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Stochastic Modeling of A Non Maintained System With Two Stages of Deterioration

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Abstract

This paper presents the reliability and mean time to failure estimation of a deteriorating system that deteriorates with time. The system is two unit active parallel system where the units operates simultaneously. In this study maintenance is not allowed to the system. Laplace transforms is being utilize to solve the mathematical model developed. Expressions for reliability and Mean time to failure (MTTF) have been computed. The objective is to see the effect of deterioration rates on system reliability and MTTF of a non maintained system. It also the objective of this study to see the effect of time on system reliability. Graphical study of the system through reliability and MTTF is obtained to highlight important results. The results have shown that reliability and MTTF of non maintained system decrease as time and deterioration rates increase. Models developed in this paper are important to engineers, maintenance managers, and plant management for proper maintenance decision and for safety of the system as a whole.

Keywords: *Reliability, MTTF, Laplace transformation, deterioration*

1 Introduction

Systems are subject to deterioration meaning that in course of time their condition falls from higher to lower, and possibly even to unacceptable, levels. Obviously, the condition level and the performance of the system are related. Generally the lower the condition level the worse the performance of the system. Maintenance actions such as inspection, local repair and replacement should be done to retain the system in or restore it to an acceptable operating condition.

Many technical systems are subjected to aging and degradation during their lifetime. Most of these systems are repairable. Deterioration has a great influence not only on inventory management, but on every area of production where items are stocked or forced to wait due to technical matters, variability or disruptions / stochastic influences in the production process. For a deteriorating system, it is reasonable to assume that the successive operating times of the system after repairs are stochastically decreasing and the consecutive repair times of the system after failures are stochastically increasing.

Because deterioration is uncertain over time, it should ideally be represented as a stochastic process. Most engineering assets experience some kind of deterioration before failing. Asset failures can be predicted based on these deterioration processes which are revealed by various deterioration indicators.

Studies on stochastic analysis on systems can be found in Ahmad and Singh [1], Ahmad et al [2], Gupta et al [3], kishan and Kumar [4], Mokaddis et al [5], Ritu et al [6], Singh and Singh [7], Sridharan and Kalyan [8],Mathew et al [9] studies the reliability of continuous casting plant system operating with full installed capacity using semi Markov processes and regenerative point techniques. While research on maintenance and replacement of deterioration system can be found in Chen and Wu [10], Pierskalla and Voelker [11], Valderz and Feldman [12], Wang [13] and Abdel-Hameed [14].

This paper studied reliability and mean time to failure (MTTF) estimation of two unit active parallel system and the effect of the time and deterioration rates on reliability and the effect of deterioration rates on MTTF using Laplace transformation.

2 Model Formulation

2.1 Assumption

1. State of the system can be: Perfect (S_1), Minor deterioration (S_2), Major deterioration (S_3), Failure (S_4)
2. At any given time t the system is either in the operating state, deteriorating state or in the failed state.
3. The units operate simultaneously
4. State S_4 can be access from the previous state
5. The state of the system changes as time progresses
6. System/units work in S_1, S_2, S_3 and fail in S_4
7. the deteriorate stages can be minor or major

2.2 Notations and Nomenclature

$\lambda_{12}, \lambda_{13}, \lambda_{23}$: Deterioration rate

λ_{14}: Failure rate of the system while in S_1

λ_{24}: Failure rate of the system while in S_2

λ_{34}: Failure rate of the system while in S_3

$R(t)$: Reliability of the system at time t

MTTF: Mean time to failure of the system

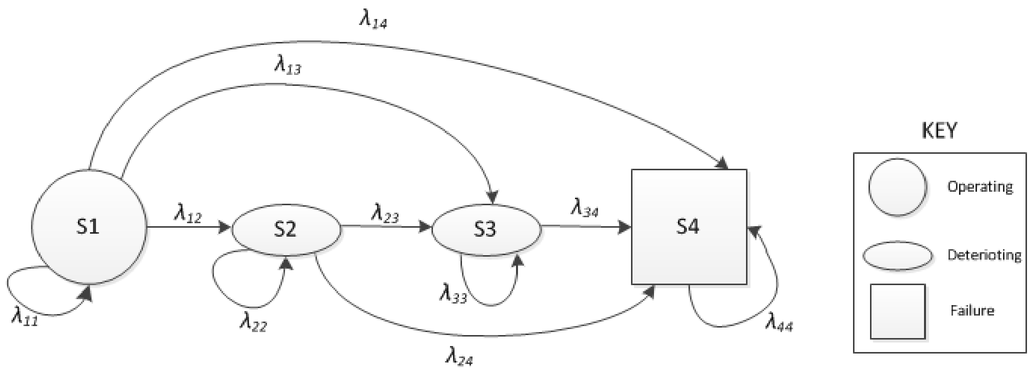

Fig 1: Transition diagram of non maintained system

From fig. 1 above, we obtained the following first order differential equations

$$\frac{dp_1(t)}{dt} = -(\lambda_{12} + \lambda_{13} + \lambda_{14}) P_1(t)$$

$$\frac{dp_2(t)}{dt} = \lambda_{12} P_1(t) - (\lambda_{23} + \lambda_{24}) P_2(t)$$

$$\frac{dp_3(t)}{dt} = \lambda_{13} P_1(t) + \lambda_{23} P_2(t) - \lambda_{34} P_3(t) \qquad (1)$$

$$\frac{dP_4(t)}{dt} = \lambda_{14} P_1(t) + \lambda_{24} P_2(t) + \lambda_{34} P_3(t)$$

Subject to the condition $P_1(0) = 1,\ P_2(0) = P_3(0) = P_4(0) = 0$

Taking the Laplace transformation of (1) we have

$$P_1(s) = \frac{1}{s+a}$$

$$P_2(s) = \frac{-\lambda_{12}}{(a-b)(s+a)} + \frac{\lambda_{12}}{(a-b)(s+b)}$$

$$P_3(s) = \frac{\lambda_{12}}{(a-c)(s+c)} - \frac{\lambda_{12}}{(a-c)(s+a)} + \frac{\lambda_{12}\lambda_{23}}{(a-b)(b-c)(s+c)} - \frac{\lambda_{12}\lambda_{23}}{(a-b)(b-c)(s+b)} - \frac{\lambda_{12}\lambda_{23}}{(a-b)(a-c)(s+c)} + \frac{\lambda_{12}\lambda_{23}}{(a-b)(a-c)(s+a)} \qquad (2)$$

Where $a = \lambda_{12} + \lambda_{13} + \lambda_{14}$, $b = \lambda_{23} + \lambda_{24}$, and $c = \lambda_{34}$

From the inverse Laplace transformation of (2) we have

$$P_1(t) = e^{-(\lambda_{12}+\lambda_{13}+\lambda_{14})t}$$

$$P_2(t) = \frac{\lambda_{12}}{(\lambda_2 + \lambda_{13} + \lambda_{14} - \lambda_{23} - \lambda_{24})} e^{-(\lambda_{23}+\lambda_{24})t} - \frac{\lambda_{12}}{(\lambda_2 + \lambda_{13} + \lambda_{14} - \lambda_{23} - \lambda_{24})} e^{-(\lambda_{12}+\lambda_{13}+\lambda_{14})t}$$

$$\begin{aligned} P_3(t) = & \frac{-\lambda_{12}\lambda_{23}}{(-\lambda_{34} + \lambda_{23} + \lambda_{24})(\lambda_{12} + \lambda_{13} + \lambda_{14} - \lambda_{23} - \lambda_{24})} e^{-(\lambda_{23}+\lambda_{24})t} \\ & - \frac{(\lambda_{13}\lambda_{12} + \lambda_{13}^2 + \lambda_{13}\lambda_{14} - \lambda_{13}\lambda_{23} - \lambda_{13}\lambda_{24} - \lambda_{23}\lambda_2)}{(-\lambda_{34} + \lambda_{12} + \lambda_{13} + \lambda_{14})(\lambda_{12} + \lambda_{13} + \lambda_{14} - \lambda_{23} - \lambda_{24})} e^{-(\lambda_{12}+\lambda_{13}+\lambda_{14})t} \\ & + \frac{(\lambda_{23}\lambda_{12} - \lambda_{13}\lambda_{34} + \lambda_{13}\lambda_{23} + \lambda_{13}\lambda_{24})}{(-\lambda_{34} + \lambda_{23} + \lambda_{24})(-\lambda_{34} + \lambda_{12} + \lambda_{13} + \lambda_{14})} e^{-\lambda_{34}t} \end{aligned} \tag{3}$$

$$R(t) = P_1(t) + P_2(t) + P_3(t) \tag{4}$$

$$M.T.T.F = \int_0^\infty R(t)\,dt = \frac{\lambda_{23}\lambda_{34} + \lambda_{24}\lambda_{34} + \lambda_{12}\lambda_{34} + \lambda_{13}\lambda_{24} + \lambda_{13}\lambda_{23} + \lambda_{23}\lambda_{12}}{\lambda_{34}(\lambda_{12} + \lambda_{13} + \lambda_{14})(\lambda_{23} + \lambda_{24})} \tag{5}$$

3 Results

Fig. 2 is the relationship between reliability and time t

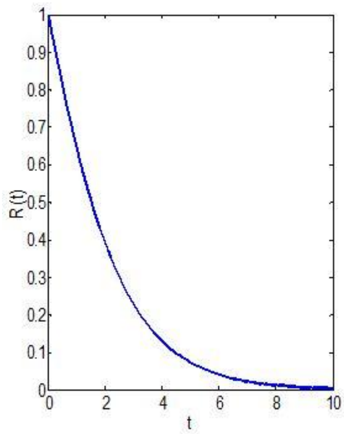

fig. 2 plot of *R(t)* against time t

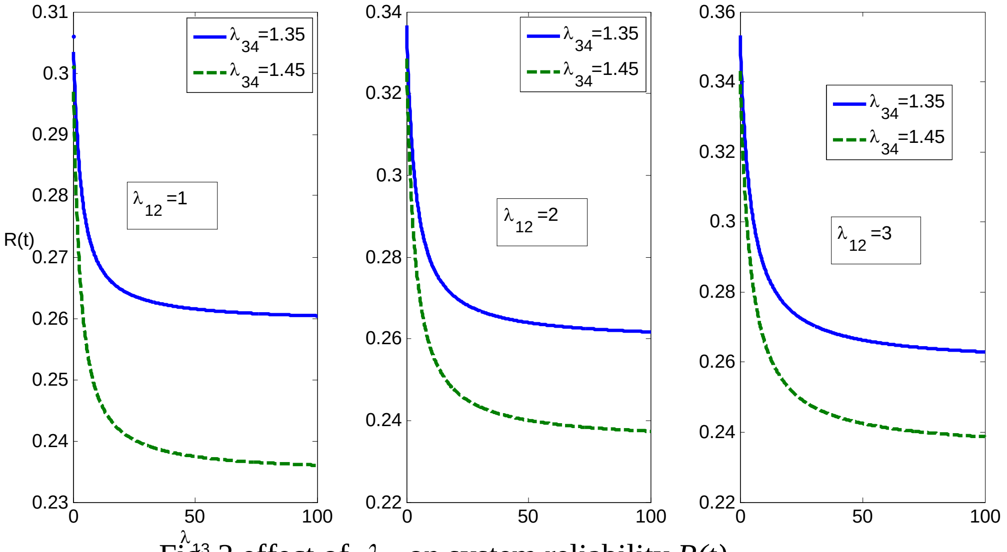

Fig. 3 effect of λ_{13} on system reliability *R(t)*

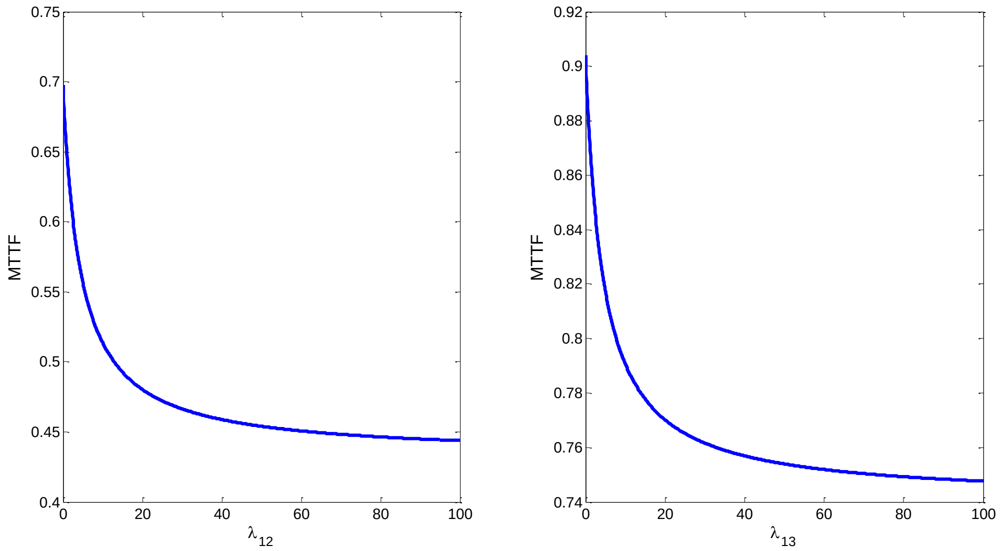

Fig. 4 effect of deterioration rates on MTTF

4 Conclusion

In this study,explicit expression for system reliability and mean time to failure of a non maintained system was developed. It has been found that reliability decreases with time and deterioration rates. That Reliability decreases with time and deterioration rate is consistent according the simulation of the model. It was found from the simulation that MTTF decreases with increase in deterioration rates.

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