International Journal of Applied Mathematical Research, 6 (1) (2017) 7-13

SPC

International Journal of Applied Mathematical Research

Website: www.sciencepubco.com/index.php/IJAMR
doi: 10.14419/ijamr.v6i1.6917
Research paper

Reliability and failure functions of some weighted systems

Imad I. H. Nashwan *

Faculty of Technology & Applied Science, Al Quds Open University, Palestine
**Corresponding author E-mail: inashwan@qou.edu*

Abstract

In modern systems, not only the number and the position of the functioning or the failed components are the main factors that keep the system into the functioning state or transform it into failure state, but the contribution or the working probability of the component is also a very important one. This contribution is usually called the component weight. In this paper, the reliability and the failure functions of the consecutive weighted *k*-out-of-*n*: F linear and circular system, the weighted *f*-out-of-*n*: F system, the weighted (n, f, k) linear and circular system and the weighted $\langle n, f, k\rangle$ linear and circular system are obtained.

Keywords: *Consecutive weighted k-out-of-n: F system, Weighted f-out-of-n: F system, Weighted* (n, f, k) *system, Weighted* $\langle n, f, k\rangle$ *system, Modular arithmetic.*

Notation

L(C)	: Linear (Circular)
$\vee(\wedge)$	: or (and)
$\mathbb{I}_j^i$	$=\{i, i+1, \ldots, j\}$

$X=\{x_1<\ldots<x_j\}$: a set $X=\{x_1, x_2, \ldots, x_j\} \subseteq \mathbb{I}_n^1$ such that $x_i<x_h$ for all $i<h$

$\overline{X}$	: The complement of X
$A-B$	$=A \cap \overline{B}$
$P\left(\mathbb{I}_n^1\right)$	: The power set of $\mathbb{I}_n^1$.
$\Psi_{L(C)}^2\left(\Theta_{L(C)}^2\right)$	: The space of consecutive (nonconsecutive) failures of the linear (circular) system.
D	$=\{W, f, W \vee f, W \wedge f\}$, where W means the weighted consecutive *k*-out-of-*n*: F system, *f* means the weighted *f*-out-of-*n*: F system, $W \vee f$ means the weighted (n, f, k) system, $W \wedge f$ means the weighted $\langle n, f, k\rangle$ system.
$\Psi_D^{L(C)}\left(\Theta_D^{L(C)}\right)$	: The failure (functioning) space of the D linear (circular) system.
$\mathbb{F}_D^{L(C)}\left(\mathbb{R}_D^{L(C)}\right)$	: The failure (reliability) function of the D linear (circular) system
g_n^α	: The composite function α times
W_X	$=\sum_{i \in X} w_i$, the total weight of the set X.
$p_i\ (q_i)$	:The reliability (unreliability) of the i^{th} components
$\mathbb{W}_{[X]}$	: The corresponding weights for all $Y \in [X]$

$F(X)=R(X)=p_X=\prod_{i \in X} p_i \prod_{j \in \overline{X}} q_j$.

$p_n^s=p(n, s)=p^{n-s} q^s$

$\mathbb{F}[X]_D=\sum_{Y \in[X]_D} F(Y), \qquad \mathbb{R}[X]_D=\sum_{Y \in[X]_D} R(Y)$

1. Introduction

The consecutive *k*-out-of-*n*: F system consists of *n* sequentially connected components, and ordered linearly or circularly, the system fails if *k* consecutive components are in the failure state. The reliability models of the consecutive system have been proposed for the design of integrated circuits, microwave relay stations in telecommunications, oil pipeline systems [4], vacuum systems in accelerators, computer ring networks (*k* loop), and spacecraft relay [14] etc. see [3-6], [12], and therein.

Many variations and generalizations of the consecutive *k*-out-of-*n*: F system appeared, such as, the weighted consecutive *k*-out-of-*n*: F system. It consists of *n* components with different contributions to the system, which are defined as weights, each component has its own working probability and a positive weight such that, the whole system fails if the total weight of some failed consecutive components reach a fixed threshold *k*. Actually, when all weights equal 1, then the consecutive weighted *k*-out-of-*n*: F system becomes the ordinary consecutive *k*-out-of-*n*: F system. As well as, the consecutive weight system has two types according to the connection between components, the linear and circular system. Wu & Chen [15-16] were the 1st who studied such a system; they found the minimal cut sets of the system to compute the reliability of the linear and circular system. Chang et al [1] studied only the circular consecutive weight system and introduced a simpler and more efficient algorithm, while Eryilmaz and Tutuncu [7] provided recursive formulas for the reliability of consecutive weighted *k*-out-of-*n*: F system.

Nowadays, in the modern system, the industrials design electronic system under multiple functioning criteria; they combine various system models with each other to produce a more reliable system.

It is worth mentioning that, the *f*-out-of-*n* system structure is very popular in combined system; it consists of *n* component, and fails if and only if at least *f* of the *n* components fail. Both parallel and series systems are special cases of the *f*-out-of-*n*: F system, when

$f = 1, n$ respectively. The *f*-out-of-*n* system structure finds wide applications in both industrial and military systems, fault-tolerant systems include the multi display system in a cockpit, the multiengine system in an airplane, and the multi pump system in a hydraulic control system [16].

On the whole, the combination of reliability models of the *f*-out-of-*n*: F system and the consecutive *k*-out-of-*n*: F system forms a new system called, "the $(n, f, k)(\langle n, f, k\rangle)$ system", it also consists of *n* components ordered in a line or a cycle, while the system fails if and only if, the event "at least *f* failed components **or (and)** at least *k* consecutive failed components" occurs, Tung [13] was the 1st introduced the (n, f, k) system, Chang et al. [2] used the Markov Chain to find its reliability, while Kamalja & Shinde [9] study the reliability of the $(n, f, k)(\langle n, f, k\rangle)$ systems. The infrared (IR) detecting and signal processing portion of a system, automatic payment systems in banks [2], evaluation of reliabilities for furnace systems [17] are the direct application of the (n, f, k) system.

Eryilmaz and Aksoy [8] introduced and studied the linear weighted (n, f, k) system and computed the reliability of the system using a recursive equation, the system can be used in electronic equipment.

The weighted $(n, f, k)(\langle n, f, k\rangle)$ linear and circular system is the same as the $(n, f, k)(\langle n, f, k\rangle)$ system but the criteria of failure depends on the weights of the components, where the system fails if, and only if, the total weight of any failed components are at least *f* **or (and)** the total weight of any consecutive failed components is at least *k*.

In this paper, a new algorithm to find the reliability and failure functions of the consecutive weighted *k*-out-of-*n*: F linear and circular system, the weighted *f*-out-of-*n*: F system, the weighted (n, f, k) linear and circular system, and the weighted $\langle n, f, k\rangle$ linear and circular system are obtained, the failure and the functioning space of the ordinary consecutive 2-out-of-*n*: F circular system plays a basic role to determine the failure space of the weighted systems.

The following assumptions are assumed to be satisfied by all mentioned systems:

1) The state of the component and the system is either "functioning" or "failed".
2) All the components are mutually statistically independent.

2. The consecutive *k*-out-of-*n*: f system

The consecutive *k*-out-of-*n*: F linear (circular) system consists of *n* components; the system fails if, and only if, at least *k* consecutive components are in the failed state. We present the system using the Index Structure Function [10], where $\mathbb{I}_n^1$ denotes the indices of the components of the system, hence the failure space of the components is $P(\mathbb{I}_n^1)$. The set $X \in P(\mathbb{I}_n^1)$ presents the system, which consists of all indices of failed components, is called "failed set" if *k* consecutive indices of failed components are included in *X*, otherwise, *X* is called "functioning set". If $\Theta_{L(C)}^k \left(\Psi_{L(C)}^k\right)$ consists of all functioning (failed) set of the consecutive *k*-out-of-*n*: F linear (circular) system, then $P(\mathbb{I}_n^1) = \Theta_{L(C)}^k \cup \Psi_{L(C)}^k$. For example, the set $X = \{1, 2, 5\} \subset \mathbb{I}_6^1$ represents the consecutive 2-out-of 6: F linear or circular system, for simply *X*=125, indicates that the 1st, the 2nd and the 5th components are only the failed components, hence *X* is a failed set, but the set *X*=135 is a functioning set.

Many researchers consider the consecutive *k*-out-of-*n*: F circular system is a generalization of the consecutive *k*-out-of-*n*: F linear system, due to the connection between the 1st and the *n*th components, which generates more possible consecutive failure states, consequently extra system failures states. This leads to the fact that $\Psi_L^k \subseteq \Psi_C^k$, for any $k \le n$. Also, since any consecutive *k* failed components includes consecutive *r* failed components, for any $r \le k$ but vice versa is not true, i.e. $\Psi_{L(C)}^k \subseteq \Psi_{L(C)}^r$. Consequently, $\Psi_{L(C)}^k \subseteq \Psi_{L(C)}^2 \subseteq \Psi_C^2$ for all $2 \le k \le n$. Note that, the failure space of the consecutive 2-out-of-*n*: F linear (circular) system $\Psi_{L(C)}^2$ represents the collection of all consecutive failures of the components.

Nashwan [10-11] simulated the symmetric property of the circular consecutive system using a bijection function $g_n : \mathbb{I}_n^1 \to \mathbb{I}_n^1$, where $g_n(x) = (x \bmod n) + 1$ for any $x \in \mathbb{I}_n^1$, and introduced an algorithm to partition the $P(\mathbb{I}_n^1), \Psi_C^k, \Theta_C^k$ into finite mutually pairwise disjoint equivalences classes, the algorithm to find these classes, when *k*=2 is as follows:

If *j* is the total number of failed components, and $X = \{x_1 < \ldots < x_j\}$ represents the consecutive 2-out-of-*n*: F circular system, define $d_X = (d_1^X, d_2^X, \ldots, d_j^X)$ the rotations of set *X*, where $d_i \ge 1$ is the minimum integer number such that $g_n^{d_i}(x_i) = x_{i+1}$, for *i*=1,2,…, *j*-1, and $g_n^{d_j}(x_j) = x_1$.

1. Finding Ψ_C^2

1.1. For *j*=2,3, …,*n*, find all rotation $d_X = (d_1, d_2, \ldots, d_j)$, such that $n = \sum_{i=1}^{j} d_i$ and at least one of d_i equals 1.

1.2. Find the corresponding set $X \in P(\mathbb{I}_n^1)$ and $[X] = \{g_n^\alpha(X) : \alpha \in \mathbb{Z}\}$.

1.3. If $d_Y = d_X^m = (d_{j-m+1}^X, \ldots, d_j^X, d_1^X, \ldots, d_{j-m}^X)$ for some *m*, and $X, Y \in P(\mathbb{I}_n^1)$ then $Y \in [X]$, furthermore $[Y] = [X]$.

1.4. For Ψ_L^2, applied the steps above and omit any set in Ψ_C^k such that $d_i > 1$ for all $i \ne j$ and $d_j = 1$.

2. Finding Θ_C^2

2.1. For *j*=0, the only class is $[\varnothing]$, and for *j*=1, the only class is $[1]$.

2.2. For *j*=2, …, *M*, do the same steps above but $d_i \succ 1$ for all $i \in \mathbb{I}_j^1$, where $M = \begin{cases} n - [n/2], & n \text{ is even} \\ n - 1 - [n/2], & n \text{ is odd} \end{cases}$.

2.3. For the linear system Θ_L^2, add the omitted states which are resulted from step 1.4.

3. Weighted systems

In this section, the Index Structure Function [10] is used to represent the weighted systems below, where $\mathbb{I}_n^1$ denotes the indices of the components, $w_1, w_2, \ldots, w_n < k, f$ are the weights of these components, and the set $X \in P(\mathbb{I}_n^1)$ represents the system and consists of all indices of the failed components as in section 2.

3.1. The consecutive weighted *k*-out-of-*n*: f system

The consecutive weighted *k*-out-of-*n*: F linear (circular) system consists of *n* components. It fails if the total weight of some consecutive failed components is at least *k*. If $X=\{x_1<...<x_j\}$ represents the system, then *X* is called failed set if there exists $\mathbb{I}_j^i \subseteq X$ such that $W_{\mathbb{I}_j^i} \ge k$ for any $1\le i<j\in \mathbb{I}_n^2$ in the linear system (for the circular system, $1\le i<j\in \mathbb{I}_{n+1}^2$, where the *n*+1$^{\text{th}}$ component is the 1$^{\text{st}}$ component), and since $\Psi_{L(C)}^2$ is the collection of all consecutive failures, then $\Psi_W^{L(C)} \subseteq \Psi_{L(C)}^2$, which implies that the failure space of the system is $\Psi_W^{L(C)}=\left\{X\in\Psi_{L(C)}^2 : \mathbb{I}_j^i \subseteq X \ \wedge\ W_{\mathbb{I}_j^i}\ge k\right\}$ for all $1\le i<j\in \mathbb{I}_n^2\left(\mathbb{I}_{n+1}^2\right)$.

Note that, for all $X\in\Theta_{L(C)}^2$, *X* has non-consecutive failed components, which applies that $X\in\Theta_W^{L(C)}$.

3.2. The weighted *f*-out-of-*n*: f system

The weighted *f*-out-of-*n*: F system, consists of *n* components, and fails if the total weight of the failed components is at least *f*. i.e. if $X=\{x_1<...<x_j\}$ represents the system, then *X* is called failed set if $W_X\ge f$, i.e. the failure space of the system is $\Psi_f^{L(C)}=\left\{X\in P\left(\mathbb{I}_n^1\right):\ W_X\ge f\right\}$.

3.3. Weighted $(n,f,k)(\langle n,f,k\rangle)$ systems

The weighted $(n,f,k)(\langle n,f,k\rangle)$ linear and circular system consists of *n* components and fails if, and only if the total weight of the failed components is at least *f* **or (and)** the total weight of any consecutive failed components is at least *k*. If *X* represents the system, then *X* is called failed set if $W_X\ge f$ **or (and)** there exist $\mathbb{I}_j^i\subseteq X$ such that $W_{\mathbb{I}_j^i}\ge k$, then the failure space of systems is $\Psi_{W\vee f}^{L(C)}\left(\Psi_{W\wedge f}^{L(C)}\right)=\left\{X\in P\left(\mathbb{I}_n^1\right):\ W_X\ge f\ \vee(\wedge)\ \mathbb{I}_j^i\subseteq X \text{ where } W_{\mathbb{I}_j^i}\ge k\right\}$.

Note that $\Psi_{W\vee f}^{L(C)}=\Psi_f^{L(C)}\cup\Psi_W^{L(C)}$ and $\Psi_{W\wedge f}^{L(C)}=\Psi_f^{L(C)}\cap\Psi_W^{L(C)}$.

4. The proposed algorithm

Consider the weighted systems in section 3, and proceed from the fact that, $\Psi_{L(C)}^2$ consists of all consecutive failures in the linear (circular) system, where $P\left(\mathbb{I}_n^1\right)=\Theta_{L(C)}^2\cup\Psi_{L(C)}^2$. According to Nashwan [11] algorithm, $\Psi_{L(C)}^2$ and $\Theta_{L(C)}^2$ respectively consists of a finite mutual pairwise disjoint classes, such that $\Psi_{L(C)}^2=\bigcup_{t=1}^s[X_t]$ and $\Theta_{L(C)}^2=\bigcup_{t=s+1}^r[X_t]$.

In this context, for *t*=1,2,…,*r* define:

$$[X_t]_W=\left\{Z\in[X_t]:\mathbb{I}_j^i\subseteq Z\ \wedge\ W_{\mathbb{I}_j^i}\ge k\right\},\quad [X'_t]_W=[X_t]-[X_t]_W$$

$$[X_t]_f=\{Z\in[X_t]:W_Z\ge k\},\quad [X'_t]_f=[X_t]-[X_t]_f$$

$$[X_t]_{W\vee f}=[X_t]_f\cup[X_t]_W,\quad [X'_t]_{W\vee f}=[X_t]-[X_t]_{W\vee f}$$

$$[X_t]_{W\wedge f}=[X_t]_f\cap[X_t]_W,\quad [X'_t]_{W\wedge f}=[X_t]-[X_t]_{W\wedge f}$$

Note that

1) For *t*=*s*+1,*s*+2, …, *r*, $[X_t]_W=[X_t]_{W\wedge f}=\varnothing$, and then $[X'_t]_W=[X'_t]_{W\wedge f}=[X_t]$.

2) $[X'_t]_{W\vee f}=[X'_t]_f\cap[X'_t]_W$, $[X'_t]_{W\wedge f}=[X'_t]_f\cup[X'_t]_W$

3) Anyone can easily check that $\Psi_W^{L(C)},\Psi_f^{L(C)},\Psi_{W\vee f}^{L(C)},\Psi_{W\wedge f}^{L(C)}$ and $\Theta_W^{L(C)},\Theta_f^{L(C)},\Theta_{W\vee f}^{L(C)},\Theta_{W\wedge f}^{L(C)}$ are a union of finite pairwise disjoint classes as well as $\Psi_{L(C)}^2,\Theta_{L(C)}^2$. (See the following theorem).

Theorem 4.1: *According the above definitions and notes, then*

1) $\Psi_W^{L(C)}=\bigcup_{t=1}^s[X_t]_W$

2) $\Psi_f^{L(C)}=\bigcup_{t=1}^r[X_t]_f$

3) $\Psi_{W\vee f}^{L(C)}=\left(\bigcup_{t=1}^s[X_t]_{W\vee f}\right)\cup\left(\bigcup_{t=s+1}^r[X_t]_f\right)$

4) $\Psi_{W\wedge f}^{L(C)}=\bigcup_{t=1}^s[X_t]_{W\wedge f}$

5) $\Theta_W^{L(C)}=\bigcup_{t=1}^r[X'_t]_W$

6) $\Theta_f^{L(C)}=\bigcup_{t=1}^r[X'_t]_f$

7) $\Theta_{W\vee f}^{L(C)}=\left(\bigcup_{t=1}^s[X'_t]_{W\vee f}\right)\cup\left(\bigcup_{t=s+1}^r[X'_t]_f\right)$

8) $\Theta_{W\wedge f}^{L(C)}=\bigcup_{t=1}^r[X'_t]_{W\wedge f}$

Proof:

1) For all $Y\in\Psi_W^{L(C)}\subseteq\Psi_{L(C)}^2=\bigcup_{t=1}^s[X_t]$, there exist $[X_t]\in\Psi_{L(C)}^2$ such that $Y\in[X_t]$, and since $Y\in\Psi_W^{L(C)}$ there exist $\mathbb{I}_j^i\subseteq Y\ \wedge\ W_{\mathbb{I}_j^i}\ge k$, then $Y\in[X_t]_W$, hence $\Psi_W^{L(C)}\subseteq\bigcup_{t=1}^s[X_t]_W$. Conversely, $[X_t]_W\subseteq\Psi_W^{L(C)}$ for all $[X_t]\in\Psi_{L(C)}^2$, i.e. $\bigcup_{[X]\in\Psi_{L(C)}^2}[X]_W\subseteq\Psi_W^{L(C)}$.

2) Let $Y\in\Psi_f^{L(C)}\subseteq P\left(\mathbb{I}_n^1\right)=\bigcup_{t=1}^r[X_t]$, there exist $[X_t]\in\Theta_{L(C)}^2\vee\Psi_{L(C)}^2$ such that $Y\in[X_t]$, but $W_Y\ge f$, which implies $Y\in[X_t]_f$, hence $\Psi_f^{L(C)}\subseteq\bigcup_{t=1}^r[X_t]_f$. Conversely, $[X_t]_f\subseteq\Psi_f^{L(C)}$ for all $[X_t]\in P\left(\mathbb{I}_n^1\right)$, which implies that $\bigcup_{t=1}^r[X_t]_f\subseteq\Psi_f^{L(C)}$

3)
$$\begin{aligned}\Psi_{W\vee f}^{L(C)}&=\Psi_f^{L(C)}\cup\Psi_W^{L(C)}=\left(\bigcup_{t=1}^r[X_t]_f\right)\cup\left(\bigcup_{t=1}^s[X_t]_W\right)\\&=\bigcup_{t=1}^s\left([X_t]_W\cup[X_t]_f\right)\cup\left(\bigcup_{t=s+1}^r[X_t]_f\right)\\&=\bigcup_{t=1}^s\left([X_t]_{W\vee f}\right)\cup\left(\bigcup_{t=s+1}^r[X_t]_f\right)\end{aligned}$$

4)
$$\begin{aligned}\Psi_{W\wedge f}^{L(C)}&=\Psi_f^{L(C)}\cap\Psi_W^{L(C)}=\left(\bigcup_{t=1}^r[X_t]_f\right)\cap\left(\bigcup_{t=1}^s[X_t]_W\right)\\&=\bigcup_{t=1}^s\left([X_t]_W\cap[X_t]_f\right)\cup\left(\bigcup_{t=s+1}^r[X_t]_W\cap\bigcup_{t=1}^s[X_t]_W\right)\\&=\bigcup_{t=1}^s\left([X_t]_{W\wedge f}\right)\cup\varnothing=\bigcup_{t=1}^s\left([X_t]_{W\wedge f}\right)\end{aligned}$$

5) For *t*=1,2…,*s*, $[X'_t]_W=[X_t]-[X_t]_W\Rightarrow$

$$\bigcup_{t=1}^s[X'_t]_W=\bigcup_{t=1}^s[X_t]-\bigcup_{t=1}^s[X_t]_W=\bigcup_{t=1}^s[X_t]-\bigcup_{t=1}^s[X_t]_W=\Psi_{L(C)}^2-\Psi_W^{L(C)},$$

6) For that for t=s+1, s+2,…,r , $[X'_t]_W=[X_t]$ hence, $\Theta^2_{L(C)}=\bigcup_{t=s+1}^{r}[X'_t]_W=\bigcup_{t=s+1}^{r}[X_t]$, this implies that $\bigcup_{t=1}^{r}[X'_t]_W=\left(\Psi^2_{L(C)}-\Psi^{L(C)}_W\right)\cup\Theta^2_{L(C)}=P\left(\mathbb{I}^1_n\right)-\Psi^{L(C)}_W=\Theta^{L(C)}_W$.

(The proof of 6-8 is the same as the proof of 5).

Theorem 4.2: *Recall Theorem 4.1, then*

1) $\mathbb{F}^{L(C)}_W=\sum_{t=1}^{s}\sum_{Y\in[X_t]_W}F(Y)$

2) $\mathbb{F}^{L(C)}_f=\sum_{t=1}^{r}\sum_{Y\in[X_t]_f}F(Y)$

3) $\mathbb{F}^{L(C)}_{W\vee f}=\sum_{t=1}^{s}\sum_{Y\in[X_t]_{W\vee f}}F(Y)+\sum_{t=s+1}^{r}\sum_{Z\in[X_t]_f}F(Z)$

4) $\mathbb{F}^{L(C)}_{W\wedge f}=\sum_{t=1}^{s}\sum_{Y\in[X_t]_{W\wedge f}}F(Y)$

5) $\mathbb{R}^{L(C)}_W=\sum_{t=1}^{r}\sum_{Y\in[X'_t]_W}F(Y)$

6) $\mathbb{R}^{L(C)}_f=\sum_{t=1}^{r}\sum_{Y\in[X'_t]_f}F(Y)$

7) $\mathbb{R}^{L(C)}_{W\vee f}=\sum_{t=1}^{s}\sum_{Y\in[X'_t]_{W\vee f}}F(Y)+\sum_{t=s+1}^{r}\sum_{Z\in[X'_t]_f}F(Z)$

8) $\mathbb{R}^{L(C)}_{W\wedge f}=\sum_{t=1}^{r}\sum_{Y\in[X'_t]_{W\wedge f}}F(Y)$

Proof:

1) Using theorem 4.1, $\Psi^{L(C)}_W=\bigcup_{t=1}^{s}[X_t]_W$, then

$$\mathbb{F}^{L(C)}_W=\mathbb{F}\left(\bigcup_{t=1}^{s}[X_t]_W\right)=\sum_{t=1}^{s}\mathbb{F}[X_t]_W=\sum_{t=1}^{s}\sum_{Y\in[X_t]_W}F(Y)$$

2) The same proof of 1, if we put r, f instead of s, W respectively.

3) Using 1 & 2 above,

$$\mathbb{F}^{L(C)}_{W\vee f}=\mathbb{F}\left(\Psi_{W\vee f}\right)=\sum_{t=1}^{s}\mathbb{F}[X_t]_{W\vee f}+\sum_{t=s+1}^{r}\mathbb{F}[X_t]_f$$
$$=\sum_{t=1}^{s}\sum_{Y\in[X_t]_{W\vee f}}F(Y)+\sum_{t=s+1}^{r}\sum_{Y\in[X_t]_f}F(Y)$$

4) The same proof of 1, if we put $W\wedge f$ instead of W.

5) using theorem 4.1,

$$\Theta^{L(C)}_W=\bigcup_{t=1}^{r}[X'_t]_W\Rightarrow\mathbb{R}^{L(C)}_W=\mathbb{R}\left(\bigcup_{t=1}^{r}[X'_t]_W\right)$$
$$=\sum_{t=1}^{r}\mathbb{R}[X_t]_W=\sum_{t=1}^{s}\sum_{Y\in[X_t]_W}R(Y)$$

(The proof of 6-8 is the same as the proof of 5).

4.1. The proposed algorithm

- Use Nashwan [11] algorithm in section 2, and determine the classes $[X_t]:t=1,2,...,r$ of each consecutive failures space $\Psi^2_{L(C)}$ and nonconsecutive failures space $\Theta^2_{L(C)}$.
- For any $[X_t]:t=1,2,...,r$, $\mathbb{W}_{[X_t]}$ find the corresponding weights of all $Y\in[X_t]$.
- For $t=1,2,...,r$, find $[X_t]_W,[X_t]_f,[X_t]_{W\vee f},[X_t]_{W\wedge f}$ and $[X'_t]_W,[X'_t]_f,[X'_t]_{W\vee f},[X'_t]_{W\wedge f}$.
- Use theorem 4.1 to find $\Psi^{L(C)}_W,\Psi^{L(C)}_f,\Psi^{L(C)}_{W\vee f},\Psi^{L(C)}_{W\wedge f}$ and $\Theta^{L(C)}_W,\Theta^{L(C)}_f,\Theta^{L(C)}_{W\vee f},\Theta^{L(C)}_{W\wedge f}$.
- Use theorem 4.2 to find $\mathbb{F}^{L(C)}_W,\mathbb{F}^{L(C)}_f,\mathbb{F}^{L(C)}_{W\vee f},\mathbb{F}^{L(C)}_{W\wedge f}$ and $\mathbb{R}^{L(C)}_W,\mathbb{R}^{L(C)}_f,\mathbb{R}^{L(C)}_{W\vee f},\mathbb{R}^{L(C)}_{W\wedge f}$.

Example 4.1: Find the reliability and the failure functions of all circular weighted systems in section 3, when n=7, f=6, k=5 with weights $1,2,1,2,3,2,1$

1. For consecutive failures i.e. Ψ^2_C

For j=2,

$(1,6)\Leftrightarrow[12]=\left[\mathbb{I}^1_2\right]=\left\{g^{\alpha}_7(12):\alpha\in\mathbb{Z}\right\}=\{12,23,34,45,56,67,17\}$
$\mathbb{W}_{[12]}=3,3,3,5,5,3,2$
$[12]_W=\{45,56\}$ $[12]_f=\{\ \}$
$[12]_{W\vee f}=\{45,56\}$ $[12]_{W\wedge f}=\{\ \}$
$\mathbb{F}[12]_W=2p^2_7$ $\mathbb{F}[12]_f=0$ $\mathbb{F}[12]_{W\vee f}=2p^2_7$ $\mathbb{F}[12]_{W\wedge f}=0$

$[12']_W=\{12,23,34,67,17\}$, $[12']_f=[12]$,
$[12']_{W\vee f}=\{12,23,34,67,17\}$, $[12']_{W\wedge f}=[12]$
$\mathbb{R}[12]_W=5p^2_7$ $\mathbb{R}[12]_f=7p^2_7$
$\mathbb{R}[12]_{W\vee f}=5p^2_7$ $\mathbb{R}[12]_{W\wedge f}=7p^2_7$

For j=3,

$(1,1,5)\Leftrightarrow\left[\mathbb{I}^1_3\right]=\{123,234,345,456,567,167,127\}$
$\mathbb{W}_{[123]}=4,5,6,7,6,4,4$
$[123]_W=\{234,345,456,567\}$, $[123]_f=\{345,456,567\}$
$[123]_{W\vee f}=[123]_W$, $[123]_{W\wedge f}=[123]_f$
$\mathbb{F}[123]_W=4p^3_7$ $\mathbb{F}[123]_f=3p^3_7$
$\mathbb{F}[123]_{W\vee f}=4p^3_7$ $\mathbb{F}[123]_{W\wedge f}=3p^3_7$
$[123']_W=\{123,167,127\}$, $[123']_f=\{123,234,167,127\}$
$[123']_{W\vee f}=[123']_W$, $[123']_{W\wedge f}=[123']_f$
$\mathbb{R}[123']_W=3p^3_7$ $\mathbb{R}[123']_f=4p^3_7$
$\mathbb{R}[123']_{W\vee f}=3p^3_7$ $\mathbb{R}[123']_{W\wedge f}=4p^3_7$
$(1,2,4)\Leftrightarrow[124]=\{124,235,346,457,156,267,137\}$
$\mathbb{W}_{[124]}=5,6,6,5,6,6,5,3$
$[124]_W=\{457,156\}$, $[124]_f=\{235,457,156\}$
$[124]_{W\vee f}=[124]_f$, $[124]_{W\wedge f}=[124]_W$
$\mathbb{F}[124]_W=2p^3_7$ $\mathbb{F}[124]_f=3p^3_7$
$\mathbb{F}[124]_{W\vee f}=3p^3_7$ $\mathbb{F}[124]_{W\wedge f}=2p^3_7$

$[124']_W=\{124,235,346,267,137\}$, $[124']_f=\{124,346,267,137\}$
$[124']_{W\vee f}=[124']_W$, $[124']_{W\wedge f}=[124']_f$
$\mathbb{R}[124']_W=5p^3_7$ $\mathbb{R}[124']_f=4p^3_7$
$\mathbb{R}[124']_{W\vee f}=4p^3_7$ $\mathbb{R}[124']_{W\wedge f}=5p^3_7$
$(1,3,3)\Leftrightarrow[125]=\{125,236,347,145,256,367,147\}$
$\mathbb{W}_{[125]}=6,5,4,6,7,4,4$
$[125]_W=\{145,256\}$, $[125]_f=\{125,236,145,256\}$
$[125]_{W\vee f}=[125]_f$, $[125]_{W\wedge f}=[125]_W$
$\mathbb{F}[125]_W=2p^3_7$ $\mathbb{F}[125]_f=4p^3_7$
$\mathbb{F}[125]_{W\vee f}=4p^3_7$ $\mathbb{F}[125]_{W\wedge f}=2p^3_7$
$[125']_W=\{125,236,347,367,147\}$, $[125']_f=\{347,367,147\}$
$[125']_{W\vee f}=[125']_f$, $[125']_{W\wedge f}=[125']_W$
$\mathbb{R}[125']_W=5p^3_7$ $\mathbb{R}[125']_f=3p^3_7$
$\mathbb{R}[125']_{W\vee f}=3p^3_7$ $\mathbb{R}[125']_{W\wedge f}=5p^3_7$

$(1,4,2) \Leftrightarrow [126] = \{126, 237, 134, 245, 356, 467, 157\}$

$\mathbb{W}_{[126]} = 5, 4, 4, 7, 6, 5, 5$

$[126]_w = \{245, 356\}, \ [126]_f = \{245, 356\}$

$[126]_{w \vee f} = [126]_{w \wedge f} = [126]_f = [126]_w$

$\mathbb{F}[126]_w = 2p_7^3 \qquad \mathbb{F}[126]_f = 2p_7^3$

$\mathbb{F}[126]_{w \vee f} = 2p_7^3 \qquad \mathbb{F}[126]_{w \wedge f} = 2p_7^3$

$[126']_w = \{126, 237, 134, 467, 157\}$

$[126']_f = \{126, 237, 134, 467, 157\},$

$[126']_{w \vee f} = [126']_{w \wedge f} = [126']_f = [126']_w$

$\mathbb{R}[126']_w = 5p_7^3 \qquad \mathbb{R}[126']_f = 5p_7^3$

$\mathbb{R}[126']_{w \vee f} = 5p_7^3 \qquad \mathbb{R}[126']_{w \wedge f} = 5p_7^3$

For j=4.

$(1,1,1,4) \Leftrightarrow \left[\mathbb{I}_4^1\right] = \{1234, 2345, 3456, 4567, 1567, 1267, 1237\}$

$\mathbb{W}_{[1234]} = 6, 8, 8, 8, 7, 6, 5$

$\left[\mathbb{I}_4^1\right]_w = \left[\mathbb{I}_4^1\right]_f = \left[\mathbb{I}_4^1\right]_{w \vee f} = \left[\mathbb{I}_4^1\right]_{w \wedge f} = \left[\mathbb{I}_4^1\right]$

$\mathbb{F}\left[\mathbb{I}_4^1\right]_w = \mathbb{F}\left[\mathbb{I}_4^1\right]_f = \mathbb{F}\left[\mathbb{I}_4^1\right]_{w \vee f} = \mathbb{F}\left[\mathbb{I}_4^1\right]_{w \wedge f} = 7p_7^4$

$\left[\mathbb{I}_4^{1\prime}\right]_w = \left[\mathbb{I}_4^{1\prime}\right]_f = \left[\mathbb{I}_4^{1\prime}\right]_{w \vee f} = \left[\mathbb{I}_4^{1\prime}\right]_{w \wedge f} = \varnothing$

$\mathbb{R}\left[\mathbb{I}_4^{1\prime}\right]_w = \mathbb{R}\left[\mathbb{I}_4^{1\prime}\right]_f = \mathbb{R}\left[\mathbb{I}_4^{1\prime}\right]_{w \vee f} = \mathbb{R}\left[\mathbb{I}_4^{1\prime}\right]_{w \wedge f} = 0$

$(1,1,2,3) \Leftrightarrow [1235] = \{1235, 2346, 3457, 1456, 2567, 1367, 1247\}$

$\mathbb{W}_{[1235]} = 7, 7, 6, 8, 8, 5, 6$

$[1235]_w = \{2346, 3457, 1456, 2567\},$

$[1235]_f = \{1235, 2346, 3457, 1456, 2567, 1247\}$

$[1235]_{w \vee f} = [1235]_f, \ [1235]_{w \wedge f} = [1235]_w$

$\mathbb{F}[1235]_w = 4p_7^4 \qquad \mathbb{F}[1235]_f = 6p_7^4$

$\mathbb{F}[1235]_{w \vee f} = 6p_7^4 \qquad \mathbb{F}[1235]_{w \wedge f} = 4p_7^4$

$[1235']_w = \{1235, 1367, 1247\}, \qquad [1235']_f = \{1367\}$

$[1235']_{w \vee f} = [1235']_f, \ [1235']_{w \wedge f} = [1235']_w$

$\mathbb{R}[1235']_w = 3p_7^4 \qquad \mathbb{R}[1235']_f = p_7^4$

$\mathbb{R}[1235']_{w \vee f} = p_7^4 \qquad \mathbb{R}[1235']_{w \wedge f} = 3p_7^4$

$(1,1,3,2) \Leftrightarrow [1236] = \{1236, 2347, 1345, 2456, 3567, 1467, 1257\}$

$\mathbb{W}_{[1236]} = 6, 6, 7, 9, 7, 6, 7$

$[1236]_w = \{2347, 1345, 2456, 3567\}, \ [1236]_f = [1236]$

$[1236]_{w \vee f} = [1236]_f, \ [1236]_{w \wedge f} = [1236]_w$

$\mathbb{F}[1236]_w = 4p_7^4 \qquad \mathbb{F}[1236]_f = 7p_7^4$

$\mathbb{F}[1236]_{w \vee f} = 7p_7^4 \qquad \mathbb{F}[1236]_{w \wedge f} = 4p_7^4$

$[1236']_w = \{1236, 1467, 1257\}, \ [1236']_f = \varnothing$

$[1236']_{w \vee f} = \varnothing, \ [1236']_{w \wedge f} = [1236]_w$

$\mathbb{R}[1236']_w = 3p_7^4 \qquad \mathbb{R}[1236']_f = 0$

$\mathbb{R}[1236']_{w \vee f} = 0 \qquad \mathbb{R}[1236']_{w \wedge f} = 3p_7^4$

$(1,2,1,3) \Leftrightarrow [1245] = \{1245, 2356, 3467, 1457, 1256, 2367, 1347\}$

$\mathbb{W}_{[1245]} = 8, 8, 6, 7, 9, 7, 6, 7$

$[1245]_w = \{1457, 1256\}, \ [1245]_f = \{1245, 2356, 3467, 1457, 1256, 2367\}$

$[1245]_{w \vee f} = [1245]_f, \ [1245]_{w \wedge f} = [1245]_w$

$\mathbb{F}[1245]_w = 2p_7^4 \qquad \mathbb{F}[1245]_f = 6p_7^4$

$\mathbb{F}[1245]_{w \vee f} = 6p_7^4 \qquad \mathbb{F}[1245]_{w \wedge f} = 2p_7^4$

$[1245']_w = \{1245, 2356, 3467, 2367, 1347\}, \ [1245']_f = \{1347\}$

$[1245']_{w \vee f} = \{1347\}, \ [1245']_{w \wedge f} = [1245']_w$

$\mathbb{R}[1245']_w = 5p_7^4 \qquad \mathbb{R}[1245']_f = p_7^4$

$\mathbb{R}[1245']_{w \vee f} = p_7^4 \qquad \mathbb{R}[1245']_{w \wedge f} = 5p_7^4$

$(1,2,2,2) \Leftrightarrow [1246] = \{1246, 2357, 1346, 2457, 1356, 2467, 1357\}$

$\mathbb{W}_{[1246]} = 7, 7, 6, 8, 7, 7, 6$

$[1246]_w = \{2457, 1356\}, \ [1246]_f = [1246]$

$[1246]_{w \vee f} = [1246]_f, \ [1246]_{w \wedge f} = [1246]_w$

$\mathbb{F}[1246]_w = 2p_7^4 \qquad \mathbb{F}[1246]_f = 7p_7^4$

$\mathbb{F}[1246]_{w \vee f} = 7p_7^4 \qquad \mathbb{F}[1246]_{w \wedge f} = 2p_7^4$

$[1246']_w = \{1246, 2357, 1346, 2467, 1357\} \ \ [1246']_f = \varnothing,$

$[1246']_{w \vee f} = \varnothing, \ [1246']_{w \wedge f} = [1246']_w$

$\mathbb{R}[1246']_w = 5p_7^4 \qquad \mathbb{R}[1246']_f = 0$

$\mathbb{R}[1246']_{w \vee f} = 0 \qquad \mathbb{R}[1246']_{w \wedge f} = 5p_7^4$

For j=5.

$(1,1,1,1,3) \Leftrightarrow$

$\left[\mathbb{I}_5^1\right] = \{12345, 23456, 34567, 14567, 12567, 12367, 12347\}$

$\Rightarrow \mathbb{W}_{\left[\mathbb{I}_5^1\right]} = 9, 10, 9, 9, 9, 7, 7$

$\left[\mathbb{I}_5^1\right]_w = \left[\mathbb{I}_5^1\right]_f = \left[\mathbb{I}_5^1\right]_{w \vee f} = \left[\mathbb{I}_5^1\right]_{w \wedge f} = \left[\mathbb{I}_5^1\right]$

$\mathbb{F}\left[\mathbb{I}_5^1\right]_w = 7p_7^5 \qquad \mathbb{F}\left[\mathbb{I}_5^1\right]_f = 7p_7^5$

$\mathbb{F}\left[\mathbb{I}_5^1\right]_{w \vee f} = 7p_7^5 \qquad \mathbb{F}\left[\mathbb{I}_5^1\right]_{w \wedge f} = 7p_7^5$

$\left[\mathbb{I}_5^{1\prime}\right]_w = \left[\mathbb{I}_5^{1\prime}\right]_{w \vee f} = \left[\mathbb{I}_5^{1\prime}\right]_{w \wedge f} = \left[\mathbb{I}_5^{1\prime}\right] = \varnothing$

$\mathbb{R}\left[\mathbb{I}_5^{1\prime}\right]_w = \mathbb{R}\left[\mathbb{I}_5^{1\prime}\right]_f = \mathbb{R}\left[\mathbb{I}_5^{1\prime}\right]_{w \vee f} = \mathbb{R}\left[\mathbb{I}_5^{1\prime}\right]_{w \wedge f} = 7p_7^5$

$(1,1,1,2,2) \Leftrightarrow$

$[12346] = \{12346, 23457, 13456, 24567, 13567, 12467, 12357\}$

$\mathbb{W}_{[12346]} = 8, 9, 9, 10, 8, 8, 8$

$[12346]_w = [12346]_f = [12346]_{w \vee f} = [12346]_{w \wedge f} = [12346]$

$\mathbb{F}[12346]_w = 7p_7^5 \qquad \mathbb{F}[12346]_f = 7p_7^5$

$\mathbb{F}[12346]_{w \vee f} = 7p_7^5 \qquad \mathbb{F}[12346]_{w \wedge f} = 7p_7^5$

$[12346']_w = [12346']_f = [12346']_{w \vee f} = [12346']_{w \wedge f} = \varnothing$

$\mathbb{R}[12346']_w = \mathbb{R}[12346']_f = \mathbb{R}[12346']_{w \vee f} = \mathbb{R}[12346']_{w \wedge f} = 0$

$(1,1,2,1,2) \Leftrightarrow$

$[12356] = \{12356, 23467, 13457, 12456, 23567, 13467, 12457\}$

$\mathbb{W}_{[12356]} = 9, 8, 8, 10, 9, 7, 9$

$[12356]_w = \{12356, 23467, 13457, 12456, 23567, 12457\}$

$[12356]_f = [12356]$

$[12356]_{w \vee f} = [12356]_f, \ [12356]_{w \wedge f} = [12356]_w$

$\mathbb{F}[12356]_w = 6p_7^5 \qquad \mathbb{F}[12356]_f = 7p_7^5$

$\mathbb{F}[12356]_{w \vee f} = 7p_7^5 \qquad \mathbb{F}[12356]_{w \wedge f} = 6p_7^5$

$[12356']_w = \{13467\}, \ [12356']_f = [12356']_{w \vee f} = \varnothing$

$[12356']_{w \wedge f} = \{13467\}$

$\mathbb{R}[12356']_w = p_7^5, \ \mathbb{R}[12356']_f = 0,$

$\mathbb{R}[12356']_{w \vee f} = 0, \ \mathbb{R}[12356']_{w \wedge f} = p_7^5$

For j=6.

$$\left(1,1,1,1,1,2\right)\Leftrightarrow$$
$$\left[\mathbb{I}_6^1\right]=\{123456,234567,134567,124567,123567,123467,123457\}$$
$$\mathbb{W}_{[12346]}=11,11,10,11,10,9,10$$
$$\left[\mathbb{I}_6^1\right]_W=\left[\mathbb{I}_6^1\right]_f=\left[\mathbb{I}_6^1\right]_{W\vee f}=\left[\mathbb{I}_6^1\right]_{W\wedge f}=\left[\mathbb{I}_6^1\right]$$
$$\mathbb{F}\left[\mathbb{I}_6^1\right]_W=\mathbb{F}\left[\mathbb{I}_6^1\right]_f=\mathbb{F}\left[\mathbb{I}_6^1\right]_{W\vee f}=\mathbb{F}\left[\mathbb{I}_6^1\right]_{W\wedge f}=7p_7^6$$

$$\left[\mathbb{I}_6^{1\prime}\right]_W=\left[\mathbb{I}_6^{1\prime}\right]_f=\left[\mathbb{I}_6^{1\prime}\right]_{W\vee f}=\left[\mathbb{I}_6^{1\prime}\right]_{W\wedge f}=\varnothing$$
$$\mathbb{R}\left[\mathbb{I}_6^{1\prime}\right]_W=\mathbb{R}\left[\mathbb{I}_6^{1\prime}\right]_f=\mathbb{R}\left[\mathbb{I}_6^{1\prime}\right]_{W\vee f}=\mathbb{R}\left[\mathbb{I}_6^{1\prime}\right]_{W\wedge f}=0$$

For j=7.

$$\left(1,1,1,1,1,1,1\right)\Leftrightarrow\left[\mathbb{I}_7^1\right]=\{1234567\}\Rightarrow\mathbb{W}_{\left[\mathbb{I}_7^1\right]}=12$$
$$\left[\mathbb{I}_7^1\right]_W=\left[\mathbb{I}_7^1\right]_f=\left[\mathbb{I}_7^1\right]_{W\vee f}=\left[\mathbb{I}_7^1\right]_{W\wedge f}=\left[\mathbb{I}_7^1\right]$$
$$\mathbb{F}\left[\mathbb{I}_7^1\right]_W=\mathbb{F}\left[\mathbb{I}_7^1\right]_f=\mathbb{F}\left[\mathbb{I}_7^1\right]_{W\vee f}=\mathbb{F}\left[\mathbb{I}_7^1\right]_{W\wedge f}=7p_7^6$$
$$\left[\mathbb{I}_7^{1\prime}\right]_W=\left[\mathbb{I}_7^{1\prime}\right]_f=\left[\mathbb{I}_7^{1\prime}\right]_{W\vee f}=\left[\mathbb{I}_7^{1\prime}\right]_{W\wedge f}=\varnothing$$
$$\mathbb{R}\left[\mathbb{I}_7^{1\prime}\right]_W=\mathbb{R}\left[\mathbb{I}_7^{1\prime}\right]_f=\mathbb{R}\left[\mathbb{I}_7^{1\prime}\right]_{W\vee f}=\mathbb{R}\left[\mathbb{I}_7^{1\prime}\right]_{W\wedge f}=0$$

$$\mathbb{F}_W^C=\sum_{t=1}^{s}\sum_{Y\in[X_t]_W}F(Y)=p_7^7+7p_7^6+20p_7^5+19p_7^4+10p_7^3+2p_7^2$$
$$\mathbb{F}_{W\wedge f}^C=\sum_{t=1}^{s}\sum_{Y\in[X_t]_{W\wedge f}}F(Y)=p_7^7+7p_7^6+20p_7^5+19p_7^4+9p_7^3$$

2. For nonconsecutive failures i.e. Θ_C^2 ,

$7=\sum_{i=1}^{j}d_i, d_i>1:\forall i$, and for j=0 to M=3

$$\Theta_C^2=\{[\varnothing],[1],[13],[135]\}\qquad W=\{1,2,1,2,3,2,1\}$$
$$[\varnothing]=\{\varnothing\}\Rightarrow\mathbb{W}_{[1]}=\{0\}$$
$$[\varnothing]_f=\varnothing,\ [\varnothing']_W=[\varnothing']_{W\wedge f}=[\varnothing]$$
$$[\varnothing]_f=[\varnothing']_f=\{\ \}$$
$$\mathbb{F}[\varnothing]_f=0,\ \mathbb{R}[\varnothing]_f=\mathbb{R}[\varnothing']_W=\mathbb{R}[\varnothing']_{W\wedge f}=p_7^7$$

$$[1]=\{1,2,3,4,5,6,7\}\Rightarrow\mathbb{W}_{[1]}=1,2,1,2,3,2,1\Rightarrow$$
$$[1]_f=\{\ \}\Rightarrow\mathbb{F}[1]_f=\mathbb{F}[1]_{W\vee f}=0$$
$$[1']_W=\{\ \},\ [1']_f=\{\ \},\ [1']_{W\vee f}=\{\ \},\ [1']_{W\wedge f}=\{\ \}\Rightarrow$$
$$\mathbb{R}[1]_W=\mathbb{R}[1]_f=\mathbb{R}[1]_{W\vee f}=\mathbb{R}[1]_{W\wedge f}=p_7^7$$

$$(2,5)\Leftrightarrow[13]=\{13,24,35,46,57,16,27\}$$
$$\mathbb{W}_{[13]}=2,4,4,4,4,3,3$$
$$[13]_f=\{\ \}\Rightarrow\mathbb{F}[13]_f=\mathbb{F}[13]_{W\vee f}=0$$
$$[13']_W=\{\ \},\ [13']_f=\{\ \},$$
$$[13']_{W\vee f}=\{\ \},\ [13']_{W\wedge f}=\{\ \}\Rightarrow$$
$$\mathbb{R}[13']_W=\mathbb{R}[13']_f=\mathbb{R}[13']_{W\vee f}=\mathbb{R}[13']_{W\wedge f}=7p_7^5$$

$$(2,2,3)\Leftrightarrow[135]=\{135,246,357,146,257,136,247\}$$
$$\mathbb{W}_{[135]}=5,6,5,5,6,4,5$$
$$[135]_f=\{246,257\}\Rightarrow\mathbb{F}[135]_f=\mathbb{F}[135]_{W\vee f}=2p_7^3$$
$$[135']_W=[135],\ [135']_f=\{135,357,146,136,247\}$$
$$[135']_{W\vee f}=\{135,357,146,136,247\},\ [135']_{W\wedge f}=[135]$$
$$\mathbb{R}[135']_W=7p_7^3,\ \mathbb{R}[135']_f=5p_7^3$$
$$\mathbb{R}[135']_{W\vee f}=5p_7^3,\ \mathbb{R}[135']_{W\wedge f}=7p_7^3$$

$$\mathbb{F}_f^C=\sum_{t=1}^{r}\sum_{Y\in[X_t]_f}F(Y)=p_7^7+7p_7^6+21p_7^5+33p_7^4+14p_7^3$$
$$\mathbb{F}_{W\vee f}^C=\sum_{t=1}^{s}\sum_{Y\in[X_t]_{W\vee f}}F(Y)+\sum_{t=s+1}^{r}\sum_{Z\in[X_t]_f}F(Z)$$
$$=p_7^7+7p_7^6+21p_7^5+33p_7^4+15p_7^3+2p_7^2$$
$$\mathbb{R}_W^C=p_7^0+7p_7^1+12p_7^2+25p_7^3+16p_7^4+p_7^5$$
$$\mathbb{R}_f^C=p_7^0+7p_7^1+14p_7^2+21p_7^3+2p_7^4$$
$$\mathbb{R}_{W\vee f}^C=p_7^0+7p_7^1+12p_7^2+20p_7^3+2p_7^4$$
$$\mathbb{R}_{W\wedge f}^C=p_7^0+7p_7^1+14p_7^2+26p_7^3+16p_7^4+p_7^5$$

5. Conclusion

In this paper a new algorithm to find the reliability and the failure functions of the consecutive weighted k-out-of-n: F linear and circular system, the weighted f-out-of-n: F system, the weighted (n,f,k) linear and circular system, and the weighted $\langle n,f,k\rangle$ linear and circular system is obtained. The keystone of this, is defining the space of consecutive and non-consecutive failures of all weighted systems.

References

[1] Chang C. J., Chen R. J. and Hwang F. K., "A fast reliability algorithm for the circular weighted-k-out-of-n: F system", *IEEE Transactions on Reliability*, Vol. 47, pp.472-474, 1998. https://doi.org/10.1109/24.756092.

[2] Chang, G. J., Lirong C. and K. H., "Reliabilities for (n; f; k) systems", Statistics and Probability Letters, Vol. 43, pp. 237-242, 1999. https://doi.org/10.1016/S0167-7152(98)00263-6.

[3] Chao M. T., Fu J. C. and Koutras M. V. "Survey of reliability studies of consecutive k-out-of-n: F and related system", IEEE Transactions on Reliability, Vol. 44, No. 1, pp. 120-127, 1995. https://doi.org/10.1109/24.376531.

[4] Chiang, D. T., and Niu, S.-C., "Reliability of consecutive k-out-of-n: F system" ", IEEE Transactions on Reliability, Vol. 30, No.1, pp. 87-89, 1981. https://doi.org/10.1109/TR.1981.5220981.

[5] Chiang, D. T., and Niu, S.-C., "Reliability of consecutive k-out-of-n: F system" ", IEEE Transactions on Reliability, Vol. 30, No.1, pp. 87-89, 1981. https://doi.org/10.1109/TR.1981.5220981.

[6] Derman C, Lieberman G.J., and Ross S.M. "On the consecutive k-out-of-n: F system", IEEE Transactions on Reliability, Vol. 31, pp. 57-63, 1982. https://doi.org/10.1109/TR.1982.5221229.

[7] Erylmaz, S. and Tutuncu G. Y., "Reliability evaluation of linear consecutive weighted-k-out-of-n: F system, Asia-Pacific Journal of Operational Research, in press, Vol. 26, pp. 805-816, 2009. https://doi.org/10.1142/S0217595909002481.

[8] Eryilmaz S., Aksoy T., "Reliability of linear (n, f, k) systems with weighted components", Journal of Systems Science and Systems Engineering, Vol. 19, pp. 277-284, 2010. https://doi.org/10.1007/s11518-010-5131-0.

[9] Kamalja K. K. and Shinde R. L., "On the reliability of $(n,f,k)(\langle n,f,k\rangle)$ systems", Communication in Statistics-Theory and Methods, Vol. 43, pp. 1649-1665, 2014. https://doi.org/10.1080/03610926.2012.673674.

[10] Nashwan I. I. H, "Reliability of some consecutive system", Ph. D thesis, Ain Shams University, pp. 65-70, 2012.

[11] Nashwan, I. I. H, "New algorithms to find reliability and unreliability functions of the consecutive k-out-of-n: F linear & circular System", The International Arab Conference on Information Technology, ACIT 2015.

[12] Sfakianakis, M. E., and Papastavridis S. G." Reliability of a general consecutive k-out-of-n: F System", IEEE Transactions on Reliability, Vol. 42, pp. 491-496, 1993. https://doi.org/10.1109/24.257837.

[13] Tung, S. S., "Combinatorial analysis in determining reliability", Proceedings of annual Reliability & Maintainability Symposium, pp. 262-266, 1982.

[14] Way, k. Zou M., "optimal reliability modeling principals and applications", JOHN WILEY & SONS, pp. 231-232, 2003.

[15] Wu, J. and Chen R., "An algorithm for computing the reliability of weighted-k-out-of-n systems, IEEE Transactions on Reliability, Vol. 43, 327-328, 1994. Wu J. and Chen R., "Efficient Algorithms for k-

out-of-*n* & Consecutive-Weighted-*k*-out-of-*n*: F System", IEEE Transaction on Reliability, Vol. 43, pp. 650-655, 1994.

[16] Zuo, M.J., Lin, D. & Wu, Y., "Reliability evaluation of combined *k*-out-of-*n*: F, consecutive *k*-out-of-*n*: F, and linear connected (*r*,*s*)-out-of-(*m*,*n*): F system structures", IEEE Transactions on Reliability, Vol. 49, pp. 99–104, 2000. https://doi.org/10.1109/24.855542.