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Measuring the Effectiveness of a Complex Repairable Series-Parallel System involving Four Types of Failures

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Abstract

Many authors have developed models on two units' cold standby system, but little or no attention is paid on series-parallel system involving four types of failures. In this study, measures of system effectiveness such as mean time to system failure (MTSF), steady state availability, busy period and profit function were discussed. The system is analyzed using Kolmogorov's forward equations method. The result has shown that MTSF, steady state availability and profit function increases with repair and decreases with failure rate.

Keywords: *MTSF, availability, effectiveness, profit function.*

1 Introduction

Due to the importance of series-parallel systems in various industries, determination of their availability has become an increasingly important issue. System availability represents the percentage of time the system is available to users. Failure is an unavoidable phenomenon which can be dangerous and costly and bring about less production and profit. Proper maintenance planning plays a role in achieving high system reliability, availability and production output. It is therefore important to keep the equipments/systems always available and to lay emphasis on system availability at the highest order.

A large volume of literature exists on the issue of predicting performance evaluation of various systems. Kumar et al [1] discussed the reliability analysis of the Feeding system in the paper industry, Kumar el al.[2] discussed the availability analysis of the washing system in the paper industry, Kumar el al. [3] deal with reliability, availability and operational behavior analysis for different systems in paper plant. Kumar el al. [4] discussed the behavior analysis of Urea decomposition in the fertilizer industry under the general repair policy. Kumar et al.[5] studied the design and cost analysis of a refining system in a Sugar industry. Srinath [6] has explained a Markov model to determine the availability expression for a simple system consisting of only one component. Gupta el al. [7] has evaluated the reliability parameters of butter manufacturing system in a diary plant considering exponentially distributed failure rates of various components. Gupta et al. [8] studied the behavior of Cement manufacturing plant. Arora and Kumar [9] studied the availability analysis of the cool handling system in paper plant by dividing it into three subsystems. Singh and Garg [10] perform the availability analysis of the core veneer manufacturing system in a plywood manufacturing system under the assumption of constant failure and repair rates.

In the present paper, we study a series-parallel system consisting of five different subsystems arranged in series. Through the transition diagram obtained in this study, systems of differential equations are developed and solved recursively via probabilistic approach. Availability matrices for each subsystem have been developed to provide various performance values for different combinations of failure and repair rates of all subsystems. Performance of each subsystem of series-parallel system is evaluated.

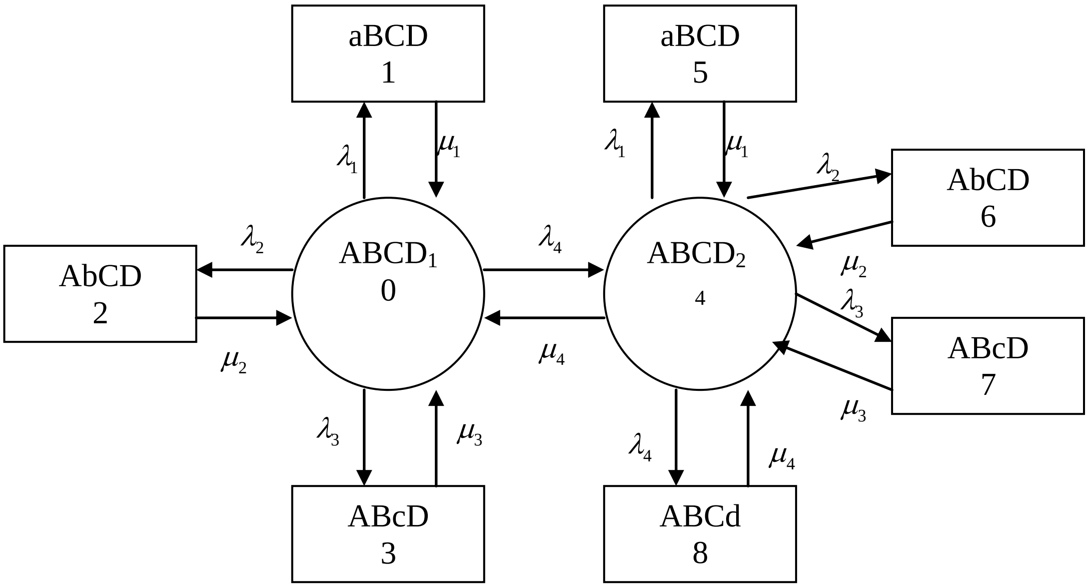

Fig. 1: schematic diagram of the system

2 Mean Time to System Failure for System

Let $P(t)$ be the probability row vector at time t, then the initial conditions for this problem are as follows:

$$
\begin{aligned}
&P(0)=\left[P_0(0),P_1(0),P_2(0),P_3(0),P_4(0),P_5(0),P_6(0),P_7(0),P_8(0)\right]=\left[1,0,0,0,0,0,0,0,0\right]\\
&P_0'(t)=-(\lambda_1+\lambda_2+\lambda_3+\lambda_4)P_0(t)+\mu_1P_1(t)+\mu_2P_2(t)+\mu_3P_3(t)+\mu_4P_4(t)\\
&P_1'(t)=-\mu_1P_1(t)+\lambda_1P_0(t)\\
&P_2'(t)=-\mu_2P_2(t)+\lambda_2P_0(t)\\
&P_3'(t)=-\mu_3P_3(t)+\lambda_3P_0(t)\\
&P_4'(t)=-(\mu_4+\lambda_1+\lambda_2+\lambda_3+\lambda_4)P_4(t)+\lambda_4P_0(t)+\mu_1P_5(t)+\mu_2P_6(t)+\mu_3P_7(t)+\mu_4P_8(t)\\
&P_5'(t)=-\mu_1P_5(t)+\lambda_1P_4(t)\\
&P_6'(t)=-\mu_2P_6(t)+\lambda_2P_4(t)\\
&P_7'(t)=-\mu_3P_3(t)+\lambda_3P_4(t)\\
&P_8'(t)=-\mu_4P_8(t)+\lambda_4P_4(t)
\end{aligned}
\tag{1}
$$

$$
A=\begin{bmatrix}
-(\lambda_1+\lambda_2+\lambda_3+\lambda_4) & \mu_1 & \mu_2 & \mu_3 & \mu_4 & 0 & 0 & 0 & 0\\
\lambda_1 & -\mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
\lambda_2 & 0 & -\mu_2 & 0 & 0 & 0 & 0 & 0 & 0\\
\lambda_3 & 0 & 0 & -\mu_3 & 0 & 0 & 0 & 0 & 0\\
\lambda_4 & 0 & 0 & 0 & -(\mu_4+\lambda_1+\lambda_2+\lambda_3+\lambda_4) & \mu_1 & \mu_2 & \mu_3 & \mu_4\\
0 & 0 & 0 & 0 & \lambda_1 & -\mu_1 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & \lambda_2 & 0 & -\mu_2 & 0 & 0\\
0 & 0 & 0 & 0 & \lambda_3 & 0 & 0 & -\mu_3 & 0\\
0 & 0 & 0 & 0 & \lambda_4 & 0 & 0 & & -\mu_4
\end{bmatrix}
$$

It is difficult to evaluate the transient solutions hence following El-said[14], and Haggag [15] we delete the rows and columns of absorbing state of matrix A and take the transpose to produce a new matrix, say Q.

The expected time to reach an absorbing state is obtained from

$$
E\left[T_{P(0)\to P(absorbing)}\right]=P(0)(-Q^{-1})\begin{pmatrix}1\\1\\1\end{pmatrix}
$$

$$
MTSF=P(0)(-Q^{-1})\begin{pmatrix}1\\1\\1\end{pmatrix}
\tag{2}
$$

where

$$Q=\begin{bmatrix} -(\lambda_1+\lambda_2+\lambda_3+\lambda_4) & \lambda_4 \\ \mu_4 & -(\mu_4+\lambda_1+\lambda_2+\lambda_3+\lambda_4) \end{bmatrix}$$

$$MTSF=\frac{\mu_4+\lambda_1+\lambda_3+2\lambda_4}{\mu_4\lambda_1+\lambda_1^2+2\lambda_1\lambda_2+2\lambda_1\lambda_3+2\lambda_1\lambda_4+\mu_4\lambda_2+\lambda_2^2+2\lambda_2\lambda_3+2\lambda_2\lambda_4+\mu_4\lambda_3+\lambda_3^2+2\lambda_3\lambda_4+\lambda_4^2}$$

3 Steady state availability Analysis for System

For the availability case of Fig. 1 following El-said and El-Hamid (2006), the initial conditions for this system are:
$P(0)=[P_1(0),P_2(0),P_3(0),P_4(0),P_5(0),P_6(0),P_7(0),P_8(0)]=[1,0,0,0,0,0,0,0,0]$
The system of differential equations in for System 1 above can be expressed as:

$$\begin{bmatrix} \dot{P}_0 \\ \dot{P}_1 \\ \dot{P}_2 \\ \dot{P}_3 \\ \dot{P}_4 \\ \dot{P}_5 \\ \dot{P}_6 \\ \dot{P}_7 \\ \dot{P}_8 \end{bmatrix}=\begin{bmatrix} -(\lambda_1+\lambda_2+\lambda_3+\lambda_4) & \mu_1 & \mu_2 & \mu_3 & \mu_4 & 0 & 0 & 0 & 0 \\ \lambda_1 & -\mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_2 & 0 & -\mu_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_3 & 0 & 0 & -\mu_3 & 0 & 0 & 0 & 0 & 0 \\ \lambda_4 & 0 & 0 & 0 & -(\mu_4+\lambda_1+\lambda_2+\lambda_3+\lambda_4) & \mu_1 & \mu_2 & \mu_3 & \mu_4 \\ 0 & 0 & 0 & 0 & \lambda_1 & -\mu_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_2 & 0 & -\mu_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_3 & 0 & 0 & -\mu_3 & 0 \\ 0 & 0 & 0 & 0 & \lambda_4 & 0 & 0 & & -\mu_4 \end{bmatrix}\begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{bmatrix}$$

In the steady state the derivatives of state probabilities become zero which enable us to compute steady state probabilities. Thus, the availability for System is

$$AV=P_0(\infty)+P_4(\infty) \tag{3}$$

and

$$AP(\infty)=0 \tag{4}$$

which in matrix form as

$$\begin{bmatrix} -(\lambda_1+\lambda_2+\lambda_3+\lambda_4) & \mu_1 & \mu_2 & \mu_3 & \mu_4 & 0 & 0 & 0 & 0 \\ \lambda_1 & -\mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_2 & 0 & -\mu_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_3 & 0 & 0 & -\mu_3 & 0 & 0 & 0 & 0 & 0 \\ \lambda_4 & 0 & 0 & 0 & -(\mu_4+\lambda_1+\lambda_2+\lambda_3+\lambda_4) & \mu_1 & \mu_2 & \mu_3 & \mu_4 \\ 0 & 0 & 0 & 0 & \lambda_1 & -\mu_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_2 & 0 & -\mu_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_3 & 0 & 0 & -\mu_3 & 0 \\ 0 & 0 & 0 & 0 & \lambda_4 & 0 & 0 & & -\mu_4 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

using the normalizing condition

$$P_0(\infty)+P_1(\infty)+P_2(\infty)+P_3(\infty)+P_4(\infty)+P_5(\infty)+P_6(\infty)+P_7(\infty)+P_8(\infty)=1 \qquad (5)$$

we substitute (5) in one of the redundant rows of (4). The resulting matrix is

$$\begin{bmatrix} -(\lambda_1+\lambda_2+\lambda_3+\lambda_4) & \mu_1 & \mu_2 & \mu_3 & \mu_4 & 0 & 0 & 0 & 0 \\ \lambda_1 & -\mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_2 & 0 & -\mu_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_3 & 0 & 0 & -\mu_3 & 0 & 0 & 0 & 0 & 0 \\ \lambda_4 & 0 & 0 & 0 & -(\mu_4+\lambda_1+\lambda_2+\lambda_3+\lambda_4) & \mu_1 & \mu_2 & \mu_3 & \mu_4 \\ 0 & 0 & 0 & 0 & \lambda_1 & -\mu_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_2 & 0 & -\mu_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_3 & 0 & 0 & -\mu_3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

We solve the system of linear equations in matrix above to obtain the state probabilities $P_0(\infty), P_4(\infty)$

Expression for AV thus is

$$AV = \frac{N_1}{D_1}$$

$$N_1 = \mu_1\mu_2\mu_3\mu_4^2 + \mu_1\mu_2\mu_3\mu_4\lambda_4$$

$$D_1 = \mu_1\mu_2\mu_3\lambda_3\lambda_4 + \lambda_3\mu_1\mu_2\mu_4^2 + \lambda_4^2\mu_1\mu_2\mu_3 + \mu_1\mu_2\mu_3\mu_4\lambda_4 + \mu_1\mu_3\mu_4\lambda_2\lambda_4 \\ +\mu_1\mu_2\mu_3\mu_4^2 + \mu_1\mu_3\mu_4^2\lambda_2 + \mu_2\mu_3\mu_4\lambda_1\lambda_4 + \mu_2\mu_3\mu_4^2\lambda_1$$

4 Busy Period Analysis for System

Using the same initial condition in System1 above as for the reliability case:

$$P(0) = [P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0), P_7(0), P_8(0)] = [1,0,0,0,0,0,0,0,0]$$

Using the system of differential equations above, in the steady state, the derivatives of the state probabilities become zero.
The system of differential equations in for System 1 above can be expressed as:

$$\begin{bmatrix} \overset{\bullet}{P}_0 \\ \overset{\bullet}{P}_1 \\ \overset{\bullet}{P}_2 \\ \overset{\bullet}{P}_3 \\ \overset{\bullet}{P}_4 \\ \overset{\bullet}{P}_5 \\ \overset{\bullet}{P}_6 \\ \overset{\bullet}{P}_7 \\ \overset{\bullet}{P}_8 \end{bmatrix} = \begin{bmatrix} -(\lambda_1+\lambda_2+\lambda_3+\lambda_4) & \mu_1 & \mu_2 & \mu_3 & \mu_4 & 0 & 0 & 0 & 0 \\ \lambda_1 & -\mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_2 & 0 & -\mu_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_3 & 0 & 0 & -\mu_3 & 0 & 0 & 0 & 0 & 0 \\ \lambda_4 & 0 & 0 & 0 & -(\mu_4+\lambda_1+\lambda_2+\lambda_3+\lambda_4) & \mu_1 & \mu_2 & \mu_3 & \mu_4 \\ 0 & 0 & 0 & 0 & \lambda_1 & -\mu_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_2 & 0 & -\mu_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_3 & 0 & 0 & -\mu_3 & 0 \\ 0 & 0 & 0 & 0 & \lambda_4 & 0 & 0 & & -\mu_4 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{bmatrix}$$

and

$$AP(\infty) = 0$$

Which in matrix form as

$$\begin{bmatrix} -(\lambda_1+\lambda_2+\lambda_3+\lambda_4) & \mu_1 & \mu_2 & \mu_3 & \mu_4 & 0 & 0 & 0 & 0 \\ \lambda_1 & -\mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_2 & 0 & -\mu_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_3 & 0 & 0 & -\mu_3 & 0 & 0 & 0 & 0 & 0 \\ \lambda_4 & 0 & 0 & 0 & -(\mu_4+\lambda_1+\lambda_2+\lambda_3+\lambda_4) & \mu_1 & \mu_2 & \mu_3 & \mu_4 \\ 0 & 0 & 0 & 0 & \lambda_1 & -\mu_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_2 & 0 & -\mu_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_3 & 0 & 0 & -\mu_3 & 0 \\ 0 & 0 & 0 & 0 & \lambda_4 & 0 & 0 & & -\mu_4 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Using the normalizing condition
$P_0(\infty)+P_1(\infty)+P_2(\infty)+P_3(\infty)+P_4(\infty)+P_5(\infty)+P_6(\infty)+P_7(\infty)+P_8(\infty)=1$
We substitute () in one of the redundant rows of (). The resulting matrix is

$$\begin{bmatrix} -(\lambda_1+\lambda_2+\lambda_3+\lambda_4) & \mu_1 & \mu_2 & \mu_3 & \mu_4 & 0 & 0 & 0 & 0 \\ \lambda_1 & -\mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_2 & 0 & -\mu_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_3 & 0 & 0 & -\mu_3 & 0 & 0 & 0 & 0 & 0 \\ \lambda_4 & 0 & 0 & 0 & -(\mu_4+\lambda_1+\lambda_2+\lambda_3+\lambda_4) & \mu_1 & \mu_2 & \mu_3 & \mu_4 \\ 0 & 0 & 0 & 0 & \lambda_1 & -\mu_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_2 & 0 & -\mu_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda_3 & 0 & 0 & -\mu_3 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0(\infty) \\ P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \\ P_5(\infty) \\ P_6(\infty) \\ P_7(\infty) \\ P_8(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

We solve the system of linear equations in matrix above to obtain the state probabilities $P_0(\infty), P_4(\infty)$
and this will enable us to compute steady state busy :

$$B(\infty) = 1 - (P_0(\infty) + P_4(\infty)) = \frac{N_2}{D_1} \tag{6}$$

$$N_2 = \mu_1\mu_2\mu_3\lambda_3\lambda_4 + \lambda_3\mu_1\mu_2\mu_4^2 + \lambda_4^2\mu_1\mu_2\mu_3 + \mu_1\mu_2\mu_3\mu_4\lambda_4 + \mu_1\mu_3\mu_4\lambda_2\lambda_4 \\ + \mu_1\mu_3\mu_4^2\lambda_2 + \mu_2\mu_3\mu_4\lambda_1\lambda_4 + \mu_2\mu_3\mu_4^2\lambda_1 + \mu_2\mu_3\mu_4^2\lambda_1$$

5 Profit Analysis

Following El-said (2008) and Haggag (2009) the expected profit per unit time incurred to the system in the steady-state is given by:
Profit=total revenue generated – cost incurred for repairing the failed units.

$$PF = C_1 AV(\infty) - C_2 B(\infty) \tag{7}$$

Where PF : is the profit incurred to the system

C_1 : is the revenue per unit up time of the system

C_2 : is the cost per unit time which the system is under repair

6 Results

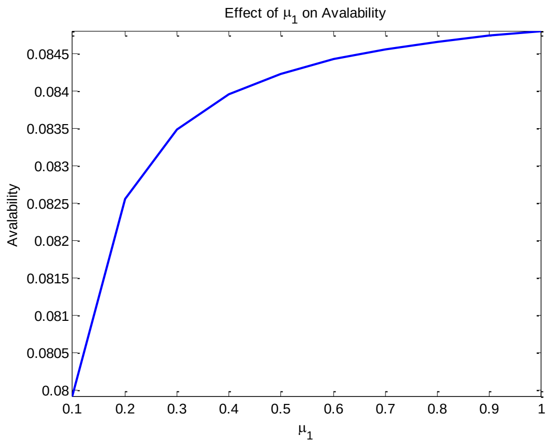

Fig. 2: plot of availability vs μ_1

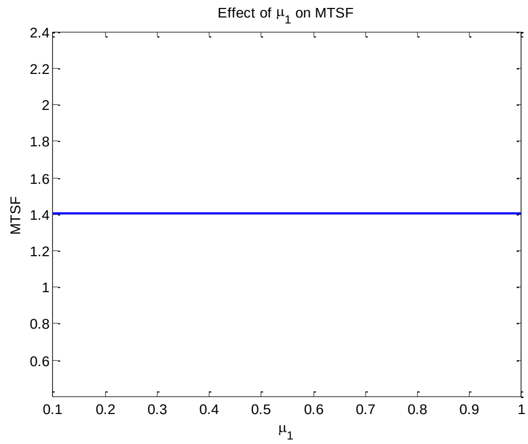

Fig. 3: plot of MTSF versus μ_1

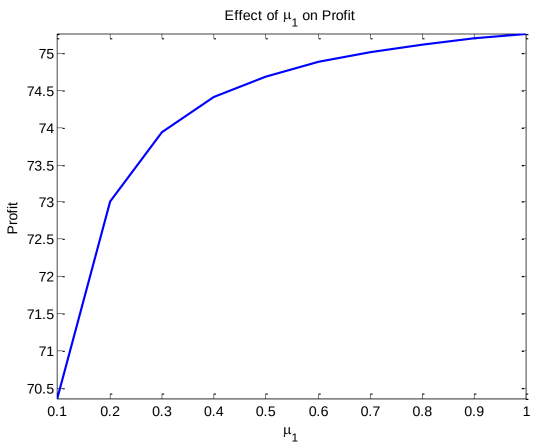

Fig. 4: plot of profit versus μ_1

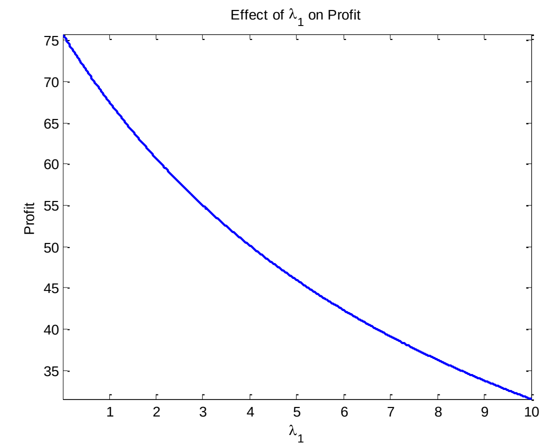

Fig. 5: Plot of profit versus λ_1

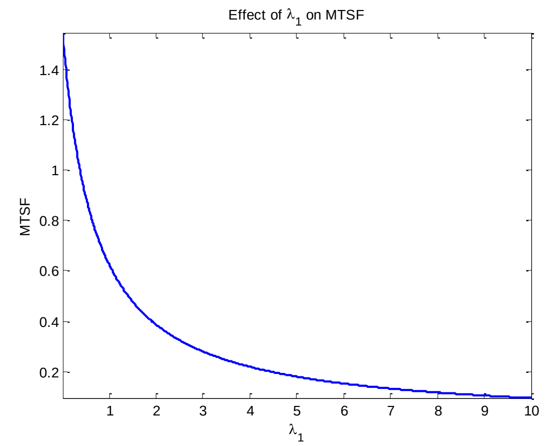

Fig. 6: Plot of MTSF versus λ_1

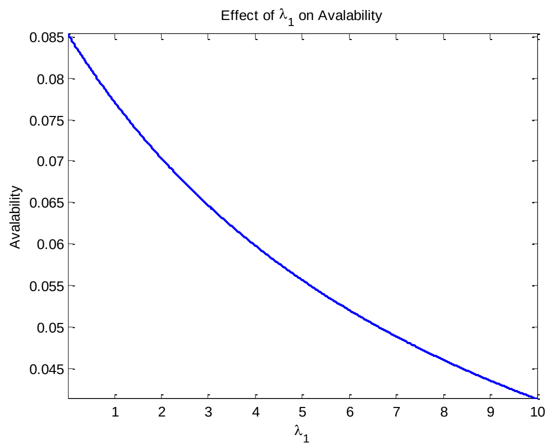

Fig. 7: plot of availability versus λ_1

From Fig. 2 to 7 above, it is clear profit, system availability and MTSF decreases as λ_1 increase, while profit and system availability increases with increase in the value of μ_1 and constant with MTSF as can be observe from Fig. 3.

7 Conclusion

In this paper, we developed explicit expressions for measures of system effectiveness such as MTSF, system availability and profit function. Graphs were plotted to highlight important results. Results have shown that measures of system effectiveness such MTSF, system availability and profit increases with repair rates and decreases with failure rates.

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