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Research Paper



Pendulum state estimation using nonlinear state estimators

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Abstract

The convergence over Non-linear state estimation is not satisfied by Kalman filter. For nonlinear state estimation problems, the present research work on the performance analysis of linearized kalman filter and extended kalman filter for a simple nonlinear state estimation problem. The simple pendulum is the best example for simple nonlinear state dynamics. The performance analysis based on the root mean square errors of the estimates also specified through Monte-Carlo simulation.

Keywords: *Extended Kalman Filter; Linearized Kalman Filter; Non-Linear; Simple Pendulum; State estimation*

1. Introduction

A simple pendulum is a physical object identifying the phenomenon, simple harmonic motion. A suspended metal massy object (bob) with a string attached to a rigid support is the simple pendulum schematic shown in fig 1[1,9]. The path of bob having three major points, equilibrium position and two maximum positions as back and forth. The moment of bob is less than 15degrees.Then that is the best example for simple harmonic motion where the angular displacement equals to its acceleration. If bob moves away from its equilibrium more than 15degrees then the acceleration becomes the sinusoidal function of the displacement angle. The measurement of angular position of bob is linear and following a Gaussian model. The measurement along the horizontal position of the bob leads to a nonlinear model. The simple one-dimensional nonlinear state space model considered for this research[2].

The Kalman[3] filter proposed for linear time-varying systems having noise characteristic follows Gaussian distribution. But for nonlinear model like a simple pendulum, the Kalman filter is no more sufficient. The linearized Kalman[3] filter (LKF) is good but the linearization of nonlinearities in trajectory is along the nominal trajectory, which is unknown. Kalman and Bucy proposed Extended Kalman Filter[4,5] also known as Kalman-Bucy filter uses Taylor series expansion to linearize the nonlinearities in the dynamic system which is suitable for nonlinear systems.

2. The Linearization

Extended Kalman Filter discretizes the linearized models of nonlinear systems through Taylor series expansion using Gaussian approximations [6,7,8]. Now the general Kalman filter is applied to those linearized results. Now the expression for nonlinear continuous time dynamic system and measurements is given by

$$y = h(m, v, t) \tag{1}$$
$$w(t) \sim (0, Q)$$
$$v \sim (0, R)$$

Here m is a state vector, u is a control vector, w is the state dynamic noise which follows Gaussian distribution with zero mean and covariance Q, y is the measurement defined through state vector and measurement noise which is zero mean white Gaussian noise with covariance R[6]. The linearization and discretization are as follows,

$$m_k = f_{k-1}(m_{k-1}, u_{k-1}, w_{k-1})$$
$$y_k = h_k(x_k, v_k) \tag{2}$$
$$w_k \sim (0, Q_k)$$
$$v_k \sim (0, R_k)$$

The suffix k represents the discrete form of equations. The initialization of filter is given by the priori estimates and priori estimate error covariance as follows

$$mm = E(m) \tag{3}$$
$$PP = E[(m - mm)(m - mm)']$$

The Linearized Kalman Filter linearizes along the nominal trajectory estimate given by, the nominal trajectory taken such that the angular displacement is zero that is the equilibrium position.

$$F = \frac{\partial f}{\partial m_{nominal}} \tag{4}$$
$$H = \frac{\partial h}{\partial m_{nominal}}$$

For k=1,2,…; Extended Kalman filter needs Jacobians of state transition and measurement model matrices, such that the state model and measurement model matrices must be differentiable, that is given by

$$F = \frac{\partial f}{\partial m} \tag{5}$$

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$$H = \frac{\partial h}{\partial m}$$

Time update of the state estimate and estimation error covariance is

$$m = f \tag{6}$$
$$P = FPF' + Q$$
$$S = HPH' + R$$

The measurement update of the state estimation and estimated error covariance is given by

$$K = PH'/S$$
$$m = m + K(y(k) - h) \tag{7}$$
$$P = P - KSK'$$

3. Mathematical Modeling

Acceleration of a simple pendulum with a unit mass and suspended by a unit length (L=1) string is defined through a continuous time nonlinear differential equation which is given by [1,7]

$$\frac{d^2\theta}{dt^2} = -g\sin(\theta) + w(t) \tag{8}$$

Where θ is the angle, g is the gravitational acceleration and w(t) is a random noise process. The angle, velocity and acceleration of metal bob are defined through a state space model as follows

$$\frac{d}{dt}(m_1; m_2) = (m_2; -g\sin(m_1)) + (0; 1)w(t) \tag{9}$$

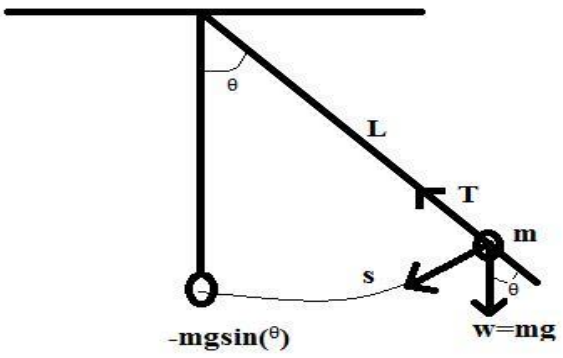


Fig 1. A simple pendulum

Where $m_1 = \theta$ and $m_2 = \frac{d\theta}{dt}$.The continuous time nonlinear dynamic model is simplified as follows

$$\frac{dm}{dt} = f(m) + w \tag{10}$$

The linear Gaussian measurement model of the pendulum which is measured along the angular position of the pendulum is

$$y_k = m(t_k) + noise \tag{11}$$

The measurement along the horizontal position of the pendulum leads to a nonlinear measurement model, given by

$$y_k = \sin(\theta(t_k)) + noise \tag{12}$$

Now, the continuous nonlinear state and discrete nonlinear measurements are written in the form of discretized nonlinear state space models as

$$m_k = f(m_{k-1}, q_{k-1}) \tag{13}$$

$$y_k = h(x_k, r_k)$$

Where y_k is the measurement vector, $q_{k-1} \sim N(0, Q)$ and $r_k \sim$ N(0,R). The pendulum model is simply discretized as follows

$$(m_{1,k}; m_{2,k}) = (m_{1,k-1} + m_{2,k-1} * DT; m_{2,k-1} - g * \sin(m_{1,k-1}) * DT) + q_{k-1} y_k = \sin(m_{1,k}) + r_k \tag{14}$$

Where

$$m_k = (m_{1,k}; m_{2,k})$$
$$f(m_{k-1}) = (m_{1,k-1} + m_{2,k-1} * DT; m_{2,k-1} - g * \sin(m_{1,k-1}) * DT) h(x_k) = \sin(m_{1,k}) \tag{15}$$

Jacobian matrices of f and h for first order EKF are as follows;

$q_{k-1} \sim N(0, Q)$ and $r_k \sim$ N(0,R)

$$Q = q^c \left(\frac{DT^3}{3} \frac{DT^2}{2}; \frac{DT^2}{2} DT\right) \tag{16}$$

q^c is the spectral density of continuous time process noise. Now, substitute the above equations in extended Kalman filter equations 4,5 and 6.

4. Simulation and Results

The simple pendulum dynamics simulated, the root mean square error (rmse) value calculated for the measurements obtained through simulation. Now the linearized kalman filter and extended kalman filter applied to the simulated data and then the rmse with respect to both the filters has taken. The same simulation is done for 100 random noise sequences in Monte-Carlo simulation and the average rmse is compared.

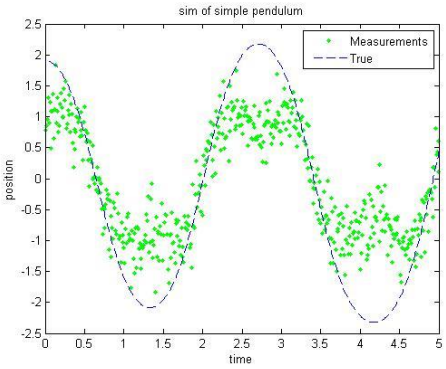


Fig 2. Simulation of a simple pendulum,

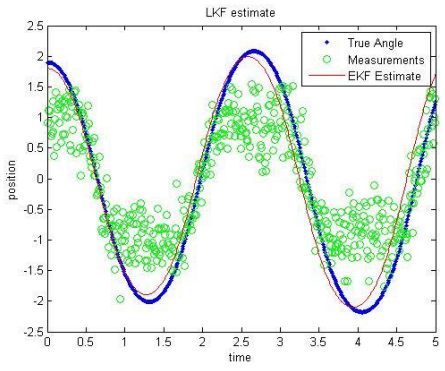


Fig 3. Linearized Kalman Filter Estimate.

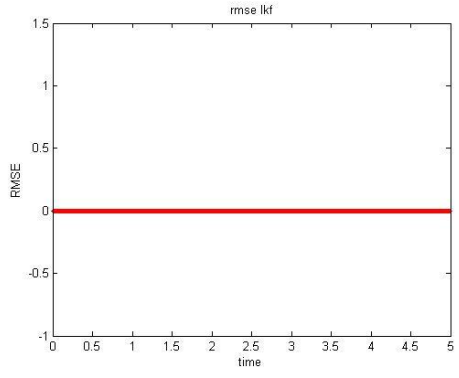


Fig 4. RMS Error of LKF estimate.

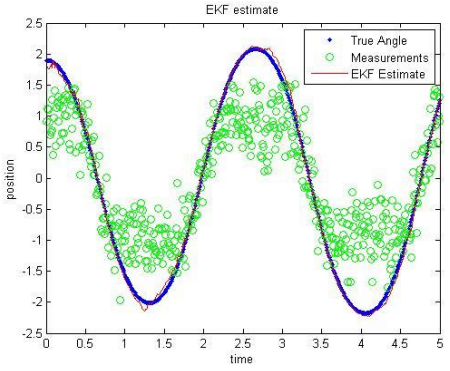


Fig 5. EKF Estimate.

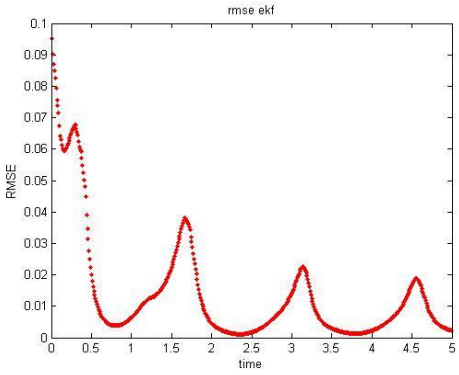


Fig 6. RMSE of EKF estimate

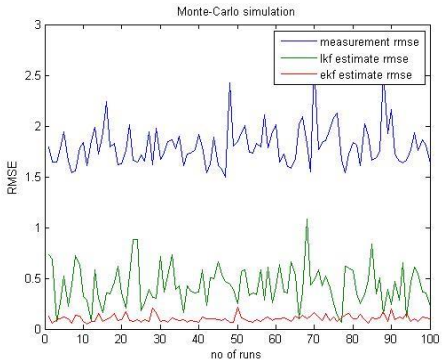


Fig 7.A Monte Carlo simulation for RMSE of measurement, LKF estimate and EKF estimate

Table 1. Comparison result of non-linear estimators.

	Simulated Rmse	LKF estimate Rmse	EKF estimate Rmse
For random noise sequence	1.7836	0.6370	0.1300
Avg of 100 Monte-Carlo runs	1.807254	0.430148	0.10301

5. Conclusion

The result shows that the extended kalman filter shows better results compared to linearized kalman filter because the actual trajectory is more differ from nominal trajectory. The future work comprised of applying other estimation algorithms and analysing the performance. The enhancement of applications using these algorithms may be analyzed.

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