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Research paper

Conservation law and exact solutions for ion acoustic waves in a magnetized quantum plasma

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Abstract

The head on collision of ion- acoustic solitary waves (IASWs) in a magnetized plasma are considered. The two- sides Korteweg-de Vries (KdV) equations in generic case as well as the two- sides modified Korteweg-de Vries (mKdV) equations in a special case are obtained. The analytical phase shifts and the trajectories after the Head-on collisions of two IASWs in a three species quantum plasma are derived by using the extended version of Poincaré-Lighthill-Kuo (PLK) method for both the situations. The conservation laws for KdV and mKdV equations are obtained. By applying the extended direct algebraic method, we found the traveling wave solutions for the two-sides KdV and mKdV equations.

Keywords: *Head-on collision. Ion- acoustic solitary waves. Magnetized quantum plasma. PLK method. Direct algebraic method. KdV and mKdV equations*

1. Introduction

Study of nonlinear waves in plasma is one of the hottest topics in plasma physics, specifically the dusty plasma. A dusty plasma is ordinary plasma with embedded solid microparticles. In dusty plasma, there are essential two types of acoustic waves: high frequency dust ion acoustic wave (DIAWs) involving mobile ions and static dust grains, and a low frequency dust acoustic wave (DAWs) involving mobile dust grains. Both modes have been studied theoretically and experimentally [1-9]. Dusty Plasmas are present in astrophysical and space environments, like the tails of comets, interstellar medium and planetary rings [4-5]. The reason for their presence in these regions is the presence of high-density dust grains different types of plasma wave modes. Ref. [6] was the first report recorded theoretically the existence of DIA waves. The study showed that an unmagnetized weakly coupled dusty supports the DIAW whose phase velocity is much smaller (larger) than the electron (ion) thermal speed. The frequency of DIAWs is munch larger (smaller) than the dust (ion) plasma frequency. At the same time, Rao et al. [7] theoretically predicted the existence of DAWs, which is provided by the inertia dust particles mass and are providing power restoration due to pressures of less inertia of electrons and ion. The DAWs have supported a great deal of interest in understanding the basic characteristics and properties of local electrostatic perturbations in space and laboratory dusty plasmas.

Recently numerous investigations have been made to observe the head-on collision of two, electron-acoustic [10-11], DA [12-13], DIA [14-15], IA [16-17] solitary waves. Few theoretical investigations have already been made by some researcher on magnetized as well as un magnetized quantum plasmas. For instance,[18] have been studied the characteristics of head-on collision between two quantum ion- acoustic solitary waves (QIASWs) in a dense electrons, positrons and classical ions plasma. Using the extended version of PLK method they derived the two sided KdV equations and the phase shifts after collision. Later on [19] have been examined the propagation and interaction of ion- acoustic solitary waves in a quantum electron-positron-ion plasma. We have a theoretical investigation about the head-on collision of ion-acoustic solitary waves in magnetized quantum dusty plasmas [20] and we observe that the quantum diffraction parameter, the ion cyclotron frequency and the density ratio of electrons to ions have significant effects on phase shifts. Several authors. cited above on head- on collision, the authors are interested to find the KdV solitons with their phase shifts and the effects of different parameters on it, avoiding the detail discussion on the critical composition, but after the detail discussion on the critical composition, but after the poineer work of [21-22], there have been a surge of interest to deduce the mKdV solitons also for the critical composition, where the cubic nonlinearity rather than the quadratic nonlinearity of the KdV equations will appear in the evolution equations. It is important to note here that [23] have been investigated ion-acoustic shock waves and their head-on collision in a dense electron-positron-ion quantum plasma [10].

Nonlinear partial differential equations (NLPDEs) can describe certain phenomena in plasma physics, fluid mechanics and other fields. Methods used to construct the analytic and exact solutions of the NLPDEs have been proposed, such as the Hirota bilinear method [33], Bäcklund transformation (BT) [34-35], inverse scattering transform [36], and Painlevé analysis [37-39]. BT is the connection of several analytic solutions and auto-BT can connect the different solutions of the same NLPDE [40-43]. In Refs.[41-44], the homogeneous balance (HB) method was improved to investigate the BT, Lax pairs, symmetries, and exact solutions for some nonlinear

PDEs [45]. He also showed that there is a close connection between the HB method and WTC method. By using his extended homogeneous balance method, the BTs of many nonlinear PDEs have been successfully obtained.
The paper is organized as follows: The introduction is presented in Sec. 1. In Sec. 2, the basic equations are considered and the problem formulation is derived by using the PLK method. In Sec. 3, the conservation laws of the typical KdV and mKdV equations are obtained. In Sec. 4, the stability condition is presented. In Sec. 5, the exact solutions of the typical KdV and typical mKdV are obtained by using an extended homogeneous balance method. Finally, the conclusion is presented in section 6.

2. Problems formulations

We consider a three species dense quantum plasma composed of electrons, positrons and singly charged positive ions. The plasma is considered to be in the uniform external magnetic field $\mathbf{B}_0 = B_0\hat{z}$, where B_0 is the strength of the magnetic field and $\hat{z}$ is the unit vector in the z-direction. The nonlinear dynamics of ion acoustic waves in space in such a magnetized quantum plasma system is described by the following set of dimensionless equations [31].

$$\frac{\partial n_i}{\partial t} + \nabla.(n_i u_i) = 0, \tag{1}$$

$$\frac{\partial u_i}{\partial t} + u_i.\nabla.u_i = -\nabla\phi + u_i \times \hat{z}, \tag{2}$$

$$\Omega\nabla^2\phi = n_e - n_p - n_i, \tag{3}$$

$$n_e = \mu_e\left(1 + 2\phi + H_e^2\frac{\nabla^2\sqrt{n_e}}{\sqrt{n_e}}\right)^{1/2}, \tag{4}$$

$$n_p = \mu_p\left(1 - 2\sigma\phi + \sigma H_e^2\frac{\nabla^2\sqrt{n_p}}{\sqrt{n_p}}\right)^{1/2}. \tag{5}$$

Here n_i, u_i, m_i and ϕ are the ion number density with the equilibrium value n_{i0}, the ion mass, and the electrostatic potential respectively. $\sigma = T_{Fe}/T_{Fp}$ is the electron to positron Fermi temperature ratio, $H_e = eB_0\hbar/2c\sqrt{m_i m K_B T_{Fe}}$ is the quantum diffraction parameter, $\omega_{ci} = eB_0/m_i c$ and $\omega_{pi} = \sqrt{4\Pi e^2 n_{i0}/m_i}$ are the ion gyrofrequency and the ion plasma frequency, $\Omega = \omega_{ci}^2/\omega_{pi}^2$ where $\hbar$ is the Plank constant divided by 2π, c is the speed of light in vacuum. The electron/positrons are considered to be degenerate owing their small mass relative to the ions. The physical quantities in Eqs. (1)-(5) have been appropriately normalized by the transformations $n_{e,p,i} \to n_{e,p,i}/n_{i0}, u_i \to u_i/C_s, t \to t\omega_{ci}, \nabla \to \nabla C_s/\omega_{ci}$ and $\phi \to e\phi/2K_B T_{Fe}$ with the ion- sound Fermi speed $C_s = \sqrt{2K_B T_{Fe}/m_i}$. According to PLK method the dependent variables are expanded in different powers of ε in the form

$$\psi = \psi_0 + \sum \varepsilon^n \psi^{(n)}, \tag{6}$$

where $\psi = (n_e, n_i, n_p, u_{ix}, u_{iy}, u_{iz}, \phi)$ with $\psi_0 = (\mu_e, 1, \mu_p, 0, 0, 0, 0)$. We introduce the stretched independent variables as

$$\xi = \varepsilon\big(l_x x + l_y y + l_z z - \lambda t\big) + \varepsilon^2 P(\xi, \eta, \tau) + ...,$$

$$\eta = \varepsilon\big(l_x x + l_y y + l_z z + \lambda t\big) + \varepsilon^2 Q(\xi, \eta, \tau) + ...,$$

$$\tau = \varepsilon^3 t. \tag{7}$$

Where ξ and η denote the trajectories of the two solitary waves during head-on collision in the direction having directional velocities (l_x, l_y, l_z) with equal but opposite directional velocities. Introducing the operators

$$\hat{X} = \frac{\partial}{\partial_\xi} + \frac{\partial}{\partial_\eta}, \quad \hat{X}' = \Big(\frac{\partial p}{\partial_\xi} + \frac{\partial p}{\partial_\eta}\Big)\frac{\partial}{\partial_\xi} + \Big(\frac{\partial Q}{\partial_\xi} + \frac{\partial Q}{\partial_\eta}\Big)\frac{\partial}{\partial_\eta}, \tag{8}$$

$$\hat{T} = \frac{\partial}{\partial_\xi} - \frac{\partial}{\partial_\eta}, \quad \hat{T}' = \lambda\Big(\frac{\partial p}{\partial_\xi} - \frac{\partial p}{\partial_\eta}\Big)\frac{\partial}{\partial_\xi} + \lambda\Big(\frac{\partial Q}{\partial_\xi} - \frac{\partial Q}{\partial_\eta}\Big)\frac{\partial}{\partial_\eta}. \tag{9}$$

We write operators $\partial/\partial_{x,y,z}$ and ∂/∂_t in compact from as follows

$$\frac{\partial}{\partial_{x,y,z}} = l_{x,y,z}\big(\varepsilon\hat{X} + \varepsilon^3\hat{X}' + ...\big), \quad \frac{\partial}{\partial_t} = \lambda\varepsilon\hat{T} + \lambda\varepsilon^3\hat{T}' + \varepsilon^3\frac{\partial}{\partial_t} + ... \tag{10}$$

and by using the version of PLK method, from Eqs. (1)-(5) the two-sides KdV equations can be derived as

$$\frac{\partial\phi_\xi^{(2)}}{\partial_t} + A\phi_\xi^{(2)}\frac{\partial\phi_\xi^{(2)}}{\partial\xi} + B\frac{\partial^3\phi_\xi^{(2)}}{\partial\xi^3} = 0,$$

$$\frac{\partial\phi_\eta^{(2)}}{\partial_t} - A\phi_\eta^{(2)}\frac{\partial\phi_\eta^{(2)}}{\partial t} - B\frac{\partial^3\phi_\eta^{(2)}}{\partial\eta^3} = 0, \tag{11}$$

with

$$A = \frac{\lambda^3}{2l_z^2}\left(\mu_e - \sigma^2\mu_p + \frac{3l_z^4}{\lambda^4}\right),$$

$$B = \frac{\lambda^3}{2l_z^2}\left[1 + \Omega - l_z^2 - \frac{H_e^2}{4\mu_e\mu_p}(\mu_p + \sigma^2\mu_e)\right]. \tag{12}$$

And at the critical composition when $A = 0$, the quadratic non linearty in the KdV Eq. (11) will disappear and the cubic nonlinearity will appear. we can cancel (Verheest et al. 2012a) the solutions of the linear operator without loss of generality, hence,

$$\phi_\xi^{(2)} = \phi_\eta^{(2)} = 0 \tag{13}$$

then the two-sides modified Korteweg-de Vries (mKdV) equations can be derived as

$$\frac{\partial\phi_\xi^{(1)}}{\partial t} + A_1(\phi_\xi^{(1)})^2\frac{\partial\phi_\xi^{(1)}}{\partial\xi} + B\frac{\partial^3\phi_\xi^{(1)}}{\partial\xi^3} = 0,$$

$$\frac{\partial\phi_\eta^{(1)}}{\partial t} + A_1(\phi_\eta^{(1)})^2\frac{\partial\phi_\eta^{(1)}}{\partial\eta} - B\frac{\partial^3\phi_\eta^{(1)}}{\partial\eta^3} = 0, \tag{14}$$

with

$$A_1 = \frac{\lambda^3}{2l_z^2}\left[\frac{15l_z^6}{2\lambda^6} - \frac{3(\mu_e + \sigma^3\mu_p)}{2}\right] \tag{15}$$

3. Conservation law for typical KdV-mKdV equations

Example 1. Consider the typical KdV equations (11)

$$E_1 = \frac{\partial\phi_\xi^{(2)}}{\partial_t} + A\phi_\xi^{(2)}\frac{\partial\phi_\xi^{(2)}}{\partial\xi} + B\frac{\partial^3\phi_\xi^{(2)}}{\partial\xi^3} = 0,$$

$$E_2 = \frac{\partial\phi_\eta^{(2)}}{\partial_t} - A\phi_\eta^{(2)}\frac{\partial\phi_\eta^{(2)}}{\partial t} - B\frac{\partial^3\phi_\eta^{(2)}}{\partial\eta^3} = 0. \tag{16}$$

The determining equations for multipliers of the form $\alpha\left(\phi_\xi^{(2)}, \phi_\eta^{(2)}, \tau, \xi, \eta\right)$ and $\lambda\left(\phi_\xi^{(2)}, \phi_\eta^{(2)}, \tau, \xi, \eta\right)$

$$\frac{\delta}{\delta\phi_{\xi}^{(2)}}\left[\alpha\left(\frac{\partial\phi_{\xi}^{(2)}}{\partial_t}+A\phi_{\xi}^{(2)}\frac{\partial\phi_{\xi}^{(2)}}{\partial\xi}+B\frac{\partial^3\phi_{\xi}^{(2)}}{\partial\xi^3}\right)+\lambda\left(\frac{\partial\phi_{\eta}^{(2)}}{\partial_t}\right.\right.$$

$$\left.\left.-A\phi_{\eta}^{(2)}\frac{\partial\phi_{\eta}^{(2)}}{\partial t}-B\frac{\partial^3\phi_{\eta}^{(2)}}{\partial\eta^3}\right)\right]=0, \tag{17}$$

$$\frac{\delta}{\delta\phi_{\eta}^{(2)}}\left[\alpha\left(\frac{\partial\phi_{\xi}^{(2)}}{\partial_t}+A\phi_{\xi}^{(2)}\frac{\partial\phi_{\xi}^{(2)}}{\partial\xi}+B\frac{\partial^3\phi_{\xi}^{(2)}}{\partial\xi^3}\right)+\lambda\left(\frac{\partial\phi_{\eta}^{(2)}}{\partial_t}\right.\right.$$

$$\left.\left.-A\phi_{\eta}^{(2)}\frac{\partial\phi_{\eta}^{(2)}}{\partial t}-B\frac{\partial^3\phi_{\eta}^{(2)}}{\partial\eta^3}\right)\right]=0. \tag{18}$$

Expansion of (17) and (18) yields

$$\alpha_{\phi_{\xi}^{(2)}}\left(\frac{\partial\phi_{\xi}^{(2)}}{\partial_t}+A\phi_{\xi}^{(2)}\frac{\partial\phi_{\xi}^{(2)}}{\partial\xi}+B\frac{\partial^3\phi_{\xi}^{(2)}}{\partial\xi^3}\right)+\lambda_{\phi_{\xi}^{(2)}}\left(\frac{\partial\phi_{\eta}^{(2)}}{\partial_t}-A\phi_{\eta}^{(2)}\frac{\partial\phi_{\eta}^{(2)}}{\partial t}\right.$$

$$\left.-B\frac{\partial^3\phi_{\eta}^{(2)}}{\partial\eta^3}\right)+a\alpha\phi_{\xi}^{(2)}-D_{\xi}(a\alpha\phi_{\xi}^{(2)})-D_{\tau}(\alpha)-D_{\xi}^3(b\alpha)=0, \tag{19}$$

and

$$\alpha_{\phi_{\eta}^{(2)}}\left(\alpha\frac{\partial\phi_{\xi}^{(2)}}{\partial_t}+A\phi_{\xi}^{(2)}\frac{\partial\phi_{\xi}^{(2)}}{\partial\xi}+B\frac{\partial^3\phi_{\xi}^{(2)}}{\partial\xi^3}\right)+\lambda_{\phi_{\eta}^{(2)}}\left(\frac{\partial\phi_{\eta}^{(2)}}{\partial_t}-A\phi_{\eta}^{(2)}\frac{\partial\phi_{\eta}^{(2)}}{\partial t}\right.$$

$$\left.-B\frac{\partial^3\phi_{\eta}^{(2)}}{\partial\eta^3}\right)+a\alpha_{\phi_{\eta}}^{(2)}-D_{\eta}(a\alpha_{\phi_{\xi}}^{(2)})-D_{\tau}(\alpha)-D_{\eta}^3(b\alpha)=0. \tag{20}$$

Eqs. (19) and (20) are separated according to different combination of derivatives of $\phi_{\xi}^{(2)}$ and $\phi_{\eta}^{(2)}$ and after some simplification following system of equations for α and λ are obtained,

$$\alpha_{\phi_{\xi}^{(2)}\phi_{\xi}^{(2)}}=0,\quad \alpha_{\phi_{\xi}^{(2)}\phi_{\eta}^{(2)}}=0,$$

$$\alpha_{\phi_{\eta}^{(2)}\phi_{\eta}^{(2)}}=0,\quad \alpha_{\phi_{\xi}^{(2)}}-\lambda_{\phi_{\eta}^{(2)}}=0,\quad \alpha_{\phi_{\eta}^{(2)}}-\lambda_{\phi_{\xi}^{(2)}}=0,$$

$$\alpha_{\tau}+a\phi_{\xi}^{(2)}\alpha_{\xi}+b\alpha_{\xi\xi\xi}=0,\quad \lambda_{\phi_{\xi}^{(2)}\phi_{\xi}^{(2)}}=0,\quad \lambda_{\phi_{\xi}^{(2)}\phi_{\eta}^{(2)}}=0,$$

$$\lambda_{\phi_{\eta}^{(2)}\phi_{\eta}^{(2)}}=0,\quad -\lambda_{\tau}+a\phi_{\eta}^{(2)}\alpha_{\eta}+b\alpha_{\eta\eta\eta}=0, \tag{21}$$

and by solving system yield

$$\alpha=c_1+a\phi_{\xi}^{(2)}(c_3+c_4\tau)-\xi c_4, \tag{22}$$

$$\lambda=c_2+a\phi_{\eta}^{(2)}(c_3+c_4\tau)+\eta c_4, \tag{23}$$

where c_1,c_2,c_3 and c_4 are constants

$$T_1^1=\phi_{\xi}^{(2)},\quad T_1^2=\phi_{\eta}^{(2)},\quad T_1^2=a\frac{(\phi_{\xi}^{(2)})^2}{2}+b(\phi_{\xi}^{(2)})_{\xi\xi},$$

$$T_2^2=-a\frac{(\phi_{\eta}^{(2)})^2}{2}+b(\phi_{\xi}^{(2)})_{\eta\eta}. \tag{24}$$

Example 2. Consider the typical mKdV Eq. (14)

$$E_1=\frac{\partial\phi_{\xi}^{(1)}}{\partial t}+A_1(\phi_{\xi}^{(1)})^2\frac{\partial\phi_{\xi}^{(1)}}{\partial\xi}+B\frac{\partial^3\phi_{\xi}^{(1)}}{\partial\xi^3}=0,$$

$$E_2=\frac{\partial\phi_{\eta}^{(1)}}{\partial t}+A_1(\phi_{\eta}^{(1)})^2\frac{\partial\phi_{\eta}^{(1)}}{\partial\eta}-B\frac{\partial^3\phi_{\eta}^{(1)}}{\partial\eta^3}=0, \tag{25}$$

the determining equations for multipliers of the form $\alpha(\phi_{\xi}^{(1)},\phi_{\eta}^{(1)},\tau,\xi,\eta)$ and $\lambda(\phi_{\xi}^{(1)},\phi_{\eta}^{(1)},\tau,\xi,\eta)$

$$\frac{\delta}{\delta\phi_{\xi}^{(1)}}\left[\alpha\left(\frac{\partial\phi_{\xi}^{(1)}}{\partial t}+A_1(\phi_{\xi}^{(1)})^2\frac{\partial\phi_{\xi}^{(1)}}{\partial\xi}+B\frac{\partial^3\phi_{\xi}^{(1)}}{\partial\xi^3}\right)\right.$$

$$\left.+\lambda\left(\frac{\partial\phi_{\eta}^{(1)}}{\partial t}+A_1(\phi_{\eta}^{(1)})^2\frac{\partial\phi_{\eta}^{(1)}}{\partial\eta}-B\frac{\partial^3\phi_{\eta}^{(1)}}{\partial\eta^3}\right)\right]=0, \tag{26}$$

$$\frac{\delta}{\delta\phi_{\eta}^{(1)}}\left[\alpha\left(\frac{\partial\phi_{\xi}^{(1)}}{\partial t}+A_1(\phi_{\xi}^{(1)})^2\frac{\partial\phi_{\xi}^{(1)}}{\partial\xi}+B\frac{\partial^3\phi_{\xi}^{(1)}}{\partial\xi^3}\right)\right.$$

$$\left.+\lambda\left(\frac{\partial\phi_{\eta}^{(1)}}{\partial t}+A_1(\phi_{\eta}^{(1)})^2\frac{\partial\phi_{\eta}^{(1)}}{\partial\eta}-B\frac{\partial^3\phi_{\eta}^{(1)}}{\partial\eta^3}\right)\right]=0. \tag{27}$$

Expansion of (26)and (27) yields

$$\alpha_{\phi_{\xi}^{(1)}}\left(\frac{\partial\phi_{\xi}^{(1)}}{\partial t}+A_1(\phi_{\xi}^{(1)})^2\frac{\partial\phi_{\xi}^{(1)}}{\partial\xi}+B\frac{\partial^3\phi_{\xi}^{(1)}}{\partial\xi^3}\right)$$

$$+\lambda_{\phi_{\xi}^{(1)}}\left(\partial\phi_{\eta}^{(1)}\partial t+A_1(\phi_{\eta}^{(1)})^2\frac{\partial\phi_{\eta}^{(1)}}{\partial\eta}-B\frac{\partial^3\phi_{\eta}^{(1)}}{\partial\eta^3}\right)$$

$$+2a_1\alpha\phi^{(1)}\phi_{\xi}^{(1)}\partial_{\xi}\phi_{\xi}^{(1)}-D_{\xi}(a_1\alpha(\phi_{\xi}^{(1)})^2)-D_{\tau}(\alpha)-D_{\xi}^3(b\alpha)=0, \tag{28}$$

and

$$\alpha_{\phi_{\eta}^{(1)}}\left(\frac{\partial\phi_{\xi}^{(1)}}{\partial t}+A_1(\phi_{\xi}^{(1)})^2\frac{\partial\phi_{\xi}^{(1)}}{\partial\xi}+B\frac{\partial^3\phi_{\xi}^{(1)}}{\partial\xi^3}\right)$$

$$+\lambda_{\phi_{\eta}^{(1)}}\left(\frac{\partial\phi_{\eta}^{(1)}}{\partial t}+A_1(\phi_{\eta}^{(1)})^2\frac{\partial\phi_{\eta}^{(1)}}{\partial\eta}-B\frac{\partial^3\phi_{\eta}^{(1)}}{\partial\eta^3}\right)$$

$$+2a_1\alpha\phi_{\eta}^{(1)}\partial_{\eta}\phi_{\eta}^{(1)}-D_{\eta}(a_1\alpha(\phi_{\xi}^{(1)})^2)-D_{\tau}(\alpha)-D_{\eta}^3(b\alpha)=0. \tag{29}$$

Eqs. (28) and (29) are separated according to different combination of derivatives of $\phi_{\xi}^{(1)}$ and $\phi_{\eta}^{(1)}$ and after some simplification following system of equations for α and λ are obtained:

$$\alpha_{\phi_{\xi}^{(1)}\phi_{\xi}^{(1)}}=0,\quad \alpha_{\phi_{\xi}^{(1)}\phi_{\eta}^{(1)}}=0,\quad \alpha_{\phi_{\eta}^{(1)}\phi_{\eta}^{(1)}}=0,\quad \alpha_{\phi_{\xi}^{(1)}}-\lambda_{\phi_{\eta}^{(1)}}=0,$$

$$\alpha_{\phi_{\eta}^{(1)}}-\lambda_{\phi_{\xi}^{(1)}}=0,\quad \alpha_{\tau}+a_1(\phi_{\xi}^{(1)})^2\alpha_{\xi}+b\alpha_{\xi\xi\xi}=0,$$

$$\lambda_{\phi_{\xi}^{(1)}\phi_{\xi}^{(1)}}=0,\quad \lambda_{\phi_{\xi}^{(1)}\phi_{\eta}^{(1)}}=0,\quad \lambda_{\phi_{\eta}^{(1)}\phi_{\eta}^{(1)}}=0,$$

$$-\lambda_{\tau}+a_1(\phi_{\eta}^{(1)})^2\alpha_{\eta}+b\alpha_{\eta\eta\eta}=0, \tag{30}$$

and by solving system yield

$$\alpha=c_1+2a_1(\phi_{\xi}^{(1)})^2(c_3+c_4\tau)-\xi c_4, \tag{31}$$

$$\lambda=c_2+2a_1(\phi_{\eta}^{(1)})^2(c_3+c_4\tau)+\eta c_4. \tag{32}$$

Where c_1,c_2,c_3 and c_4 are constants

$$T_1^1=\phi_{\xi}^{(1)},\quad T_1^2=\phi_{\eta}^{(1)},\quad T_1^2=a_1\frac{(\phi_{\xi}^{(1)})^3}{3}+b(\phi_{\xi}^{(1)})_{\xi\xi},$$

$$T_2^2=a_1\frac{(\phi_{\eta}^{(1)})^3}{3}+b(\phi_{\xi}^{(1)})_{\eta\eta}. \tag{33}$$

4. Stability analysis

The momentum for Eqs. (11) and (14) are given by

$$M=\frac{1}{2}\int_{-\infty}^{\infty}\phi^2d\xi+\frac{1}{2}\int_{-\infty}^{\infty}\phi^2d\eta \tag{34}$$

where M is the momentum and ϕ is the electric field potential. The sufficient condition for soliton stability is

$$\frac{\partial M}{\partial\omega}>0 \tag{35}$$

where ω is the frequency

5. Direct algebraic function method and Exact solutions

Example 1. typical KdV equations

We will find the solutions of the two- sides KdV equations, by applying the direct algebraic function method. By take the transformation $\phi^{(2)}(\xi,\tau)=\phi^{(2)}(U)$ where $U=(k\xi+\omega\tau)$ and $\phi^{(2)}(\eta,\tau)=\phi^{(2)}(V)$ where $V=(k\eta+\omega\tau)$ then equation (11) becomes

$$\omega(\phi_U^{(2)})'+ka\phi_U^{(2)}(\phi_U^{(2)})'+bk^3(\phi_u^{(2)})'''=0, \tag{36}$$

$$\omega(\phi_V^{(2)})' - ka\phi_V^{(2)}(\phi_V^{(2)})' - bk^3(\phi_V^{(2)})''' = 0. \quad (37)$$

Integrating Eqs. (36)-(37) with respect to U and V and letting the integrating constant to be zero, we have

$$\omega\phi_U^{(2)} + \frac{ka}{2}(\phi_U^{(2)})^2 + bk^3(\phi_U^{(2)})'' = 0, \quad (38)$$

$$\omega\phi_V^{(2)} - \frac{ka}{2}(\phi_V^{(2)})^2 - bk^3(\phi_V^{(2)})'' = 0. \quad (39)$$

Assume that the typical KdV Eq. (11) has the following formal solution

$$\phi(\xi) = \sum_{i=0}^{N} A_i(\phi(U))^i, \quad and \quad (\phi')^2 = \alpha\phi^2 + \beta\phi^4, \quad (40)$$

$$\phi(\eta) = \sum_{i=0}^{N} B_i(\phi(V))^i, \quad and \quad (\phi')^2 = \alpha\phi^2 + \beta\phi^4. \quad (41)$$

where α, β are arbitrary constants. and to determine N balancing the nonlinear term and the highest order derivative in Eqs. (38)-(39) gives $N = 2$. The solution of Eqs. (38)-(39) is in the form

$$\phi(\xi) = A_0 + A_1\phi(U) + A_2\phi(U)^2, \quad (42)$$

$$\phi(\eta) = B_0 + B_1\phi(V) + B_2\phi(V)^2. \quad (43)$$

Substituting from (42)-(43) into (40)-(41) yields a set of algebraic equations for A_0, A_1, A_2, B_0, B_1, B_2, k, ω, α, β. Solving the system of equations by Mathematica, we obtain the following results.

Case 1.

$$A_0 = \frac{-2\omega}{ak}, \; A_1 = 0, \; A_2 = \frac{12bk^2}{A}, \; \alpha = \frac{\omega}{4bk^3}, \; \beta = -1, \quad (44)$$

$$B_0 = \frac{2\omega}{ak}, \quad B_1 = 0, \quad B_2 = \frac{12bk^2}{A}, \quad \alpha = \frac{\omega}{4bk^3}. \quad (45)$$

Case 2.

$$A_0 = A_1 = 0, \quad A_2 = \frac{12bk^2}{a}, \quad \alpha = \frac{-\omega}{4bk^3}, \quad (46)$$

$$B_0 = B_1 = 0, \quad B_2 = \frac{12bk^2}{a}, \quad \alpha = \frac{-\omega}{4bk^3}. \quad (47)$$

Substituting from Eqs. (44)-(47) into (11), then we can obtained the solutions as:

$$\phi_{(\xi,\tau)} = \frac{-2\omega}{ak} + \frac{3\omega}{ak} sech^2\left[\sqrt{\frac{\omega}{4bk^3}}(k\xi + \omega\tau)\right], \quad (48)$$

$$\phi_{(\eta,\tau)} = \frac{2\omega}{ak} + \frac{3\omega}{ak} sech^2\left[\sqrt{\frac{\omega}{4bk^3}}(k\eta + \omega\tau)\right], \quad (49)$$

and

$$\phi_{(\xi,\tau)} = -\frac{3\omega}{ak} sech^2\left[\sqrt{-\frac{\omega}{4bk^3}}(k\xi + \omega\tau)\right], \quad (50)$$

$$\phi_{(\eta,\tau)} = \frac{3\omega}{ak} sech^2\left[\sqrt{-\frac{\omega}{4bk^3}}(k\eta + \omega\tau)\right], \quad (51)$$

then we have

$$\phi^2 = \frac{3\omega}{ak} sech^2\left[\sqrt{\frac{\omega}{4bk^3}}(k\xi + \omega\tau)\right] + \frac{3\omega}{ak} sech^2\left[\sqrt{\frac{\omega}{4bk^3}}(k\eta + \omega\tau)\right]. \quad (52)$$

The electric field $\overrightarrow{E} = -\nabla\phi = -\frac{\partial\phi}{\partial\xi}\widehat{e_\xi} - \frac{\partial\phi}{\partial\eta}\widehat{e_\eta}$,

$$\overrightarrow{E} = \frac{3\omega}{a}\sqrt{\frac{\omega}{bk^3}} sech^2\left[\sqrt{\frac{\omega}{4bk^3}}(k\xi + \omega\tau)\right] \tanh\left[\sqrt{\frac{\omega}{4bk^3}}(k\xi + \omega\tau)\right]\widehat{e_\xi}$$

$$-\frac{3\omega}{a}\sqrt{\frac{\omega}{bk^3}} sech^2\left[\sqrt{\frac{\omega}{4bk^3}}(k\eta + \omega\tau)\right] \tanh\left[\sqrt{\frac{\omega}{4bk^3}}(k\eta + \omega\tau)\right]\widehat{e_\eta} \quad (53)$$

From Eq. (3), the difference between the number density of ions and electrons can be obtained as

$$(n_e - n_p - n_i) = \frac{1}{\Omega}\nabla^2\phi = \frac{-3\omega^2}{2\Omega abk^2} sech^4\left[\sqrt{\frac{\omega}{4bk^3}}(k\xi + \omega\tau)\right]\Big(-2 + \cosh\left[\sqrt{\frac{\omega}{4bk^3}}(k\xi + \omega\tau)\right]\Big)$$

$$\frac{-3\omega^2}{2\Omega abk^2} sech^4\left[\sqrt{\frac{\omega}{4bk^3}}(k\eta + \omega\tau)\right]\Big(-2 + \cosh\left[\sqrt{\frac{\omega}{4bk^3}}(k\eta + \omega\tau)\right]\Big). \quad (54)$$

Example 2. Typical mKdV equations

Now we will find the solutions of the two- sides mKdV equations, by applying the direct algebraic function method. By take the transformation $\phi^{(1)}(\xi,\tau) = \phi^{(1)}(U)$ where $U = (k\xi + \omega\tau)$ and $\phi^{(1)}(\eta,\tau) = \phi^{(1)}(V)$ where $V = (k\eta + \omega\tau)$ then Eq. (14) becomes

$$\omega(\phi_U^{(1)})' + ka_1(\phi_U^{(1)})^2(\phi_U^{(1)})' + bk^3(\phi_U^{(1)})''' = 0, \quad (55)$$

$$\omega(\phi_V^{(1)})' + ka_1(\phi_V^{(1)})^2(\phi_V^{(1)})' + bk^3(\phi_V^{(1)})''' = 0. \quad (56)$$

Integrating Eq. (55) -(56) with respect to U and V and letting the integrating constant to be zero, we have

$$\omega\phi_U^{(1)} + \frac{ka_1}{3}(\phi_U^{(1)})^3 + bk^3(\phi_U^{(1)})'' = 0, \quad (57)$$

$$\omega\phi_V^{(1)} + \frac{ka_1}{3}(\phi_V^{(1)})^3 + bk^3(\phi_V^{(1)})'' = 0. \quad (58)$$

Assume that the two- sides mKdV Eq. (14) has the following formal solution

$$\phi(\xi) = \sum_{i=0}^{N} A_i(\phi(U))^i, \quad and \quad (\phi')^2 = \alpha\phi^2 + \beta\phi^4, \quad (59)$$

$$\phi(\eta) = \sum_{i=0}^{N} B_i(\phi(V))^i, \quad and \quad (\phi')^2 = \alpha u^2 + \beta\phi^4. \quad (60)$$

Where α, β are arbitrary constants.and to determine N balancing the nonlinear term and the highest order derivative in Eqs. (57)-(58) gives $N = 1$. The solution of Eqs. (57)-(58) is in the form

$$\phi(\xi) = A_0 + A_1\phi(U), \quad (61)$$

$$\phi(\eta) = B_0 + B_1\phi(V). \quad (62)$$

Substituting from (61)-(62) into (57)-(58) yields a set of algebraic equations for $A_0, A_1, A_2, B_0, B_1, B_2$, k, ω, α, β. Solving the system of algebraic equations by Mathematica, we obtain the following results.

$$A_0 = 0, \quad A_1 = \frac{\sqrt{6}\sqrt{bk}}{\sqrt{a_1}}, \quad \alpha = \frac{-\omega}{bk^3}, \quad (63)$$

$$B_0 = 0, \quad B_1 = \frac{\sqrt{6}\sqrt{Bk}}{\sqrt{A1}}, \quad \alpha = \frac{-\omega}{bk^3}. \quad (64)$$

Substituting from (63)-(64) into (14), then we can obtained the solutions as

$$\phi_\xi^{(1)} = \sqrt{\frac{-6\omega}{a_1k}} sech\left[\sqrt{\frac{-\omega}{bk^3}}(k\xi + \omega\tau)\right], \quad (65)$$

$$\phi_\eta^{(1)} = \sqrt{\frac{-6\omega}{a_1k}} sech\left[\sqrt{\frac{-\omega}{bk^3}}(k\eta + \omega\tau)\right], \quad (66)$$

and then

$$\phi^{(1)} = \sqrt{\frac{-6\omega}{a_1k}} sech\left[\sqrt{\frac{-\omega}{bk^3}}(k\xi + \omega\tau)\right] + \sqrt{\frac{-6\omega}{a_1k}} sech\left[\sqrt{\frac{-\omega}{bk^3}}(k\eta + \omega\tau)\right] \quad (67)$$

The electric field $\overrightarrow{E} = -\nabla\phi = -\frac{\partial\phi}{\partial\xi}\widehat{e_\xi} - \frac{\partial\phi}{\partial\eta}\widehat{e_\eta}$,

$$\overrightarrow{E} = -\omega\sqrt{\frac{6}{a_1b}} ksech\left[\sqrt{\frac{-\omega}{bk^3}}(k\xi + \omega\tau)\right] \tanh\left[\sqrt{\frac{-\omega}{bk^3}}(k\xi + \omega\tau)\right]\widehat{e_\xi}$$

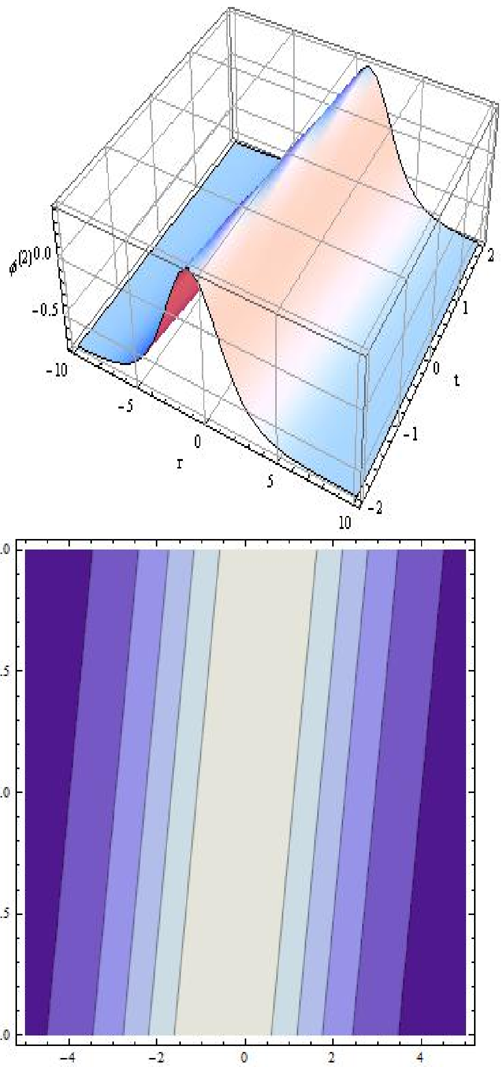
$$-\omega\sqrt{\frac{6}{a_1b}} ksech\left[\sqrt{\frac{-\omega}{bk^3}}(k\eta + \omega\tau)\right] \tanh\left[\sqrt{\frac{-\omega}{bk^3}}(k\eta + \omega\tau)\right]\widehat{e_\eta}. \quad (68)$$

From equation (3), the difference between the number density of ions and electrons can be obtained as

$$(n_e - n_p - n_i) = \frac{1}{\Omega}\nabla^2\phi = -\frac{\omega}{\Omega}\sqrt{\frac{-3\omega}{32a_1b^2k^3}} sech^3\left[\sqrt{\frac{-\omega}{bk^3}}(k\xi + \omega\tau)\right]$$

$$\left(-3+\cosh\left[2\sqrt{\frac{-\omega}{bk^3}}(k\xi+\omega\tau)\right]\right)-\frac{\omega}{\Omega}\sqrt{\frac{-3\omega}{32a_1b^2k^3}}$$

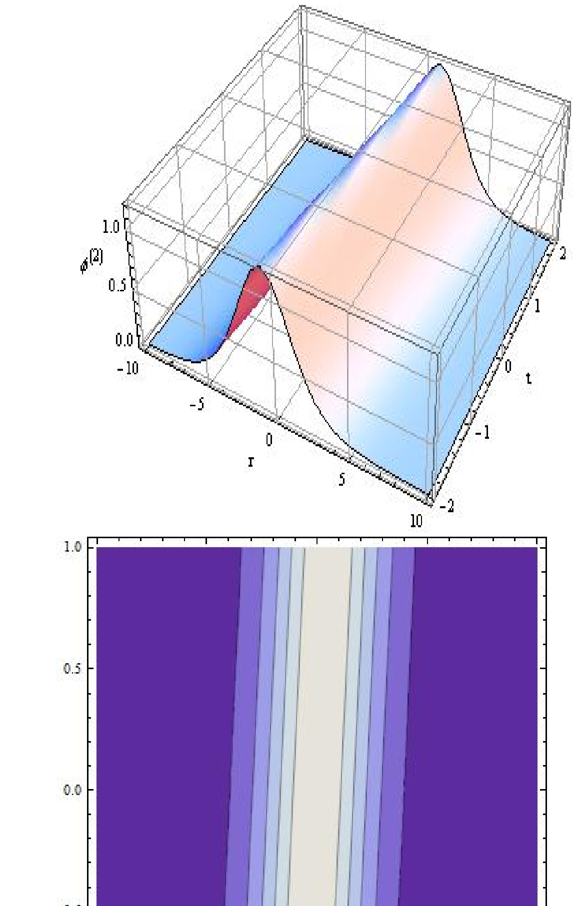
$$sech^3\left[\sqrt{\frac{-\omega}{bk^3}}\left(k\eta+\omega\tau\right)\right]\left(-3+\cosh\left[2\sqrt{\frac{-\omega}{bk^3}}(k\eta+\omega\tau)\right]\right) \qquad (69)$$



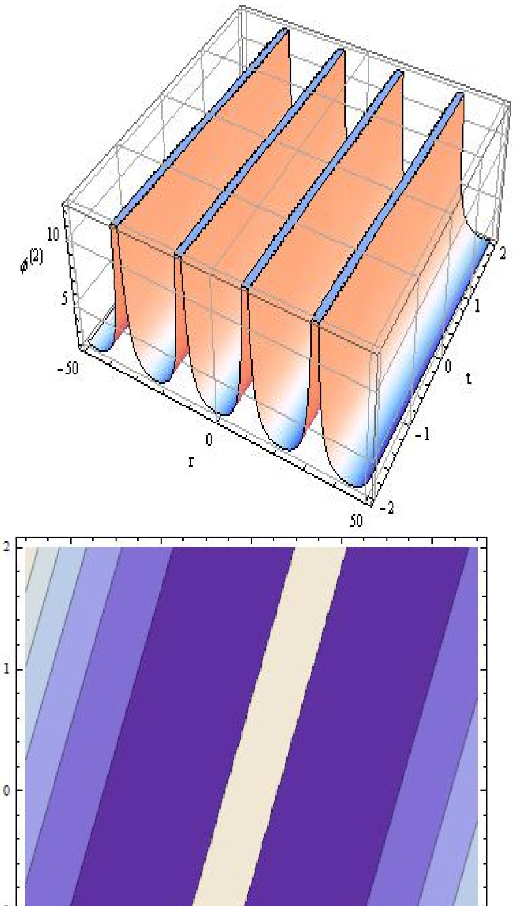
Figure(1) Head-on collision for generic plasma composition with $r=\sqrt{x^2+y^2+z^2}$, $\sigma=1.587$, $l_z=.9$, $\mu_p=1$, $\phi_A=\phi_B=0.1$, and $\varepsilon=0.1$, $k=1$, $H_e=.00624$, $\omega=0.010291$. Contour plot,in the interval [-5,5].

6. Conclusion

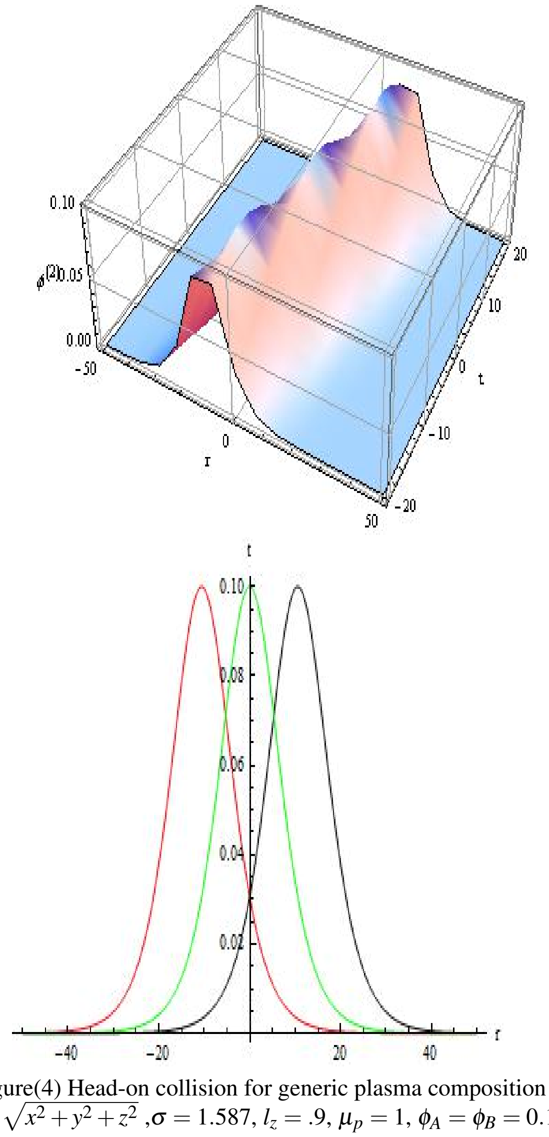
In this paper, we have studied the QIASWs propagating obliquely in a homogeneous three component dense magnetoplasma containing inertialess electron, positron, and positively charged ions. The quantum hydrodynamic (QHD) model is used to derive the two sided KdV equations for the generic case and mKdV equations for aspecial case. It is shown that at the critical values of the plasma parameter the quadratic non- linearity in the KdV equations disappears and we obtain the mKdV equations with cubic nonlinearity. The soliton solution and travelling wave solutions of the KdV equations and typical mKdV equations are derived. The conservation laws for the typical KdV equations and typical mKdV equations were established with the help of the multiplier approach. The stability analysis for the electric field potentials and electric fields are discussed with respect to the sufficient condition for soliton stability. The electric field potential and electric field are stable in the interval [-5,5]. By using the extended direct algebraic method, we found the electric field potential and electric field in the form of traveling wave solutions for the two-sides KdV equations and mKdV equations. The soliton solution and travelling wave solutions of the KdV and mKdV equation are derived.



Figure(2) Head-on collision for generic plasma composition with $r=\sqrt{x^2+y^2+z^2}$,$\sigma=1.587$, $l_z=.9$, $\mu_p=1$, $\phi_A=\phi_B=0.1$, and $\varepsilon=0.1$, $k=1$, $H_e=.00624$, $\omega=0.010291$. Contour plot in the interval [-10,10].



Figure(3) Head-on collision for special plasma composition with opposite polarities for $r=\sqrt{x^2+y^2+z^2}$, $\sigma=1.587$, $l_z=.9$, $\mu_p=.25$, $\phi_A=\phi_B=-0.1$, and $\varepsilon=0.1$, $\omega=2$. Contour plot in the interval [-5,5] ,$k=1$



Figure(4) Head-on collision for generic plasma composition with $r=\sqrt{x^2+y^2+z^2}$,$\sigma=1.587$, $l_z=.9$, $\mu_p=1$, $\phi_A=\phi_B=0.1$, and $\varepsilon=0.1$, $k=1$, $H_e=.00624$, $\omega=0.010291$.

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