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New Bayes Estimator of Parameter Weibull Distribution Using Simulation Study

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Abstract

In this paper the Jeffery prior information and the extension of Jeffery prior information for estimating the parameter Weibull distribution is presented. Through simulation study the performance of this estimator was compared to the standard Bayes with Jeffery prior information with respect to the mean square error (MSE) and mean percentage error (MPE). In the results, The new estimator with extension of Jeffery prior information $\hat{\theta}_2$ is the best estimator for Weibull Distribution, when compared it with standard Bayes with Jeffery prior information. Also depending on MSE and MPE, the $\hat{S}_2$ is the best survival function for Weibull distribution when compared it with survival function based on posterior distribution. We can easily conclude that MSE and MPE of Bayes estimators decrease with an increase of sample size.

Keywords: *Bayes estimation, Jeffrey Prior information, Extension of Jeffery prior information, Simulation study*

1 Introduction

The Weibull distribution is very useful modeling and analyzing in failure data in the field of engineering sciences and life time data in the fields of medicine and biology. Many researcher studied Bayes estimation for Weibull distribution, Sinha and Sloan [7] proposed a method based on the primary information with weighted Bayes. Also, using record statistics from Weibull model Bayesian and non-Bayesian approaches. In 2000, Shemyakin [6] suggest a modification of existing couple models using Bayesian approach and numerical results are presented for MLE and Bayesian estimation. Hossain and Zimmer [4] have discussed some comparative estimation of Weibull parameters using complete and censored samples. Besides, because of its useful applications, an array of method have been proposed for estimating parameters of the Weibull distribution. Soliman [8] carried out the comparison of the estimates. Kantar and Penoolu [5] did comparative study for the location and scale parameters of the Weibull distribution with a given shape parameter.

In 2009, Alkutubi [2], studied Jeffery prior information to get the modify Bayes estimator for exponential distribution and compared it with standard Bayes estimator and maximum likelihood estimator to find the best (less MSE and MPE). The extension of Jeffery prior information with new loss function for estimating the parameter exponential distribution of life time is presented by Alkutubi [3]. Afshari [1], obtained the Bayesian estimation and survival function, he consider Weibull distribution with unknown parameter with estimate both parameter under square error loss function.

In this study we present Bayes estimators using Jeffrey prior information and extension of Jeffrey prior information for Weibull distribution.

2 Bayes Estimators

i. Using Jeffery Prior Information

A number of n put to test and the life times of this random sample are recorded with the probability density function $f(t,\theta)$, where θ is real valued random variable. To obtain Bayes estimator, the following steps are needed. The probability density function of Weibull distribution is given by:

$$f(t,\theta) = \frac{c}{\theta}\left(\frac{t}{\theta}\right)^{c-1} e^{-\left(\frac{t}{\theta}\right)}$$

To find Bayes estimator, we need fisher information $I(\theta)$, Such that

$$E\left(\frac{\partial^2 \ln f(t,\theta)}{\partial\theta^2}\right) = \frac{-c^2}{\theta^2}$$

Then,

$$I(\theta) = -nE\left(\frac{\partial^2 \ln f(t,\theta)}{\partial\theta^2}\right) = \frac{nc^2}{\theta^2}$$

The life time probability density function $f(t,\theta)$ is regarded as a conditional probability density function $f(t/\theta)$ where the marginal probability density function of θ is given by $g(\theta)$, the Jeffery prior information

$$g(\theta) = k\sqrt{I(\theta)} = k\sqrt{\frac{nc^2}{\theta^2}} = \frac{kc\sqrt{n}}{\theta}$$

The joint probability density function is given by:

$$H(t_1,t_2,\dots,t_n,\theta)=L\left(\frac{t_1,t_2,\dots,t_n}{\theta}\right)g(\theta)=\frac{kc^2\sqrt{n}}{\theta^{n+1}}\left(\frac{\sum t_i}{\theta}\right)^{c-1}\exp\left(-\left(\frac{\sum t_i}{\theta}\right)^{c}\right)$$

The marginal probability density function of $(t_1,\dots,t_n,\theta)$ is given by:

$$P(t_1,t_2,\dots,t_n)=\int_0^\infty H(t_1,t_2,\dots,t_n,\theta)\,d\theta\ =\frac{kc\sqrt{n}}{(\sum t_i)^n}\left(\frac{n-1}{c}\right)!$$

The conditional probability density function of θ given the data $(t_1,\dots,t_n,\theta)$ is called posterior distribution of θ, given by

$$\pi(\theta,t_1,t_2,\dots,t_n)=\frac{H(t_1,t_2,\dots,t_n,\theta)}{P(t_1,t_2,\dots,t_n)}\ =\frac{c(\sum t_i)^n\left(\frac{\sum t_i}{\theta}\right)^{c-1}exp\left(-\left(\frac{\sum t_i}{\theta}\right)^{c}\right)}{\theta^{n+1}\left(\frac{n-1}{c}\right)!}$$

Bayes estimator of θ is given by using squared error loss function

$$R(\hat{\theta},\theta)=El(\hat{\theta},\theta)=\int_0^\infty c(\hat{\theta},\theta)^2\,\pi(\theta,t_1,t_2,\dots,t_n)d\theta$$

Let $\frac{\partial R(\hat{\theta},\theta)}{\partial\hat{\theta}}=0$, then Bayes estimator is

$$\hat{\theta}_1=\frac{\sum t_i}{\left(\frac{n-1}{c}\right)}$$

We can find the estimator of reliability such that

$$\widehat{S_1}(t)=\left(\frac{\sum t_i}{t_i+\sum t_i}\right)^{n+c-1}$$

ii. Using Extension of Jeffery Prior Information

The extension of Jeffery prior

$$g(\theta)=k[I(\theta)]^m=k\left(\frac{nc^2}{\theta^2}\right)^m\ =\frac{kn^mc^{2m}}{\theta^{2m}}$$

The joint probability density function is given by:

$$H(t_1,t_2,\dots,t_n,\theta)=L\left(\frac{t_1,t_2,\dots,t_n}{\theta}\right)g(\theta)$$

$$=\frac{kn^mc^{2m+1}}{\theta^{2m+n}}\left(\frac{\sum t_i}{\theta}\right)^{c-1}\exp\left(-\left(\frac{\sum t_i}{\theta}\right)^{c}\right)$$

The marginal probability density function of $(t_1,\dots,t_n,\theta)$ is given by:

$$P(t_1,t_2,\dots,t_n)=\int_0^\infty H(t_1,t_2,\dots,t_n,\theta)\,d\theta\ =\frac{kn^mc^{2m+1}}{(\sum t_i)^{2m+n-2}}\left(\frac{2m-n-2}{c}\right)!$$

The conditional probability density function of θ given the data $(t_1,\dots,t_n,\theta)$ is called posterior distribution of θ, given by

$$\pi(\theta, t_1, t_2, \dots, t_n) = \frac{H(t_1, t_2, \dots, t_n, \theta)}{P(t_1, t_2, \dots, t_n)}$$

$$= \frac{c(\sum t_i)^{2m+n-1} \left(\frac{\sum t_i}{\theta}\right)^{c-1} exp\left(-\left(\frac{\sum t_i}{\theta}\right)^{c}\right)}{\theta^{2m+n} \left(\frac{2m-n-2}{c}\right)!}$$

By using new loss function. Let $l(\hat{\theta}, \theta) = \theta^{c_1}(\hat{\theta} - \theta)^2$ In the same way, we can find the Bayes estimator depend on posterior distribution, such that

$$R(\hat{\theta}, \theta) = \int_0^{\infty} \theta^r (\hat{\theta} - \theta)^2 \pi(\theta, t_1, t_2, \dots, t_n) d\theta$$

Let $\frac{\partial R(\hat{\theta}, \theta)}{\partial \hat{\theta}} = 0$, then Bayes estimator is

$$\hat{\theta}_2 = \frac{\sum t_i}{\left(\frac{n+2m-r-2}{c}\right)!}$$

We can find the estimator of reliability such that

$$\widehat{S_2}(t) = \left(\frac{\sum t_i}{t_i + \sum t_i}\right)^{n+2m+c-2}$$

3 Simulation Study

In this simulation study, we have chosen *n = 50, 100, 150* to represent small moderate and large sample size, several values of parameter *θ = 0.5, 1.5* and *p = 0.6, 1.2*, three values of Jeffery extension c = 0.3, 0.6, 0.9. The result of this simulation is explained in Table 1 and Table 2.

4 Conclusion

The new estimator with extension of Jeffery prior information $\hat{\theta}_2$ is the best estimator for Weibull Distribution, when compared it with standard Bayes with Jeffery prior information. Also depending on MSE and MPE, the $\hat{S}_2$ is the best survival function for Weibull distribution when compared it with survival function based on posterior distribution. We can easily conclude that MSE and MPE of Bayes estimators decrease with an increased of sample size.

Reference

[1] Afshari M. (2011). Bayesian estimation distribution and survival function of records and inter record times and numerical computation for Weibull model. The journal of mathematics, 9(1): 75-81.

[2] Alkutubi H. S. & Ibrahim N.A.(2009)"a". On The Estimation of Survival Function and Parameter Exponential Life Time distribution. Journal of Mathematics and Statistics 5 (2):130-135.

[3] Alkutubi H. S. & Ibrahim N.A.(2009)"b". Bayes Estimator for Exponential Distribution with Extension of Jeffery Prior Information. Malaysian Journal of Mathematical Sciences 3(2): 295- 310.

[4] 4.Hossain, A.M. & W.J. Zimmer, (2003). Comparison of estimation methods for Weibull parameters: Complete and censored samples. J. Stat. Compute. Simul, 73:145153. DOI:10.1080/0094965021000033486.

[5] Kantar, Y.M. & B. Penoolu,(2008).A comparative study for the location and scale parameters of the Weibull distribution with given shape parameter. Compute. Geosci., 34: 1900-1909. DOI:10.1016/j.cageo.2008.04.004.

[6] Shemyakin A. & Youn H. (2000). Bayesian estimation of joint survival functions in life insurance. ISBA. Proceedings.

[7] Sinha, S. K. & J.A. Sloan, (1988). Bayes estimation of the parameters and reliability function of the 3- parameter Weibull distribution. IEEE Trans. Reliab., 37: 364-356. DOI:10.1016/ j.jspi. 2004.04.018.

[8] Soliman, A.A., A. H.Abd Ellah & K.S. Sultan, (2006).Comparison of estimates using record statistics from Weibull model: Bayesian and non-Bayesian approaches. Comput. Stat. Data Anal., 51: 2065- 2077. DOI: 10.1016/j.csda.2005.12.020

Table 1: MSR and MPE for Parameters Weibull distribution

Size n	θ	P	C	$\hat{\theta}_1$		$\hat{\theta}_2$	
				MSE	MPE	MSE	MPE
50	0.5	0.6	0.3	0.137	0.014	0.135	0.010
			0.6	0.135	0.012	0.132	0.008
			0.9	0.123	0.010	0.126	0.006
		1.2	0.3	0.138	0.014	0.136	0.011
			0.6	0.136	0.013	0.134	0.009
			0.9	0.134	0.011	0.133	0.006
	1.5	0.6	0.3	0.135	0.014	0.133	0.011
			0.6	0.134	0.012	0.131	0.011
			0.9	0.134	0.010	0.131	0.010
		1.2	0.3	0.135	0.014	0.132	0.013
			0.6	0.133	0.013	0.130	0.011
			0.9	0.131	0.011	0.130	0.011
100	0.5	0.6	0.3	0.116	0.013	0.113	0.013
			0.6	0.114	0.011	0.114	0.011
			0.9	0.111	0.011	0.110	0.011
		1.2	0.3	0.116	0.012	0.114	0.012
			0.6	0.114	0.012	0.112	0.012
			0.9	0.112	0.010	0.112	0.010
	1.5	0.6	0.3	0.113	0.011	0.112	0.011
			0.6	0.112	0.009	0.112	0.011
			0.9	0.112	0.010	0.110	0.010
		1.2	0.3	0.113	0.012	0.113	0.011
			0.6	0.111	0.011	0.110	0.009
			0.9	0.107	0.010	0.108	0.007
150	0.5	0.6	0.3	0.105	0.011	0.103	0.008
			0.6	0.104	0.010	0.104	0.007
			0.9	0.101	0.011	0.100	0.011
		1.2	0.3	0.104	0.009	0.104	0.009
			0.6	0.104	0.009	0.102	0.008
			0.9	0.101	0.008	0.102	0.007
	1.5	0.6	0.3	0.105	0.010	0.104	0.009
			0.6	0.103	0.009	0.103	0.009
			0.9	0.103	0.006	0.103	0.008
		1.2	0.3	0.103	0.009	0.102	0.007
			0.6	0.102	0.008	0.101	0.007
			0.9	0.102	0.007	0.100	0.007

Table 2: MSR and MPE for Survival function Weibull distribution

Size n	θ	P	C	$\hat{S}_1$		$\hat{S}_2$	
				MSE	MPE	MSE	MPE
50	0.5	0.6	0.3	0.238	0.118	0.236	0.114
			0.6	0.236	0.116	0.233	0.112
			0.9	0.224	0.114	0.227	0.110
		1.2	0.3	0.239	0.118	0.237	0.115
			0.6	0.237	0.117	0.235	0.113
			0.9	0.235	0.115	0.234	0.110
	1.5	0.6	0.3	0.235	0.114	0.234	0.111
			0.6	0.234	0.112	0.232	0.111
			0.9	0.234	0.110	0.232	0.110
		1.2	0.3	0.235	0.114	0.233	0.113
			0.6	0.233	0.113	0.231	0.111
			0.9	0.231	0.111	0.231	0.111
100	0.5	0.6	0.3	0.217	0.113	0.214	0.113
			0.6	0.215	0.111	0.215	0.111
			0.9	0.212	0.111	0.211	0.111
		1.2	0.3	0.217	0.112	0.213	0.112
			0.6	0.215	0.112	0.213	0.112
			0.9	0.213	0.110	0.213	0.110
	1.5	0.6	0.3	0.214	0.111	0.213	0.111
			0.6	0.213	0.109	0.213	0.111
			0.9	0.213	0.110	0.211	0.110
		1.2	0.3	0.214	0.112	0.214	0.111
			0.6	0.212	0.111	0.211	0.109
			0.9	0.208	0.110	0.209	0.107
150	0.5	0.6	0.3	0.206	0.111	0.204	0.108
			0.6	0.205	0.110	0.205	0.107
			0.9	0.202	0.111	0.201	0.111
		1.2	0.3	0.205	0.109	0.204	0.109
			0.6	0.205	0.109	0.203	0.108
			0.9	0.202	0.108	0.203	0.107
	1.5	0.6	0.3	0.206	0.110	0.104	0.109
			0.6	0.204	0.109	0.104	0.109
			0.9	0.204	0.106	0.103	0.108
		1.2	0.3	0.204	0.109	0.103	0.107
			0.6	0.203	0.108	0.102	0.107
			0.9	0.202	0.107	0.101	0.107