negative energy is subjected, means, in this regime, the production of amplifier radiation (ω_1, k_1) is estimated through the coupling of whistler pumped $(0, k_o)$ with negative energy mode (ω, k). Therefore, for the leading to the growth of radiation mode and beam space charge mode simultaneously, the free space charge mode feeding more and more negative energy to the amplifier mode.

In section 2, we study the Raman Regime operation and their relevant equations included with effect of beam space charge and neglecting boundary conditions. In section 3, an evaluation of the FEL Amplifier Gain is explained. The summary of results is briefly discussed in section 4 and their conclusion given in section 5.

2. Raman regime operation

In this regime, we seek $k_o(z)$ to be an increasing function of $'z'$, hence a negative taper in the axial guided magnetic field is considered with an interaction region $0 < z < L$. Since the guided magnetic field has a negative linear taper comprises with plasma electron density n^{o}_{op} immersed in a static magnetic field $B_s(\hat{z})$, therefore,

$$B_s = B_{os}(1 - z/L) \quad \text{and,} \quad \vec{B}_o = B_o(\hat{x} + i\hat{y})e^{ik_o dz} \tag{1}$$

A circularly polarized whistler wave is propagating through the plasma along $-\hat{z}$ direction, hence, the electric field one obtains,

$$\vec{E}_o = A_o(\hat{x} + i\hat{y})e^{-i(\omega_o t + \int k_o dz)} \tag{2}$$

Where, $k_o = \frac{\omega_o}{c}\varepsilon^{1/2}$, $\varepsilon = [1 - \frac{\omega_p^2}{\omega_o(\omega_o - \omega_c)}]$ and $\omega_c = \omega_{co}(1 - \frac{z}{L})$, $\omega_{co} = \frac{eB_{os}}{mc}$ $\omega_p = (\frac{n^{o}_{op}e^2}{\varepsilon_o m})^{1/2}$. Here ω_c and ω_{co} is electron cyclotron frequency and initial electron cyclotron frequency, ω_o is wiggler wave frequency and the wiggler wave number k_o, ω_p is plasma frequency of an electron cyclotron and n^{o}_{op} is the back ground plasma density of an electron in the medium, m and $-e$ are the rest mass and the charge of the electron, L is the magnetic field scale length and c is the light velocity in vacuum, ε and ε_o are the relative and the free space permittivity respectively.

A relativistic beam density of electron n^{o}_{ob} and beam velocity $v^{o}_{ob}\hat{z}$ that propagates through back ground plasma with interaction region. It acquires an oscillatory velocity in transverse mode due to the whistler pumped with frequency $\gamma^{o}_{ob}k_o v^{o}_{ob}$ and the axial magnetic field gives to the cyclotron motion of electrons at a frequency ω_c. Therefore, the relativistic motion of equation can be obtained as,

$$\vec{v}_{ob} = \frac{e\vec{E}_o(1 + k_o v^{o}_{ob}/\omega_o)}{imc\gamma^{o}_{ob}(\omega_o + k_o v^{o}_{ob} - \omega_c/\gamma^{o}_{ob})} \tag{3}$$

Here, $\vec{B}_o = \frac{k_o(-\hat{z} \times \vec{E}_o)}{\omega_o}$, $\vec{k}_o = -k_o\hat{z}$, $\vec{v}_{ob} = \frac{\omega_1}{k_1}$ and $\gamma^{o}_{ob} = [1 - (v^{o}_{ob}/c)^2]^{-1/2}$. γ^{o}_{ob} is the electron beams relativistic gamma factor. The plasma electrons also acquires a drift velocity, hence using Maxwell's 3rd equation and the motion of equation, the total velocity of the electron cyclotron is,

$$\vec{v}_o = \frac{e\vec{E}_o}{im(\omega_o - \omega_c)} \tag{4}$$

Now for launching a circularly polarized FEL Amplifier radiation through the length of $z = 0$ end with electric field,

$$\vec{E}_1 = A_1(\hat{x} + i\hat{y})e^{-i(\omega_1 t - k_1 z)} \tag{5}$$

Where wave number of amplifier radiation, $k_1 = \omega_1 / c$ and frequency of radiation, $\omega_1 >> \omega_p, \omega_c$. If radiated electrons move with a whistler wave, then the phase matching condition to be satisfied $\omega = \omega_1 - \omega_o$ and $k = k_o + k_1$, hence,

$$\omega_1 = \gamma^{o\,2}_{ob}\omega_o(1 + v_{ob}/c)(1 + \varepsilon^{1/2}v_{ob}/c) \tag{6}$$

The whistler wave and the FEL Amplifier radiation wave gives oscillatory velocities v_{1b} and v_{1p} to beam and plasma electrons, i.e., $\vec{v}_{1b} = \frac{e\vec{E}_1}{im\gamma^{o}_{ob}\omega_1}$ and $\vec{v}_{1p} = \frac{e\vec{E}_1}{im\omega_1}$. If the whistler wave and amplifier signal beat to exert a Ponderomotive force on beam electrons at (ω_1, k_1), assuming here, $\omega = \omega_1 - \omega_o$ and $k = |k_o| + k_1$ and $\vec{\omega}_1 >> \vec{\omega}_c$, therefore the total Ponderomotive force is as,

$$\vec{F}_{pb} = e\nabla\Phi_{pb} = -\frac{e}{2c}\vec{v}^*_{ob} \times \vec{B}_1 - \frac{e}{2c}\vec{v}_{1b} \times \vec{B}^*_o \tag{7}$$

On solving the above equation (7), one obtains,

$$\vec{F}_{pb} = -(\frac{e^2 A_o^* A_1}{imc\omega_1\omega_o\gamma^o_{ob}})[k_o + \frac{k_1(\omega_o + k_o v^o_{ob})}{(\omega_o + k_o v^o_{ob} - \omega_c / \gamma^o_{ob})}]\hat{z}e^{-i\psi} \tag{8}$$

Again we know that, $\vec{F}_{pb} = e\nabla\Phi_{pb}$ or $\vec{F}_{pb} = iek\Phi_{pb}\hat{z} = ie(k_o + k_1)\Phi_{pb}\hat{z}$, hence we have,

$$\Phi_{pb} = -\frac{eA^*_o\vec{E}_{1\perp}}{mc\omega_o\omega_1\gamma^o_{ob}(k_o + k_1)}[k_o + \frac{(\omega_o + k_o v^o_{ob})k_1}{(\omega_o + k_o v^o_{ob} - \omega_c / \gamma^o_{ob})}] \tag{9}$$

Similarly the Ponderomotive potential for electron plasma can be written as,

$$\Phi_{pp} = -\frac{eA^*_o\vec{E}_{1\perp}}{mc\omega_o\omega_1(k_o + k_1)}[k_o + \frac{\omega_o k_1}{(\omega_o - \omega_c)}] \tag{10}$$

Now the Ponderomotive force $\vec{F}_{pb} = e\vec{E}_{pb}$ i.e. $\Phi_{pb} \approx e^{-i(\omega t - kz)}$ produces z-velocity, v_2 and density, n_{2b} perturbation of the electron beam in z-direction, therefore, the electrons acquire an axial velocity as,

$$v_{2z} = -\frac{ek\Phi_{pb}}{m\gamma^{o\,3}_{ob}(\omega_1 - kv^o_{ob})} \tag{11}$$

And the resultant perturbation density can be obtained by,

$$n_{2b} = -\frac{ek^2 n^o_{ob}\Phi_{pb}}{m\gamma^{o\,3}_{ob}(\omega_1 - kv^o_{ob})^2} \tag{12}$$

Now for the generation of the nonlinear forces $-\frac{e}{2}[v_{2z}(\hat{z} \times \vec{B}_o)]$, the axial oscillatory velocity $\vec{v}_{2z}$ interacts together with the magnetic wiggler field $\vec{B}_o$ and the transverse velocity due to wiggler as,

$$\vec{v}_{2zo} = -\frac{e\vec{B}_o}{2\gamma_o mc(\omega_1 - \omega_c / \gamma^o_{ob})} \cdot \frac{n_{2b}(\omega_1 - kv^o_{ob})}{kn^o_{ob}} \tag{13}$$

In the Raman Regime, an electrostatic perturbation, i.e., $\vec{E}_{pb} = -\nabla\Phi_{pb}, \Phi_{pb} \approx e^{-i(\omega t - kz)}$ is subjected for the consideration of the electron beam density n^o_{ob} and the beam velocity $v^o_{ob}\hat{z}$, in a uniform cold electron beams for the free space charge waves, here is the consideration only of (ω, k) and real part of the forces, where $\omega = \omega_1 - \omega_0 \approx \omega_1$ and $k = k_1 + k_0$. Apart from the Ponderomotive potential, the self-consistent of free space charge potential $\Phi \approx e^{-i(\omega t - kz)}$ is also experienced on electrons to the high relativistic beam current as $\vec{E}_b \geq 40KA$ [1]. Therefore, after replacing Φ_{pb} by $(\Phi + \Phi_{pb})$ from equation (11) and equation (12) one can gets,

$$v_{2z} = -\frac{ek}{m\gamma^{o\,3}_{ob}(\omega_1 - kv^o_{ob})}(\Phi + \Phi_{pb})$$
and,
$$n_{2b} = -\frac{ek^2 n^o_{ob}}{m\gamma^{o\,3}_{ob}(\omega_1 - kv^o_{ob})^2}(\Phi + \Phi_{pb}) \tag{14}$$

The nonlinear current density at (ω_1, k_1) can be written as,

$$\begin{aligned}\vec{J}^1_{b\perp} &= -en^o_{ob}\vec{v}_{1b} - \frac{1}{2}en_{2b}\vec{v}_{ob} - en^o_{ob}\vec{v}_{2zo} \\ &= -\frac{e^2 n^o_{ob}\vec{E}_{1\perp}}{im\gamma^o_{ob}\omega_1} + \frac{1}{2}\frac{e^2k^2 n^o_{ob}\vec{v}_{ob}}{m\gamma^{o\,3}_{ob}(\omega_1 - kv^o_{ob})^2}(\Phi + \Phi_{pb}) \\ &+ \frac{e^2 n_{2b}(\omega_1 - kv^o_{ob})\vec{B}_o}{2\gamma^o_{ob}kmc(\omega_1 - \omega_c / \gamma^o_{ob})}\end{aligned} \tag{15}$$

Now from the above equation (15), putting the values of $\gamma_o = 1, \vec{v}_{ob} = 0$ and shifting n^o_{ob}, and, n_{2b} by n^o_{op}, and, n_{2p}, the nonlinear current density of the plasma in the medium is written as,

$$\vec{J}^1_{p\perp} = -\frac{e^2 n^o_{op}\vec{E}_{1\perp}}{im\omega_1} - \frac{e^3 kn^o_{op}\vec{B}_o}{2m^2\omega_1 c(\omega_1 - \omega_c)}(\Phi + \Phi_{pp}) \tag{16}$$

Also here n_{2p} and Φ_{pp} can be written from equation (10) and equation (14), we have,

$$n_{2p} = -\frac{ek^2 n^o_{op}}{m\omega_1^2}(\Phi + \Phi_{pp}),$$
$$and,$$
$$\Phi_{pp} = -\frac{eA^*_o\vec{E}_{1\perp}}{mc\omega_o\omega_1(k_o + k_1)}[k_o + \frac{\omega_o k_1}{(\omega_o - \omega_c)}] \tag{17}$$

Hence the total transverse nonlinear current density of the beams $(\vec{J}^1_{T\perp} = \vec{J}^1_{b\perp} + \vec{J}^1_{p\perp})$ due to the contribution of the background plasma at (ω_1, k_1) and radiated by the whistler pumped is as,

$$\begin{aligned}\vec{J}^1_{T\perp} &= \vec{J}^1_{b\perp} + \vec{J}^1_{p\perp} \\ &= -\frac{e^2 n^o_{ob}\vec{E}_{1\perp}}{im\gamma^o_{ob}\omega_1} \\ &+ \frac{1}{2}\frac{e^3k^2 n^o_{ob}(\omega_o + k_o v^o_{ob})(\Phi + \Phi_{pb})\vec{E}_o}{im^2\omega_o\gamma^{o\,4}_{ob}(\omega_1 - kv^o_{ob})^2(\omega_o + k_o v^o_{ob} - \omega_c / \gamma^o_{ob})} \\ &- \frac{e^3 kn^o_{ob}\vec{B}_o}{2\gamma^{o\,4}_{ob}cm^2(\omega_1 - \omega_c / \gamma^o_{ob})(\omega_1 - kv^o_{ob})}(\Phi + \Phi_{pb}) \\ &- \frac{e^2 n^o_{op}\vec{E}_{1\perp}}{im\omega_1} - \frac{e^3 kn^o_{op}\vec{B}_o}{2m^2c\omega_1(\omega_1 - \omega_c)}(\Phi + \Phi_{pp})\end{aligned} \tag{18}$$

Since the beam current is very high, the susceptibility (χ_b) is greater than unity to the medium, i.e., $(\chi_b >> 1)$, hence, therefore the self-consistent of free space charge potential (Φ) is considerable comparatively as Ponderomotive potential (Φ_{pb}), i.e., $(\Phi >> \Phi_{pb})$, therefore, rearranging equation (18) can be written as,

$$\begin{aligned}\vec{J}^1_{T\perp} &= -\frac{1}{4\pi}[\frac{\omega^2_{pb}}{i\gamma^o_{ob}\omega_1} + \frac{\omega^2_p}{i\omega_1} + \frac{1}{2}\frac{e\omega^2_p\omega_c A^*_o}{m\omega_o\omega^2_1 c(\omega_1 - \omega_c)}\{k_o + \frac{\omega_o k_1}{(\omega_o - \omega_c)}\}]\vec{E}_{1\perp} \\ &+ \frac{1}{8\pi}\frac{ek^2\omega^2_{pb}(\omega_o + k_o v^o_{ob})\vec{E}_o}{im\omega_o\gamma^{o\,4}_{ob}(\omega_1 - kv^o_{ob})^2(\omega_o + k_o v^o_{ob} - \omega_c / \gamma^o_{ob})}\Phi \\ &- \frac{1}{8\pi}\frac{k\omega^2_{pb}\omega_c}{\gamma^{o\,4}_{ob}(\omega_1 - \omega_c / \gamma^o_{ob})(\omega_1 - kv^o_{ob})}\Phi \\ &- \frac{1}{8\pi}\frac{k\omega^2_p\omega_c}{\omega_1(\omega_1 - \omega_c)}\Phi\end{aligned} \tag{19}$$

Now using the total current density $(\vec{J}^1_{T\perp})$ into the wave equation, one obtains,

$$(\omega_1^2 - k_1^2c^2)\vec{E}_{1\perp} = -4\pi i\omega_1\vec{J}^1_{T\perp} \tag{20}$$

Comparing equation (19) and equation (20), we have,

$$[\omega_1^2 - k_1^2c^2 - \frac{\omega_{pb}^2}{\gamma_{ob}^o\omega_1} - \frac{\omega_p^2}{\omega_1} - \frac{1}{2}\frac{ei\,\omega_p^2\omega_c A_o^*}{m\,\omega_o\omega_1 c(\omega_1-\omega_c)}.$$
$$\{k_o + \frac{\omega_o k_1}{(\omega_o-\omega_c)}\}]\vec{E}_{1\perp} = -\frac{k\,\omega_1\Phi}{2}.$$
$$[\frac{ek\,\omega_{pb}^2(\omega_o+k_ov_{ob}^o)\vec{E}_o}{m\,\omega_o\gamma_{ob}^{o\,4}(\omega_1-kv_{ob}^o)^2(\omega_o+k_ov_{ob}^o-\omega_c/\gamma_{ob}^o)} \tag{21}$$
$$-\frac{i\,\omega_{pb}^2\omega_c}{\gamma_{ob}^{o\,4}(\omega_1-\omega_c/\gamma_{ob}^o)(\omega_1-kv_{ob}^o)} - \frac{i\,\omega_p^2\omega_c}{\omega_1(\omega_1-\omega_c)}]$$

i.e.,

$$\mathrm{R}.\vec{E}_{1\perp} = \Phi.\mathrm{B}$$

Where,

$$R = \omega_1^2 - k_1^2c^2 - \frac{\omega_{pb}^2}{\gamma_{ob}^o\omega_1} - \frac{\omega_p^2}{\omega_1} - \frac{1}{2}\frac{ie\omega_p^2\omega_c A_o^*}{m\omega_o\omega_1(\omega_1-\omega_c)}[k_o + \frac{\omega_o k_1}{(\omega_o-\omega_c)}] \tag{22}$$

And,

$$B = -\frac{k\omega_1}{2}[\frac{ek\omega_{pb}^2(\omega_o+k_ov_{ob}^o)\vec{E}_o}{m\omega_o\gamma_{ob}^{o\,4}(\omega_1-kv_{ob}^o)^2(\omega_o+k_ov_{ob}^o-\omega_c/\gamma_{ob}^o)} - \frac{i\omega_{pb}^2\omega_c}{\gamma_{ob}^{o\,4}(\omega_1-\omega_c/\gamma_{ob}^o)(\omega_1-kv_{ob}^o)} - \frac{i\omega_p^2\omega_c}{\omega_1(\omega_1-\omega_c)}] \tag{23}$$

Now for free space and using Poisson's equation $\nabla^2\Phi = 4\pi en$ i.e., $(ik)^2\Phi = en/\varepsilon$ that yields,

$$\varepsilon\Phi = \frac{ek_bA_o^*\vec{E}_{1\perp}}{mc\omega_o\omega_1\gamma_{ob}^o(k_o+k_1)}[k_o + \frac{(\omega_o+k_ov_{ob}^o)k_1}{(\omega_o+k_ov_{ob}^o-\omega_c/\gamma_{ob}^o)}] + \frac{ek_pA_o^*\vec{E}_{1\perp}}{mc\omega_o\omega_1(k_o+k_1)}[k_o + \frac{\omega_o k_1}{(\omega_o-\omega_c)}] \tag{24}$$

Where, $\varepsilon = 1 + k_p + k_b$, $k_p = -\frac{\omega_p^{\,2}}{\omega_1^2}$ and $k_b = -\frac{\omega_{pb}^{\,2}}{\gamma_{ob}^{o\,3}(\omega_1 - kv_{ob}^o)^2}$ and $\varepsilon = 1 + \chi_b$, ε is the permittivity and χ_b is the susceptibility of the medium of the wave. Hence from equation (21) and equation (24) can be written as,

$$\mathrm{R}.\varepsilon = \mathrm{B}[\frac{ek_bA_o^*}{mc\omega_o\omega_1\gamma_{ob}^o(k_o+k_1)}.$$
$$\{k_o + \frac{(\omega_o+k_ov_{ob}^o)k_1}{(\omega_o+k_ov_{ob}^o-\omega_c/\gamma_{ob}^o)}\}$$
$$+\frac{ek_pA_o^*}{mc\omega_o\omega_1(k_o+k_1)}.\{k_o + \frac{\omega_o k_1}{(\omega_o-\omega_c)}\}] \tag{25}$$

i.e,

$$\mathrm{R}.\varepsilon = Q$$

Where,

$$Q = -\frac{k\,\omega_1}{2}[\frac{ek\,\omega_{pb}^2(\omega_o+k_ov_{ob}^o)\vec{E}_o}{m\,\omega_o\gamma_{ob}^{o\,4}(\omega_1-kv_{ob}^o)^2(\omega_o+k_ov_{ob}^o-\omega_c/\gamma_{ob}^o)}$$
$$-\frac{i\,\omega_{pb}^2\omega_c}{\gamma_{ob}^{o\,4}(\omega_1-\omega_c/\gamma_{ob}^o)(\omega_1-kv_{ob}^o)} - \frac{i\,\omega_p^2\omega_c}{\omega_1(\omega_1-\omega_c)}]$$
$$.[\frac{ek_bA_o^*}{mc\omega_o\omega_1\gamma_{ob}^o(k_o+k_1)}\{k_o + \frac{(\omega_o+k_ov_{ob}^o)k_1}{(\omega_o+k_ov_{ob}^o-\omega_c/\gamma_{ob}^o)}\}$$
$$+\frac{ek_pA_o^*}{mc\omega_o\omega_1(k_o+k_1)}\{k_o + \frac{\omega_o k_1}{(\omega_o-\omega_c)}\}]$$

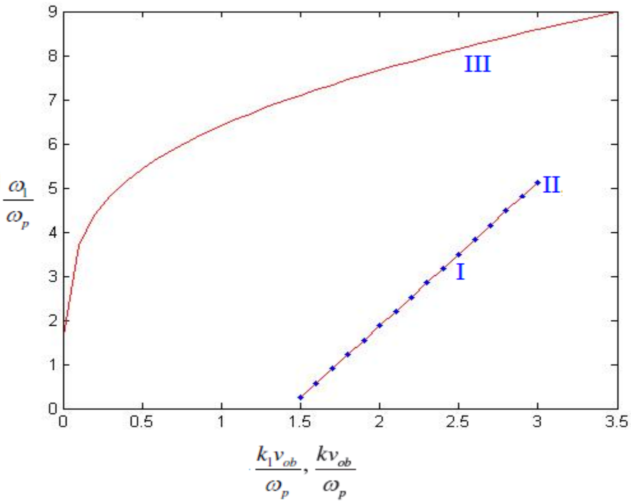

Fig. 1: Dispersion Relation Curve to the Beam Modes $(\omega_1/\omega_p vs kv_{ob}/\omega_p)$ and Radiation Modes $(\omega_1/\omega_p vs k_1v_{ob}/\omega_p)$. Curve I (Solid Line) and II (Dotted Line Validation) Represent the Beam Modes and Curve III Represent the Radiation Modes. Parameters are as: $\omega_{pb} = 0.2\omega_p, \omega_c = 4\omega_p, v_{ob} = 0.4c, k_o = 3cm^{-1}, \omega_p = 2\times10^{10}r/s$

Hence, therefore for the operation of frequency in FEL Amplifier, there are the two factors are simultaneously zero on the LHS of the above equation (25) and the equation (25), called the dispersion relation equation of FEL Amplifier. Here equation (25) plays an important role in whistler-pumped FEL Amplifier for Raman Regime operation, which gives the operation of the device i.e.,

$$\mathrm{R} = 0, \varepsilon = 0 \tag{26}$$

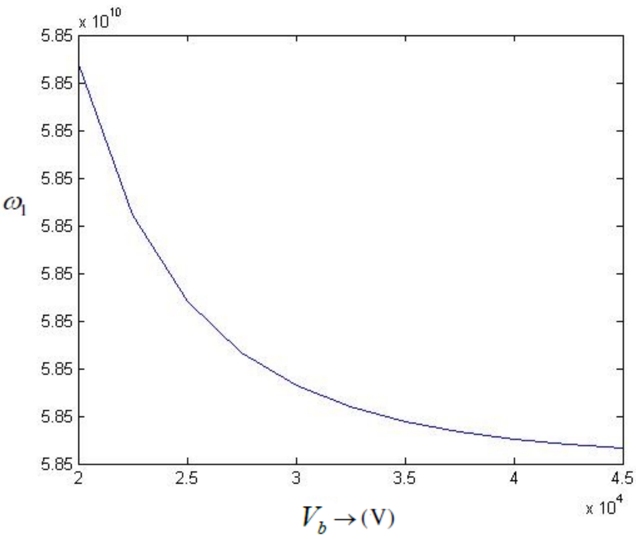

Fig. 2: Radiation Frequency (ω_1) versus Beam Voltage (V_b) for the Given Parameters: $\omega_{pb} = 0.2\omega_p, \omega_c = 4\omega_p, v_{ob} = 0.4c, k_o = 3cm^{-1}, \omega_p = 2\times10^{10}r/s$

The one with the lower sign has $\omega\partial\varepsilon/\partial\omega_1 < 0$, i.e., mode of negative energy is subjected, means, in this regime, the production of amplifier radiation (ω_1, k_1) is estimated through the coupling of whistler pumped $(0, k_o)$ with negative energy mode (ω, k). Therefore, for the leading to the growth of radiation mode and beam space charge mode simultaneously, the free space charge mode feeding more and more negative energy to the amplifier mode. Therefore the growth rate of the FEL Amplifier in this regime [1] is defined as,

$$\Gamma = [-Q\left|\frac{\partial \varepsilon}{\partial \omega_1}.\frac{\partial R}{\partial \omega_1}\right|^{-1}_{\omega_1=\omega_{1r}}]^{1/2} \quad (27)$$

Where ω_{1r} is the solution of equation (25).

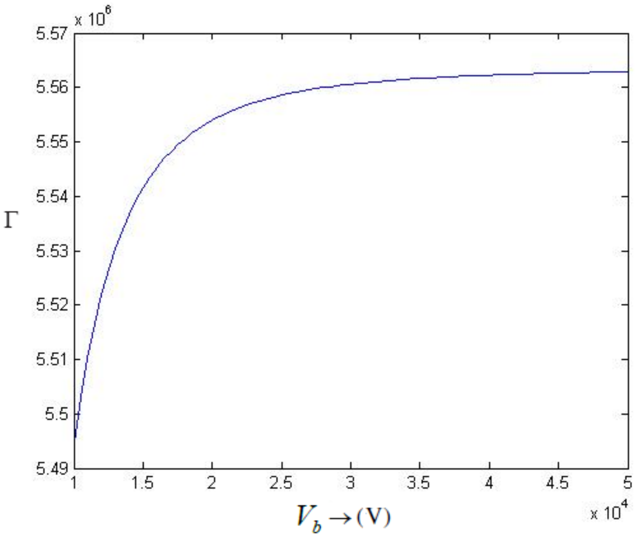

Fig. 3: Growth Rate (Γ) versus Beam Voltage (V_b) for the Given Parameters: $\omega_{pb} = 0.2\omega_p, \omega_c = 4\omega_p, v_{ob} = 0.4c, k_o = 3cm^{-1}, \omega_p = 2\times10^{10} r/s$

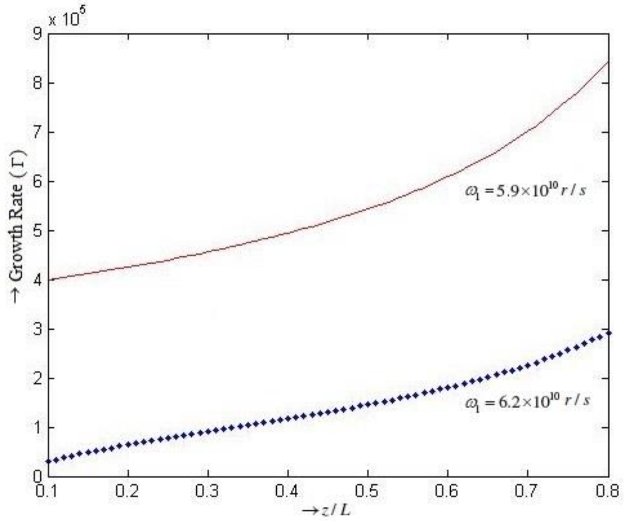

Fig. 4: Growth Rate (Γ) versus Normalized Lengths (z/L) for the Given Parameters: $\omega_{pb} = 0.2\omega_p, \omega_c = 4\omega_p, v_{ob} = 0.4c, k_o = 3cm^{-1}, \omega_p = 1.9\times10^{10} r/s,$ $\omega_p = 2\times10^{10} r/s, \omega_1 = 5.9\times10^{10} r/s, \omega_1 = 6.2\times10^{10} r/s, L = 40cm$

3. FEL amplifier gain

In the FEL Amplifier, the nonlinear state of Raman Regime (RR), amplitude of the beat wave is growing with trapped electrons; the instability may be saturated in the Ponderomotive wave. Therefore the electric field $\vec{E}_{pb}$ of the Ponderomotive wave,

$$\vec{E}_{pb} = (\frac{eA_o^* A_1}{im\omega_1\omega_o\gamma_{ob}^o})[k_o + \frac{k_1(\omega_o + k_o v_{ob}^o)}{(\omega_o + k_o v_{ob}^o - \omega_c/\gamma_{ob}^o)}]\hat{z}e^{-i\psi} \quad (28)$$

Where, $\psi = \omega t - \int(k_o + k_1)dz, \omega = \omega_1, and, k = |k_o| + k_1$, and the z-component of single particle beam momentum equation under the beat Ponderomotive force can be written as, $\frac{d\gamma_e}{dz} = \frac{1}{mc^2}F_{pbz}$ or $\frac{dP_{zb}}{dt} = v_{zb}\frac{dP_{zb}}{dz} = mc^2\frac{d\gamma_e}{dz} = F_{pbz}$. Here γ_e denotes the relativistic energy of an electron at any point. Hence from equation (8) and taking real part, we have,

$$\frac{d\gamma_e}{dz} = -\frac{eA_{pb}}{mc^2}\cos\psi \quad (29)$$

Where

$\vec{A}_{pb} = -i\vec{E}_{pb}$, $E_{pb} = (\frac{a_1 A_o^* c}{\omega_o\gamma_{ob}^o})[k_o + \frac{k_1(\omega_o + k_o v_{ob}^o)}{(\omega_o + k_o v_{ob}^o - \omega_c/\gamma_{ob}^o)}]$ and $a_1 = \frac{eA_1}{m\omega_1 c}$.

This equation represent that wiggler is in uniform state, means no resonance is possible here. In the FEL Amplifier, the nonlinear state of Raman Regime (RR), amplitude of the beat wave (i.e., free space charge wave) are growing with trapped electrons, however, the trapped electrons could not be decelerated with the beat wave continuously. Therefore neglecting the effects of free space charge, the single particle motion of equation in the Ponderomotive waves can be written as,

$$\frac{d\gamma_e}{dz} = -\frac{eA_{pb}}{mc^2}\cos(\omega t - kz) \quad (30)$$

Now defining the variables $\Delta\gamma_e = \gamma_e - \gamma_r$ and $\psi = kz - \omega t$, where, γ_e denotes the relativistic energy of an electron at any point z and γ_r is the resonant energy for a uniform wiggler which is constant. Considering here, $\gamma_e = (1 + \frac{P_\perp^2 + P_z^2}{m_o^2 c^2})^{1/2} = (1 - \frac{v_{o\perp}^2}{c^2} - \frac{v_z^2}{c^2})^{-1/2}$ and $E_p = k|\phi_p|$, P is the momentum of the electrons and has written as $\frac{dP_z}{dt} = v_z\frac{dP_z}{dz} = \frac{mc^2 d\gamma_e}{dz}$ and resonant gamma factor (γ_r) due to phase velocity i.e., $\gamma_r = (1 - \frac{v_{o\perp}^2}{c^2} - \frac{\omega^2}{k^2c^2})^{-1/2}$. Since the deviation of energy, $\Delta\gamma_e = \gamma_e - \gamma_r$ are also effective in resonant case, hence,

$$\frac{d\Delta\gamma_e}{dz} = \frac{d\gamma_e}{dz} - \frac{d\gamma_r}{dz} \quad (31)$$

Let $\Delta\gamma_e = \gamma_e - \gamma_r, \psi = kz - \omega t$ to write,

$$\frac{d\Delta\gamma_e}{dz} = -\frac{eA_{pb}}{mc^2}\cos\psi \quad (32)$$

Now the equation $\psi = \omega t - \int(k_o + k_1)dz$ is governing ψ , then differentiate w. r. to z dimensionalizing z by $\xi = z/L$, we have,

$$\frac{d\psi}{d\xi} = \frac{\omega_1 L\Delta\gamma_e}{2c(\gamma_r^2 - 1)^{3/2}} \quad (33)$$

This is called momentum of the wave i.e., $P = \frac{d\psi}{d\xi}$. Now from equation (32) and equation (33), we have,

$$\frac{dP}{d\xi} = -\frac{eA_{pb}L^2\omega_1}{2mc^3(\gamma_r^2 - 1)^{3/2}}\cos\psi \quad (34)$$

The equation (33) and equation (34) constitute the phase momentum and energy evolution equations and can be rewritten as,

$$\frac{dP}{d\xi} = -A\cos\psi$$
$$\frac{d\psi}{d\xi} = P \quad (35)$$

Where A is constant i.e., $A = \frac{eA_{pb}L^2\omega_1}{2mc^3(\gamma_r^2 - 1)^{3/2}}$. For the small radiation of single pass amplification, it is worthwhile solving equation (35) i.e., an electron can lose the energy to the wave with phase ψ is between $-\pi/2$ to $\pi/2$. Now after integration of equation (35) w.r.to ψ, we have an interesting result as,

$$P^2 = -2A\sin\psi + P_{in}^2 + 2A\sin\psi_{in} \tag{36}$$

Where, the values of electron momentum (P) and phase (ψ) at the entry point $z = 0$, are $P_{in} = P_{\xi=0}$ and $\psi_{in} = \psi_{\xi=0}$ respectively. Initially, at $z = 0$ electron lies uniformly with $P = P_{in}$ i.e., horizontal line for all times in the (P, ψ) plane. Therefore, the separatrix of electrons are trapped. If $z > 0$, means passing electrons are outside the separatrix i.e., electrons are some gain energy or lose energy, hence, the energy passing through interaction region and lost by an electron is $\Delta P \equiv P_{in} - P(\xi = 1)$ or $\Delta P \equiv -(P_1 + P_2)_{\xi=1}$.

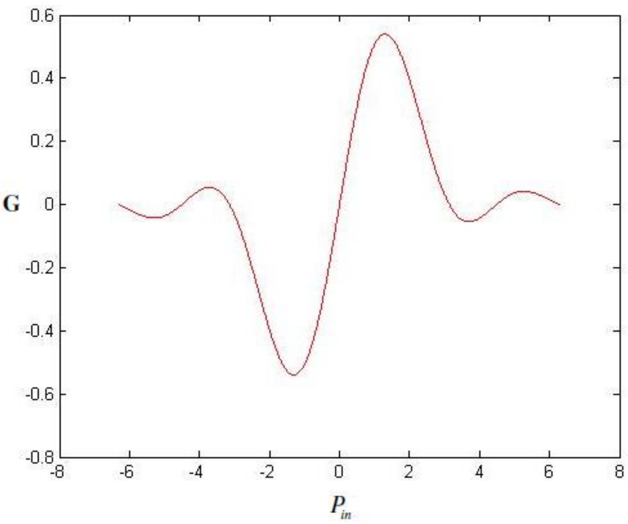

Fig. 5: Gain (G) Versus Electron Momentum (P_{in}) for the Given Parameters: $\omega_1 = 6.2\times10^{10} r/s, L = 40cm$.

Now the average values of $<\Delta P>$ over the initial phases yields,

$$<\Delta P> = -\frac{A^2}{8}\frac{d}{dx}\left(\frac{\sin^2 x}{x^2}\right) \tag{38}$$

Where, $x = \frac{P_{in}}{2}$, hence the Fig. 5, shows the gain function as, $G \equiv -\frac{d}{dx}\left(\frac{\sin^2 x}{x^2}\right)$ with initial values of P_{in} or $\gamma_o - \gamma_r$. For $\gamma_o > \gamma_r$, therefore, the net energy is transfer from electrons to the waves [1] [3]. Hence the trapped electrons efficiency radiated as,

$$\eta_{tr} = \frac{\Delta\varepsilon}{mc^2(\gamma_r(0) - 1)} \tag{39}$$

Since an electrons, which can lose the energy to the wave for a phase (ψ) is between $-\pi/2$ to $\pi/2$, i.e., π with a given length of interaction region at the exit point, therefore the total efficiency of the energy conversion in FEL amplifier is [3],

$$\eta = \frac{\gamma_r(\xi = 0) - \gamma_r(\xi = 1)}{\gamma_r(\xi = 0) - 1} \tag{40}$$

4. Results and discussion

The Normalized dispersion curve for the beam modes $(\omega_1/\omega_p vs k v_{ob}/\omega_p)$ and radiation modes $(\omega_1/\omega_p vs k_1 v_{ob}/\omega_p)$ are shown in Fig. 1, for the given parameters:
$\omega_{pb} = 0.2\omega_p, \omega_c = 4\omega_p, v_{ob} = 0.4c, k_o = 3cm^{-1}, \omega_p = 2\times10^{10} r/s$.
The curves I (solid line) and II (dotted line) represent the beam modes and curve III represent the radiation modes in Raman Regime operation of the device. The dispersion relation of the FEL Amplifier is sensitive to the tapered strong axial magnetic fields, electron cyclotron frequency and plasma frequency of electrons, which plays an important role in this configuration. For the synchronism of the pumped frequency, it should be closed to electron cyclotron frequency which is resonantly enhanced the wiggler wave number that produces the amplifier radiation for higher frequency from sub millimeter wave to optical ranges.
An emission of the FEL Amplifier frequency (ω_1) with the function of beam voltage (V_b) has been showed in Figure 2. An explanation of this figure, the guiding of radiation signal into the waveguide and charge neutralization phenomenon, the beam density should be greater than the background plasma density with tapered strong axial magnetic field, however, the device behaves similar as vacuum FEL Amplifier at very high density of electron beams. It is quite considerable that radiation signal slowed down at much higher background plasma density comparable to the density of beams and enhanced the instability growth rate also. In this device, we study the operation of Raman Regime to the generation of $\omega_1 = 6.2\times10^{10} rad/\sec$ frequency for radiation mode using with mildly REBs. It is clear that the growth rate is inversely proportional to the radiation frequency of the device, means, increasing of higher radiation frequency, the growth rate decreases and vice versa, however, in the Raman Regime operation, the growth rate is larger (e.g. Figure 3) while occurrences of radiation is quite possible at higher frequency increasing with beam voltage (V_b).
In Figure 4, Raman Regime operation, the growth rate decreases as increases with frequency of operation of the amplifier while it is unaffected in Compton Regime [1]. It is noted that as increases with background plasma density, the beat wave frequency of the Ponderomotive waves is increases thus the background plasma density can be help for enhanced the tuning of higher frequency in FEL Amplifier also. Hence the mechanism of background plasma density can serve for tenability of the higher frequency of the device. Although the frequency of radiation can be tuned by very small wiggler period and/or higher electron beams energy, however, in practically, it is not accessible more easily for high beams energy as well as shortening the wiggler periods and also typically, not all, the magnetic wiggler field are not easily accessible for very small wiggler wavelengths.
Figure 5, shows the gain function 'G', which is varying with the variation of electron momentum (P_{in}) and the net transfer energy from electron beams to the beat wave is quite considerable at $V_b \geq 40KV$, however, along the interaction region, an efficiency of the net transfer energy to the wave is enhanced adiabatically with slowing down the Ponderomotive waves also. The tapering of the strong axial guided magnetic field is a crucial role for enhancing the efficiency of the net transfer energy as well as reduction of interaction region along the axis. It is observed that, an efficiency of the transfer energy enhanced by 20 % [3] while the reduction along the interaction region of about 10 % [1] with the variation of tapering in a strong axial guided magnetic fields. On other hand an enhancement of efficiency is brought about 5% as resultant 20% change of background plasma density with presence of tapered strong axial guided magnetic field however, an intensity of an FEL Amplifier can be influenced little with dynamics of beams by tapering but it might not be detrimental to the instability in FEL Amplifier.

5. Conclusion

This paper observed the tapering effects of an axial magnetic field on whistler-Pumped FEL Amplifier in Collective Raman Regime operation. The tapering of the strong axial guided magnetic field is a crucial role for enhancing the efficiency and output power of the net transfer energy as well as reduction of interaction region along the axis and also we study the operation of Raman Regime to the generation of $\omega_1 = 6.2\times10^{10} rad/\sec$ frequency for radiation mode using with mildly REBs. It is clear that the growth rate is inversely proportional to the radiation frequency of the device, means, increasing of higher radiation frequency, the growth rate decreases and vice versa, however, in the Raman Regime operation, the growth rate is larger (e.g. Figure 3) while occurrences of radiation is quite possible at higher frequency increasing with beam voltage (V_b). There are many applications in different areas, typically not all, the science of the free electron lasers studied in many areas as classical physics, engineering physics, applied mathematics, material science, biology, medicine, life science, nuclear physics and engineering, electrical engineering, electronics engineering, mechanical engineering and many other curriculums.

Acknowledgement

One of the authors, Ram Gopal (RG), is grateful to the Indian Institute of Technology (IIT-BHU), Varanasi, India and Rajiv Gandhi National Fellowship (RGNF-UGC), New Delhi, India for financial support.

References

[1] Pant KK & Tripathi VK, "Free Electron Laser Operation in the Whistler Mode", *IEEE Transactions on plasma science*, Vol. 22, No. 3, (1994). https://doi.org/10.1109/27.297869.

[2] Sharma A & Tripathi VK, "A plasma filled gyrotron pumped free electron laser", *Phys. Plasmas*, Vol. 3, (1996), pp. 3116. https://doi.org/10.1063/1.871658.

[3] Gopal R & Jain PK, "Tapering effect of an axial magnetic field on whistler-pumped FEL Amplifier", *IRACTS-Engineering Science and Technology: An International journal (ESTIJ)*, Vol. 8, No. 2, (2018).

[4] Marshall TC, *Free Electron Lasers*, MacMillan, New York, (1985).

[5] Liu CS & Tripathi VK, *Interaction of electromagnetic waves with electron beams and plasmas*, World Scientific, (1994). https://doi.org/10.1142/2189.

[6] Botao Jia, "Study of Storage Ring Free Laser using experimental and simulation approaches", Ph.D. Dissertation, Department of Physics, Duke University, China, (2011).

[7] Pae KH & Hahn SJ, "Compact sized IR source using electromagnetic-wave wigglers", *Journal of the Korean Physical Society*, Vol. 40, No. 5, (2002), pp. 832-836.

[8] Patrick Shea GO & Freund HP, "Free Electron Lasers: Status and Applications", Vol. 292, Issue-5523, (2001), p. 1853-1858.

[9] Sharma A & Tripathi VK, "A whistler pumped free electron laser", *Phys. Fluids*, Vol. 3I, (1988), pp. 3375-3378. https://doi.org/10.1063/1.866902.

[10] Sharma A & Tripathi VK, "Kinetic theory of a whistler-pumped free electron laser," *Phys. of Fluids*, Vol. B 5, No.1, (1993).

[11] Jong RA, Scharlemann ET & Fawley WM, "Wiggler taper optimization for free electron laser amplifier (FELA) with moderate space-charge effects", *Nuclear Instruments and Methods in Physics Research*, Vol. A 272, (1988), p. 99-105.

[12] Pant KK & Tripathi VK, "Nonlocal theory of a whistler pumped free electron laser", *Physics of Plasmas*, Vol. 1, (1994), pp. 1025. https://doi.org/10.1063/1.870782.

[13] Gold SH, Hardesty DL, Kinkead AK, Barnett LR & Granatstein VL, "High gain 35-GHz free electron laser amplifier experiment", *Physical review letters*, Vol. 52, No. 14, (1984), p. 1218-1221. https://doi.org/10.1103/PhysRevLett.52.1218.

[14] Gold SH, Black WM, Freund HP, Granatstein VL & Kinkead AK, "Radiation growth in a millimetre-wave free electron laser operating in the collective Regime", *Physics of Fluids*, Vol. 27, No. 3, (1984), p. 746-754. https://doi.org/10.1063/1.864650.

[15] Gold SH, Freund HP & Bowie, "Free electron laser with tapered axial magnetic field", The United States of America as represented by the Secretary of the Navy, Washington, DC, (1987), Patent Number: 4,644,548.

[16] Orzechowski TJ, Anderson BR, Fawley WM, Prosnitz D, Scharlemann ET & Yarema SM, "High gain and high extraction efficiency from a free electron laser amplifier operating in the millimetre wave regime", *Nuclear Instruments and Methods in Physics Research*, Vol. A, No. 250, (1986), p. 144-149.

[17] Orzeehowski TJ, Anderson BR, Clark JC, Fawley WM, Paul AC, Prosnitz D, Scharlemann ET & Yarema SM, *Physical review letters*, Vol. 57, No. 17, (1986).

[18] Freund HP, "Comparison of free-electron laser amplifiers based on a step-tapered optical klystron and a conventional tapered wiggler", physical review special topics- accelerators and beams, Vol.16, 060701, (2013).

[19] Wang XJ, Freund HP, Harder D, Miner WH, Jr. Murphy JB, Qian H, Shen Y & Yang X, *Physical review letters*, Vol. 9, No. 103, (2009), 154801. https://doi.org/10.1103/PhysRevLett.103.154801.

[20] Freund HP & Ganguly AK, "Nonlinear Simulation of a High-power, Collective Free Electron Laser", *IEEE Transactions on plasma science*, Vol. 20, No. 3, (1992). https://doi.org/10.1109/27.142826.

[21] Khodyachykh S, Brunken M, Genz H, Graf HD, Hessler C, Richter A, Wesp T, Asgekar V, Saldin E, Schneidmiller E & Yurkov M, "Observation of an FEL efficiency increase caused by magnetic field tapering of the undulator," *Nuclear Instruments and Methods in Physics Research*, Vol.A, No. 530, (2004), p. 205-216.

[22] Chung TH, Kim SH & Lee JK, "Simulation of tapered FEL amplifiers in millimetre and infrared regions", *Nuclear Instruments and Methods in Physics Research*, Vol.A, No. 331, (1993), p. 482-486.

[23] J. Gardelle J, Labrouche J & Taillandier PLe," Free electron laser simulations: Effects of beam quality and space charge," *Physical Review*, Vol. 50, No. 6, (1994).

[24] Parker RK, Jackson BH, Gold SH, Freund HP, Granatstein VL, Efthimion PC, Herndon M & Kinkead AK, "Axial Magnetic-Field Effects in a Collective-Interaction Free Electron Laser at Millimetre Wavelengths", *Physical Review Letters*, Vol. 48, No. 4, (1982). https://doi.org/10.1103/PhysRevLett.48.238.

[25] Ted Scharlemann, "Physics of the Free Electron Laser," Report: E&TR, (1986), pp. 11.