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Research paper

Analysis of forced convection of Nanofluid over stretching sheet with suction effect

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Abstract

The present paper analyses boundary layer flow of incompressible nanofluid with MHD along stretching sheet with forced convection. This problem is of great practical utility. Similarity transformation has been used and PDE are converted into ODE. Equations are then solved using HAM (Homotomy Analysis Method). Many parameters are investigated and results are depicted with the help of graphs. Skin friction coefficient and rate of heat transfer are shown with the help of graphs and tables.

Keywords: *Use about five key words or phrases in alphabetical order, Separated by Semicolon*

1. Introduction

The study of forced convection involving boundary layer flow due to stretching sheet has attracted researchers during couple of years due to their practical utility. It has relevant application like electronic devices, extrusion process, emergency shutdown of nuclear reactors and so many. Mostly these fluids are Non-Newtonian in nature. Many fluids like toothpaste, shampoos, paper pulp, and slurries are examples of Non-Newtonian fluids. It is very difficult to incorporate a model which includes various characteristics like viscoelasticity, shear thinning and thickening. Thus many equations are made. Moreover, due to behaviour of Non-Newtonian fluid the equations are non-linear in nature.

The phenomenon of heat transfer occurs by three different modes namely conduction, convection and radiation. Among these mode of heat transfer through convection is widely used in industries. Sakiadis was first one to convert convection term into momentum equation. Erikson extended this work by finding effects of heat and mass transfer, momentum on surface moving with constant speeds. Nanofluids have wide variety of application in industry. Choi [2] coined the term nanofluid. He discovered that by immersing nanosized particles of metals to the base fluid increases drastically thermal conductivity and heat transfer rate. Buongirno [1] introduced model for convective heat transfer in nanofluid. He briefed about transfer of heat by convection in nanofluid. Rahman et al. explained nanofluid over permeable shrinking/stretching sheet. Haroun et al. investigated over effect of unsteady MHD mixed convection due to shrinking/stretching sheet by using spectral relaxation method. Khan et al. [4] analysed heat transfer by sisko nanofluid over nonlinear stretching sheet.

Further by introducing magnetic particles in fluid they conduct electricity. The study of behaviour of these types of fluids and their magnetic properties is called magneto hydrodynamics (MHD). The concept of MHD is that electricity is produced in conductive fluid when subjected to magnetic field. Many engineering applications requires the study of MHD. Magnetic fields are omnipresent in nature. Electrically conducting fluids lead to MHD phenomena. There are wide variety of electrically conducting fluid with different conductivities. MHD is of great importance due to its practical uses in pumps, nuclear reactors, MHD generators, heat exchangers and heat transfers. Combining boundary layer flow over stretching plate with MHD is special type of flow with lot of use in many fields.

Suction in fluid refers to flow of liquid in low pressure or partial vacuum. Suction controls formation of boundary layer. It carries away fluid particles which retards their speed due to boundary layer. Injection on other hand supplements fluid particles with extra energy to prevent their deceleration.

Almost all convection equations which deals with practical industrial problem scientific problem are nonlinear in nature. They rarely have exact solution. Liao [12-13] developed a new technique for solving nonlinear coupled equation related to boundary value problem known as homotopy analysis method (HAM). It is semi analytical method to solve highly non-linear equation. The solution converges in series. The solution obtained by HAM is compared with solution obtained by numerical method. There is good agreement between both results.

The main aim of our study is to find a better coolant. It also indicates that nanofluid is better coolant. This paper has contribution in field of cooling process of many devices. This study can be further extended to hybrid nanofluid which are proved to be much better coolant than single particle nanofluid.

Table 1: Thermophysical Properties of Water and Nanoparticles

Material	ρ(kg/m^3)	Cp(J/KgK)	K(W/mK)	B X 10 -5	σ(S/M)
Pure water	997.1	4179	0.613	21	5.5 X 10 -6
Silver (Ag)	10500	235	429	1.89	6.3 X 10^{7}
Copper(Cu)	8933	385	401	1.67	5.96 X 10^{7}
Iron(Fe)	7870	460	80	58	1.00 X 10^{7}
Aluminum(Al)	2701	902	237	2.31	3.5 X 10^{7}

Copper oxide (CuO)	6510	540	18	0.85	5.96 X 10^7
Alumina	3970	705	40	0.85	3.5 X 10^7
Titanium oxide(TiO_2)	4250	686.2	8.9538	0.9	2.38 X 10^7
Iron oxide (Fe_3O_4)	5180	670	80.4	20.6	1.12 X 10^7

2. Mathematical formulation

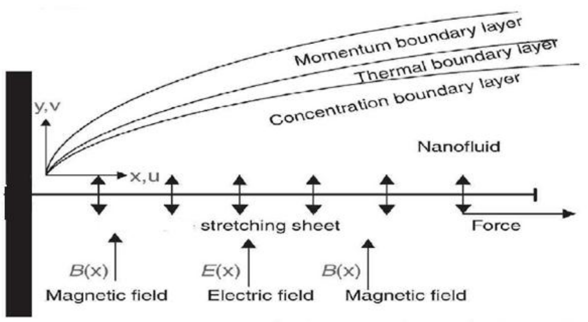

Fig. 1: Nanofluid with Forced Convection along Stretching Sheet.

The 2-D forced convection of nanofluid over stretching sheet with MHD is considered. Based on Buongiorno the equations given for heat transfer convection are given by:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2 u}{\rho_{nf}} \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2}+\frac{\mu_f}{(\rho C)_f}\left(\frac{\partial u}{\partial y}\right)^2 \quad (3)$$

Where last term in equation (2) on right hand side shows the effect of MHD. Similarly the last term of equation (3) on right hand side demonstrates viscous dissipation.
It is assumed that there is no slip on stretching sheet or velocity in direction of stretching sheet is zero. The result of suction / injection is assumed normal to stretching sheet.
Boundary conditions can be written as:

y = 0, u= Uw(x) = ax

$$y\to\infty, u\to 0, T\to T_\infty \quad (4)$$

Using similarity transformation PDE are converted to ODE:
To solve the equations following dimensionless variables are introduced:

$$\psi=u_w x(Re_x)^{-0.5}f(\eta) \quad (5)$$

$$\eta=\frac{y}{x}(Re_x)^{0.5} \quad (6)$$

Where $\boldsymbol{\psi}$(x, y) is stream function and $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$, η is similarity variable and

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \quad (7)$$

Continuity equation is satisfied and equations (2) and (3) along with boundary conditions (4.1.4) are transformed and are written as:

$$f'''+ff''-(f')^2-Mnf'=0 \quad (8)$$

$$\frac{1}{Pr}\theta''+f\theta'=0 \quad (9)$$

With boundary conditions

$$f(0)=\lambda,\ f'(0)=1, \theta(0)=1$$

$$f'(\eta)\to 0, \theta(\eta)\to 0 \ as\ \eta\to\infty \quad (10)$$

Where ‘denotes differentiation w.r.t. η. λ is suction or injection parameter ($\lambda>0$ for suction and $\lambda<0$ for injection). Parameters are defined as

$$\lambda=-\vartheta_w(x)\sqrt{\frac{2x}{\upsilon u_\infty}},\ M_n=\frac{\sigma B_0^2 x}{\rho u_\infty},\ Pr=\frac{\vartheta}{\alpha}$$

Solution by means of HAM:
As per boundary conditions and solution given by Liao

$$f(\eta)=f_0(\eta)+\sum_{i=1}^{\infty}f_i(\eta),$$

$$\theta(\eta)=\theta_0(\eta)+\sum_{i=1}^{\infty}\theta_i(\eta),$$

Where $f_0(\eta)=\lambda+1-e^{-\eta}$, $\theta_0(\eta)=e^{-\eta}$ auxiliary linear operator are given by

$$L_f=f'''-f'$$

$$L_f=\theta''+\theta$$

Which satisfies conditions

$$L_f[c_1+c_2e^{\eta}+c_3e^{-\eta}]=0$$

$$L_\theta[c_4e^{\eta}+c_5e^{-\eta}]=0$$

Where ci (i= 0, 1,2….5) are constants.

Nonlinear operators are defined:

$$N_1[\hat{f}(n;p);\hat{\theta}(n;p)]=\frac{\partial^3\hat{f}(n;p)}{\partial\eta^3}+(1-\phi)^{2.5}\Big\{\Big(\hat{f}(n;p)\frac{\partial^2\hat{f}(n;p)}{\partial\eta^2}-\frac{\partial\hat{f}(\eta;p)}{\partial\eta}^2\Big)\Big(1-\phi+\phi\frac{\rho_s}{\rho_f}\Big)+M^2\frac{\partial\hat{f}(\eta;p)}{\partial\eta}+\lambda\hat{\theta}(\eta;p)\Big(1-\phi+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\cos\alpha\Big)\Big\}=0$$

$$N_2[\hat{f}(n;p);\hat{\theta}(n;p)]=\frac{1}{Pr}\frac{k_{nf}}{k_f}\frac{\partial^2\hat{\theta}(n;p)}{\partial\eta^2}+\Big[(1-\phi)+\phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\Big]\Big(\hat{f}(n;p)\frac{\partial\hat{\theta}(\eta;p)}{\partial\eta}-\frac{\partial\hat{f}(\eta;p)}{\partial\eta}\hat{\theta}(n;p)\Big)+H_s\hat{\theta}(n;p)$$

Where $p\in[0,1]$ is embedding parameter and $\hat{f}(n;p)$ and $\hat{\theta}(n;p)$ are type of mapping functions.
Zero order deformation equations are

$$(1-p)L_f[\hat{f}(n;p)-f_0(\eta)]=phN_1[\hat{f}(n;p);\hat{\theta}(n;p)]$$

$$(1-p)L_\theta[\hat{f}(n;p)-\theta_0(\eta)]=phN_2[\hat{f}(n;p);\hat{\theta}(n;p)]$$

Along with boundary conditions:

$\hat{f}(0;p)$ = A, $\hat{f}'(0;p)$ =0, $\hat{f}'(\infty;p)$ =1

$$\hat{\theta}(0;p)=1, \hat{\theta}(\infty;p)=0$$

Substituting p=0 and p=1 in above equation we obtain

$$\hat{f}(\eta;p)=f_0(\eta), \hat{\theta}(\eta;p)=\theta_0(\eta),$$

$$\hat{f}(\eta;1)=f(\eta), \hat{\theta}(\eta;1)=\theta(\eta)$$

Where;

$$f_m(\eta) = \frac{1}{m!}\frac{\partial^m \hat{f}(\eta;p)}{\partial p^m}$$

And

$$\theta_m(\eta) = \frac{1}{m!}\frac{\partial^m \hat{\theta}(\eta;p)}{\partial p^m} \text{ at p =0.}$$

The mth order deformation equation are:

$$L_f[f_m(\eta,p) - \chi_m f_{m-1}(\eta)] = hR_m^f(\eta)$$

$$L_\theta[\theta_m(\eta,p) - \chi_m \theta_{m-1}(\eta)] = hR_m^\theta(\eta)$$

Where

$$R_m^f = f'''_{m-1} + (1-\phi)^{2.5}\{(\textstyle\sum_{n=0}^{m-1} f_{m-1-n}f''_n - \sum_{n=0}^{m-1} f'_{m-1-n}f'_n)\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right) - M^2 f'_{m-1} + \lambda\theta'_{m-1}(1-\phi) + \phi\frac{(\rho\beta)_s}{(\rho\beta)_f}\cos\alpha)\}$$

$$R_m^\theta = \left(\frac{1}{Pr_{nf}}\frac{k_{nf}}{k_f}\right)\theta''_{m-1} + \left(1-\phi+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right)(f_{i-1}\theta'_{i-1} - f'_{i-1}\theta_{i-1})$$

Where

$$\chi_m = \begin{cases} 0, m \le 1 \\ 1, m > 1 \end{cases}$$

The general solution is given by:

$$f_m(\eta) = f_m^*(\eta) + c_1^m + c_2^m\eta + c_3^m e^{-\eta}$$

$$\theta_m(\eta) = \theta_m^*(\eta) + c_4 e^{\eta} + c_5 e^{-\eta}$$

3. Results and discussions

Nanofluid (Silver/Water) with forced convection and MHD along stretching sheet is analysed. Effect of suction is also taken under consideration. HAM method was used. Equations were also solves by MATLAB. Results obtained shows clear resemblance with each other. Prandtl number was assumed to be 6.2 with volume fraction of nanoparticle as 0.1. Graphs are plotted and some inferences which can be drawn are as follows:

Figure 2 shows dimensionless velocity profile with φ = 0, 0.1, 0.2. It is observed that increase in volume fraction leads to increase in velocity of fluid.

Figure 3 denotes effect of change of Prandtl number to temperature profile. It is noticed that increase in Prandtl number reduces temperature.

Figure 4 shows effect of change of volume concentration of nanoparticles on dimensionless temperature. It was found that increase in volume fraction increases the temperature.

Figure 5 compares results obtained for velocity profile by Homotomy method with numerical method. Results tally with each other showing negligible difference in values of dimensionless velocity.

Figure 6 shows good agreement between the results obtained by semi analytical as well as numerical method. Temperature profile is almost same by both methods.

Figure 7 proves that velocity in boundary layer decreases with increase in magnetic field.

Figure 8 shows that temperature rises with rise in magnetic field. This is due to production of Lorentz force produced by MHD.

Figure 9 and Figure 10 investigates velocity profile and temperature profile for different values of injection/suction. As fluid is inserted perpendicular to the stretching plate, it tends to minimize thickness of boundary layer. Due to effect of suction the boundary layer is close to the sheet. When comparison is made between no injection (λ = 0), the momentum boundary layer are thinner and thicker for suction and injection respectively. (λ = 2, - 2).

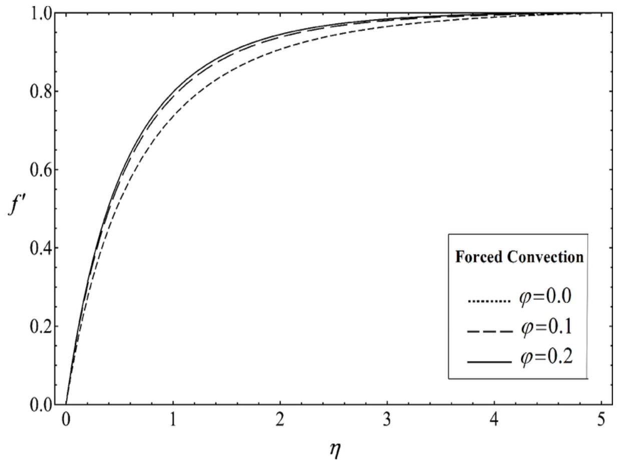

Fig. 2: Dimensionless Velocity Profile with Different Volume Concentration of Nano fluid.

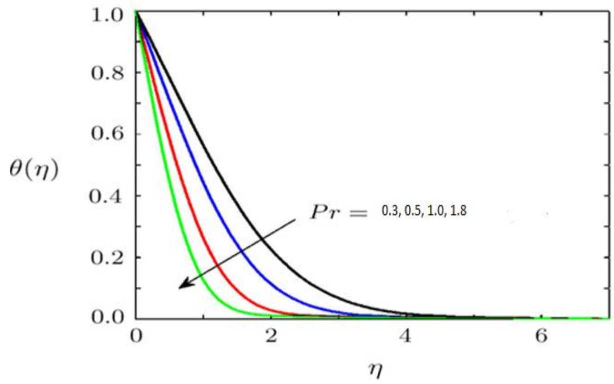

Fig. 3: Effect of Change in Prandtl Number on Dimensionless Temperature.

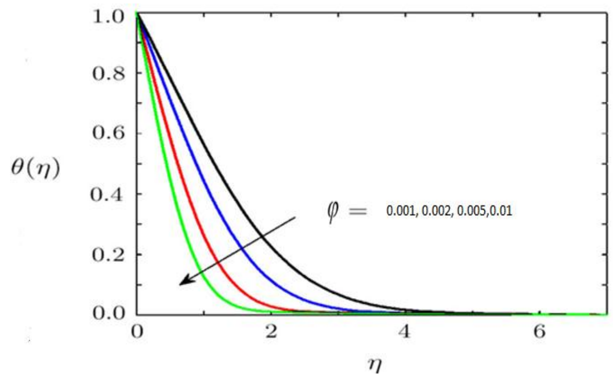

Fig. 4: Effect of Change in Volume Fraction of Nanoparticle on Dimensionless Temperature.

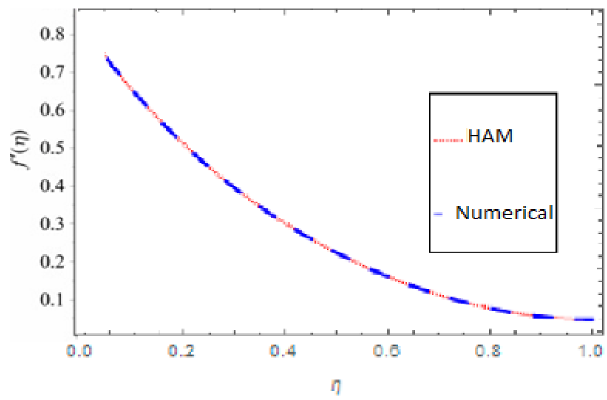

Fig. 5: Comparison of Velocity Profile by HAM and Numerical Method.

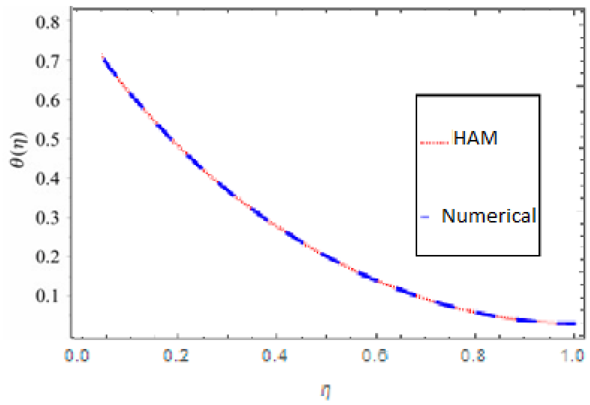

Fig. 6: Comparison of Temperature Profile by HAM and Numerical Method.

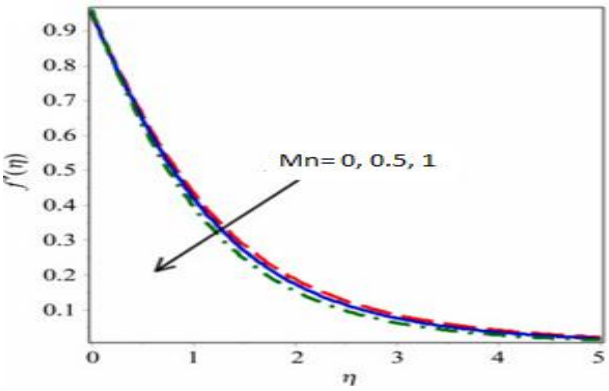

Fig. 7: Velocity Profile for Different Values of Magnetic Field.

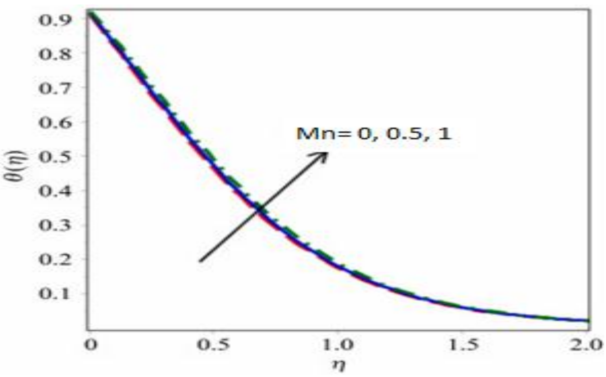

Fig. 8: Temperature Profile for Different Values of Magnetic Field.

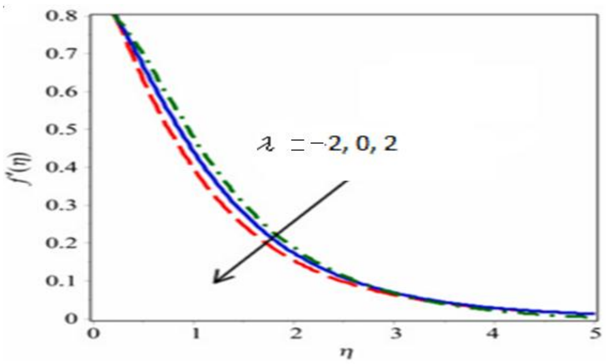

Fig. 9: Velocity Profile for Different Values of Suction Parameter.

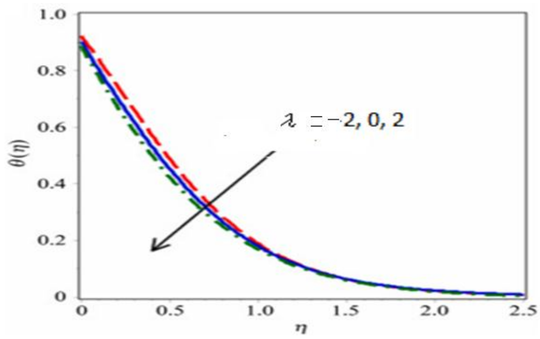

Fig. 10: Temperature Profile for Different Values of Suction Parameter.

4. Conclusion

From the above analysis following conclusions may be drawn:

- Nanofluids have better thermal conductivity and hence they have better cooling property than their base fluid.
- The increment in value of Prandtl number leads to decrease in temperature for both silver water a nanofluids.
- Increase in volume fraction results in increase of temperature.
- Changing volume fraction also changes rate of heat transfer for nanofluids. Further, increase in volume fraction causes increase in heat transfer coefficient for both types of Nano fluids.

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