



k-cordial labeling of fan and double fan

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Abstract

We discuss here k -cordial labeling of fans. We prove that fans f_n are k -cordial for all k . We divide the proof of the result into two parts namely odd k and even k . Moreover we prove that double fans Df_n are k -cordial for all odd k and $n = \frac{k+1}{2}$. The present authors are motivated by the research article entitled as 'A-cordial graphs' by A Hovey.

Keywords: Abelian Group; k -Cordial Labeling; Fan; Double fan.

1. Introduction

Throughout this work, by a graph we mean finite, connected, undirected, simple graph $G = (V(G), E(G))$ of order $|V(G)|$ and size $|E(G)|$.

Definition 1.1 A graph labeling is an assignment of integers to the vertices or edges or both subject to certain condition(s).

A latest survey on various graph labeling problems can be found in Gallian[2].

Definition 1.2 Let $\langle A, * \rangle$ be any Abelian group. A graph $G = (V(G), E(G))$ is said to be A-cordial if there is a mapping $f : V(G) \rightarrow A$ which satisfies the following two conditions when the edge $e = uv$ is labeled as $f(u) * f(v)$

(i) $|v_f(a) - v_f(b)| \leq 1$; for all $a, b \in A$,

(ii) $|e_f(a) - e_f(b)| \leq 1$; for all $a, b \in A$.

Where

$v_f(a)$ = the number of vertices with label a ;

$v_f(b)$ = the number of vertices with label b ;

$e_f(a)$ = the number of edges with label a ;

$e_f(b)$ = the number of edges with label b .

We note that if $A = \langle Z_k, +_k \rangle$, that is additive group of modulo k then the labeling is known as k -cordial labeling. The concept of A-cordial labeling was introduced by Hovey[4] and proved the following results.

- All the connected graphs are 3-cordial.
- All the trees are 3, 4, 5-cordial.
- Cycles are k -cordial for all odd k .

Here we consider the following definitions of standard graphs.

- The fan f_n is $P_n + K_1$.
- The double fan Df_n is obtained by $P_n + 2K_1$.

2. Main results

Theorem 2.1: Fans f_n are k -cordial for all odd k .

Proof: Let f_n be the fan and v_0 be the apex vertex. Let $n = mk + j$, where $m \geq 0$ and $1 \leq j \leq k - 1$. We divide the n path vertices of the fan f_n into two blocks of mk and j vertices namely $v_1, v_2, \dots, v_{mk}, v_1, v_2, \dots, v_j$. We note that $|V(G)| = n + 1$ and $|E(G)| = 2n - 1$.

Define k -cordial labeling $f : V(G) \rightarrow Z_k$ as follows.

$$f(v_0) = 0;$$

For the first block of mk vertices, if $m \geq 1$,

$$\begin{aligned} f(v_i) &= \frac{k+p_i}{2}; & i \equiv p_i \pmod{k}, p_i \text{ is odd,} \\ &= \frac{p_i}{2}; & i \equiv p_i \pmod{k}, p_i \text{ is even, } 1 \leq i \leq mk. \end{aligned}$$

The labeling pattern of second block of j vertices, where $1 \leq j \leq k - 1$ is divided into following two cases.

Case 1: $\frac{k+1}{2}$ is odd.

Subcase I: $1 \leq j \leq \frac{k+1}{2}$.

$$\begin{aligned} f(v'_i) &= \frac{k+i}{2}; & i \text{ is odd,} \\ &= \frac{k-1}{4} + \frac{i}{2}; & i \text{ is even, } 1 \leq i \leq j. \end{aligned}$$

Subcase II: $\frac{k+3}{2} \leq j \leq k - 2$.

$$\begin{aligned} f(v'_i) &= \frac{k+i}{2}; & i \text{ is odd,} \\ &= \frac{k-l}{4} + \frac{i}{2}; & i \text{ is even, } 1 \leq i \leq j, \frac{k+l-2}{2} \leq j \leq \frac{k+l}{2}, \\ & & \text{where } l = 5, 9, \dots, k - 4. \end{aligned}$$

Subcase III: $j = k - 1$.

$$\begin{aligned} f(v'_i) &= \frac{k+i}{2}; & i \text{ is odd,} \\ &= \frac{i}{2}; & i \text{ is even, } 1 \leq i \leq j. \end{aligned}$$

Case 2: $\frac{k+1}{2}$ is even.

Subcase I: $1 \leq j \leq \frac{k+3}{2}$.

$$\begin{aligned} f(v'_i) &= \frac{k+i}{2}; & i \text{ is odd,} \\ &= \frac{k-1}{4} + \frac{i}{2}; & i \text{ is even, } 1 \leq i \leq j. \end{aligned}$$

Subcase II: $\frac{k+5}{2} \leq j \leq k - 2$.

$$f(v'_i) = \frac{k+i}{2}; \quad i \text{ is odd,}$$

$$= \frac{k-l}{4} + \frac{i}{2}; \quad i \text{ is even, } 1 \leq i \leq j, \frac{k+l-2}{2} \leq j \leq \frac{k+l}{2},$$

where $l = 7, 11, \dots, k - 4$.

Subcase III: $j = k - 1$.

$$f(v'_i) = \frac{k+i}{2}; \quad i \text{ is odd,}$$

$$= \frac{i}{2}; \quad i \text{ is even, } 1 \leq i \leq j.$$

The labeling pattern defined above covers all possible arrangement of vertices. In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for k -cordial labeling. Hence fans are k -cordial for all odd k .

Illustration 2.2 The fan graph f_{23} and its 17-cordial labeling is shown in Fig.1.

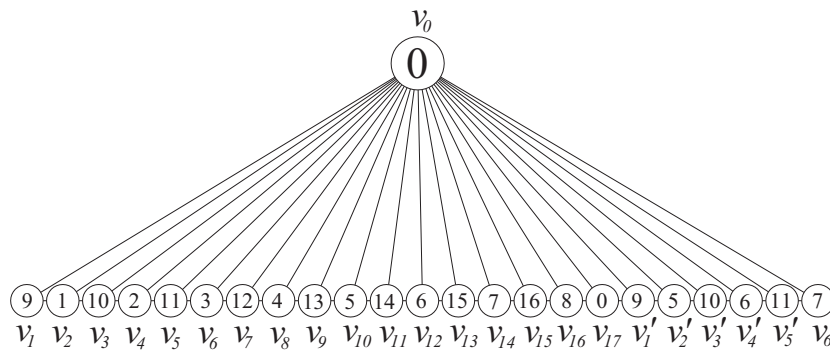


Fig.1 17-Cordial labeling of fan graph f_{23} .

Theorem 2.3: Fans f_n are k -cordial for all even k .

Proof: Let f_n be the fan and v_0 be the apex vertex. Let $n = mk + j$, where $m \geq 0$ and $1 \leq j \leq k - 1$. We divide the n path vertices of the fan f_n into two blocks of mk and j vertices namely $v_1, v_2, \dots, v_{mk}, v_1, v_2, \dots, v_j$. We note that $|V(G)| = n + 1$ and $|E(G)| = 2n - 1$.

To define k -cordial labeling $f : V(G) \rightarrow Z_k$ we consider the following four cases.

Case 1: m is odd and $\frac{k}{2}$ is odd.

$$f(v_0) = 0;$$

For the first block of mk vertices, if $m \geq 1$,

$$f(v_i) = \frac{p_i+1}{2}; \quad i \equiv p_i \pmod{k}, p_i \text{ is odd,}$$

$$= \frac{k}{2} + \frac{p_i}{2}; \quad i \equiv p_i \pmod{k}, p_i \text{ is even,}$$

$$(t - 1)k + 1 \leq i \leq tk, 1 \leq t \leq m, \text{ if } t \text{ is odd.}$$

$$f(v_i) = \frac{k}{2} + \frac{p_i+1}{2}; i \equiv p_i \pmod{k}, p_i \text{ is odd,}$$

$$= \frac{p_i}{2}; \quad i \equiv p_i \pmod{k}, p_i \text{ is even,}$$

$$(t - 1)k + 1 \leq i \leq tk, 1 \leq t \leq m, \text{ if } t \text{ is even.}$$

For the second block of j vertices, where $1 \leq j \leq k - 1$,

Subcase I: $1 \leq j \leq \frac{k+4}{2}$.

$$\begin{aligned} f(v'_i) &= \frac{k}{2} + \frac{i+1}{2}; \quad i \text{ is odd,} \\ &= \frac{k-2}{4} + \frac{i}{2}; \quad i \text{ is even, } 1 \leq i \leq j. \end{aligned}$$

Subcase II: $\frac{k+6}{2} \leq j \leq k - 2$.

$$\begin{aligned} f(v'_i) &= \frac{k}{2} + \frac{i+1}{2}; \quad i \text{ is odd,} \\ &= \frac{k-l}{4} + \frac{i}{2}; \quad i \text{ is even, } 1 \leq i \leq j, \frac{k+l}{2} \leq j \leq \frac{k+l+2}{2}, \end{aligned}$$

where $l = 6, 10, \dots, k - 4$.

Subcase III: $j = k - 1$.

$$\begin{aligned} f(v'_i) &= \frac{k}{2} + \frac{i+1}{2}; \quad i \text{ is odd, } 1 \leq i \leq j - 2, \\ &= 1; \quad i \text{ is odd, } i = j. \\ &= \frac{i}{2} + 1; \quad i \text{ is even, } 1 \leq i \leq j. \end{aligned}$$

Case 2: m is odd and $\frac{k}{2}$ is even.

$$f(v_0) = 0;$$

For the first block of mk vertices, if $m \geq 1$,

$$\begin{aligned} f(v_i) &= \frac{p_i+1}{2}; \quad i \equiv p_i \pmod{k}, p_i \text{ is odd,} \\ &= \frac{k}{2} + \frac{p_i}{2}; \quad i \equiv p_i \pmod{k}, p_i \text{ is even,} \\ &\quad (t-1)k + 1 \leq i \leq tk, 1 \leq t \leq m, \text{ if } t \text{ is odd.} \end{aligned}$$

$$\begin{aligned} f(v_i) &= \frac{k}{2} + \frac{p_i+1}{2}; \quad i \equiv p_i \pmod{k}, p_i \text{ is odd,} \\ &= \frac{p_i}{2}; \quad i \equiv p_i \pmod{k}, p_i \text{ is even,} \\ &\quad (t-1)k + 1 \leq i \leq tk, 1 \leq t \leq m, \text{ if } t \text{ is even.} \end{aligned}$$

For the second block of j vertices, where $1 \leq j \leq k - 1$,

Subcase I: $1 \leq j \leq \frac{k+2}{2}$.

$$\begin{aligned} f(v'_i) &= \frac{k}{2} + \frac{i+1}{2}; \quad i \text{ is odd,} \\ &= \frac{k}{4} + \frac{i}{2}; \quad i \text{ is even, } 1 \leq i \leq j. \end{aligned}$$

Subcase II: $\frac{k+4}{2} \leq j \leq k - 2$.

$$\begin{aligned} f(v'_i) &= \frac{k}{2} + \frac{i+1}{2}; \quad i \text{ is odd,} \\ &= \frac{k-l}{4} + \frac{i}{2}; \quad i \text{ is even, } 1 \leq i \leq j, \frac{k+l}{2} \leq j \leq \frac{k+l+2}{2}, \\ &\quad \text{where } l = 4, 8, \dots, k - 4. \end{aligned}$$

Subcase III: $j = k - 1$.

$$\begin{aligned} f(v'_i) &= \frac{k}{2} + \frac{i+1}{2}; & i \text{ is odd, } 1 \leq i \leq j-2, \\ &= 1; & i \text{ is odd, } i = j. \\ &= \frac{i}{2} + 1; & i \text{ is even, } 1 \leq i \leq j. \end{aligned}$$

Case 3: m is even and $\frac{k}{2}$ is odd.

$$f(v_0) = 0;$$

For the first block of mk vertices, if $m \geq 1$,

$$\begin{aligned} f(v_i) &= \frac{k}{2} + \frac{p_i+1}{2}; & i \equiv p_i \pmod{k}, p_i \text{ is odd,} \\ &= \frac{p_i}{2}; & i \equiv p_i \pmod{k}, p_i \text{ is even,} \\ & & (t-1)k + 1 \leq i \leq tk, 1 \leq t \leq m, \text{ if } t \text{ is odd.} \end{aligned}$$

$$\begin{aligned} f(v_i) &= \frac{p_i+1}{2}; & i \equiv p_i \pmod{k}, p_i \text{ is odd,} \\ &= \frac{k}{2} + \frac{p_i}{2}; & i \equiv p_i \pmod{k}, p_i \text{ is even,} \\ & & (t-1)k + 1 \leq i \leq tk, 1 \leq t \leq m, \text{ if } t \text{ is even.} \end{aligned}$$

For the second block of j vertices, where $1 \leq j \leq k - 1$,

Subcase I: $1 \leq j \leq \frac{k+4}{2}$.

$$\begin{aligned} f(v'_i) &= \frac{k}{2} + \frac{i+1}{2}; & i \text{ is odd,} \\ &= \frac{k-2}{4} + \frac{i}{2}; & i \text{ is even, } 1 \leq i \leq j. \end{aligned}$$

Subcase II: $\frac{k+6}{2} \leq j \leq k - 2$.

$$\begin{aligned} f(v'_i) &= \frac{k}{2} + \frac{i+1}{2}; & i \text{ is odd,} \\ &= \frac{k-l}{4} + \frac{i}{2}; & i \text{ is even, } 1 \leq i \leq j, \frac{k+l}{2} \leq j \leq \frac{k+l+2}{2}, \\ & & \text{where } l = 6, 10, \dots, k-4. \end{aligned}$$

Subcase III: $j = k - 1$.

$$\begin{aligned} f(v'_i) &= \frac{k}{2} + \frac{i+1}{2}; & i \text{ is odd, } 1 \leq i \leq j-2, \\ &= 1; & i \text{ is odd, } i = j. \\ &= \frac{i}{2} + 1; & i \text{ is even, } 1 \leq i \leq j. \end{aligned}$$

Case 4: m is even and $\frac{k}{2}$ is even.

$$f(v_0) = 0;$$

For the first block of mk vertices, if $m \geq 1$,

$$\begin{aligned} f(v_i) &= \frac{k}{2} + \frac{p_i+1}{2}; & i \equiv p_i \pmod{k}, p_i \text{ is odd,} \\ &= \frac{p_i}{2}; & i \equiv p_i \pmod{k}, p_i \text{ is even,} \\ & & (t-1)k + 1 \leq i \leq tk, 1 \leq t \leq m, \text{ if } t \text{ is odd.} \end{aligned}$$

$$\begin{aligned}
 f(v_i) &= \frac{p_i+1}{2}; & i \equiv p_i \pmod{k}, p_i \text{ is odd,} \\
 &= \frac{k}{2} + \frac{p_i}{2}; & i \equiv p_i \pmod{k}, p_i \text{ is even,} \\
 & & (t-1)k + 1 \leq i \leq tk, 1 \leq t \leq m, \text{ if } t \text{ is even.}
 \end{aligned}$$

For the second block of j vertices, where $1 \leq j \leq k - 1$,

Subcase I: $1 \leq j \leq \frac{k+2}{2}$.

$$\begin{aligned}
 f(v'_i) &= \frac{k}{2} + \frac{i+1}{2}; & i \text{ is odd,} \\
 &= \frac{k}{4} + \frac{i}{2}; & i \text{ is even, } 1 \leq i \leq j.
 \end{aligned}$$

Subcase II: $\frac{k+4}{2} \leq j \leq k - 2$.

$$\begin{aligned}
 f(v'_i) &= \frac{k}{2} + \frac{i+1}{2}; & i \text{ is odd,} \\
 &= \frac{k-l}{4} + \frac{i}{2}; & i \text{ is even, } 1 \leq i \leq j, \frac{k+l}{2} \leq j \leq \frac{k+l+2}{2}, \\
 & & \text{where } l = 4, 8, \dots, k - 4.
 \end{aligned}$$

Subcase III: $j = k - 1$.

$$\begin{aligned}
 f(v'_i) &= \frac{k}{2} + \frac{i+1}{2}; & i \text{ is odd, } 1 \leq i \leq j - 2, \\
 &= 1; & i \text{ is odd, } i = j. \\
 &= \frac{i}{2} + 1; & i \text{ is even, } 1 \leq i \leq j.
 \end{aligned}$$

The labeling pattern defined above covers all possible arrangement of vertices. In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for k -cordial labeling. Hence fans are k -cordial for all even k .

Illustration 2.4 The fan graph f_{28} and its 12- cordial labeling is shown in Fig 2.

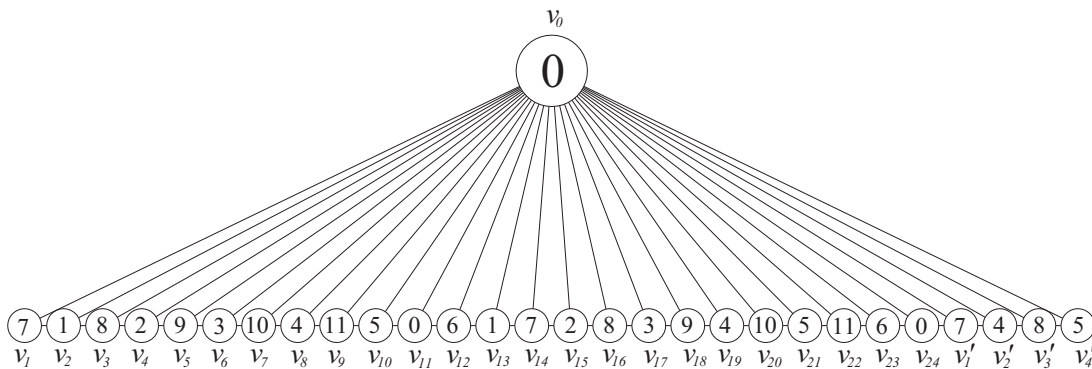


Fig 2 12-Cordial labeling of fan graph f_{28} .

Theorem 2.5 The double fans Df_n are k -cordial for all odd k and $n = \frac{k+1}{2}$.

Proof: Let Df_n be the double fan. Let v_1, v_2, \dots, v_n be the path vertices and v_0 and v'_0 be the two apex vertices of double fan Df_n . We note that $|V(G)| = n + 2$ and $|E(G)| = 3n - 1$. To define k -cordial labeling $f : V(G) \rightarrow Z_k$ we consider the following two cases.

Case 1: k is odd.

Subcase I: $\frac{k+1}{2}$ is odd.

$$\begin{aligned} f(v_0) &= 0; \\ f(v_0) &= (k-1); \\ f(v_i) &= \frac{k+i}{2}; \quad i \text{ is odd,} \\ &= \frac{k-1+2i}{4}; \quad i \text{ is even, } 1 \leq i \leq n. \end{aligned}$$

Subcase II: $\frac{k+1}{2}$ is even.

$$\begin{aligned} f(v_0) &= 0; \\ f(v_0) &= (k-1); \\ f(v_i) &= \frac{k+i}{2}; \quad i \text{ is odd,} \\ &= \frac{k-3+2i}{4}; \quad i \text{ is even, } 1 \leq i \leq n. \end{aligned}$$

Case 2: k is even.

Subcase I: $\frac{k}{2}$ is odd.

$$\begin{aligned} f(v_0) &= 0; \\ f(v_0) &= (k-1); \\ f(v_i) &= \frac{k-1+i}{2}; \quad i \text{ is odd,} \\ &= \frac{k-2+2i}{4}; \quad i \text{ is even, } 1 \leq i \leq n. \end{aligned}$$

Subcase II: $\frac{k}{2}$ is even.

$$\begin{aligned} f(v_0) &= 0; \\ f(v_0) &= (k-1); \\ f(v_i) &= \frac{k-1+i}{2}; \quad i \text{ is odd,} \\ &= \frac{k+2i}{4} - 1; \quad i \text{ is even, } 1 \leq i \leq n. \end{aligned}$$

The labeling pattern defined above covers all possible arrangement of vertices. In each possibility the graph under consideration satisfies the vertex conditions and edge conditions for k -cordial labeling. Hence the double fans Df_n are k -cordial for all odd k and $n = \frac{k+1}{2}$.

Illustration 2.6 The double fan Df_{22} and its 43-cordial labeling is shown in Fig 3.

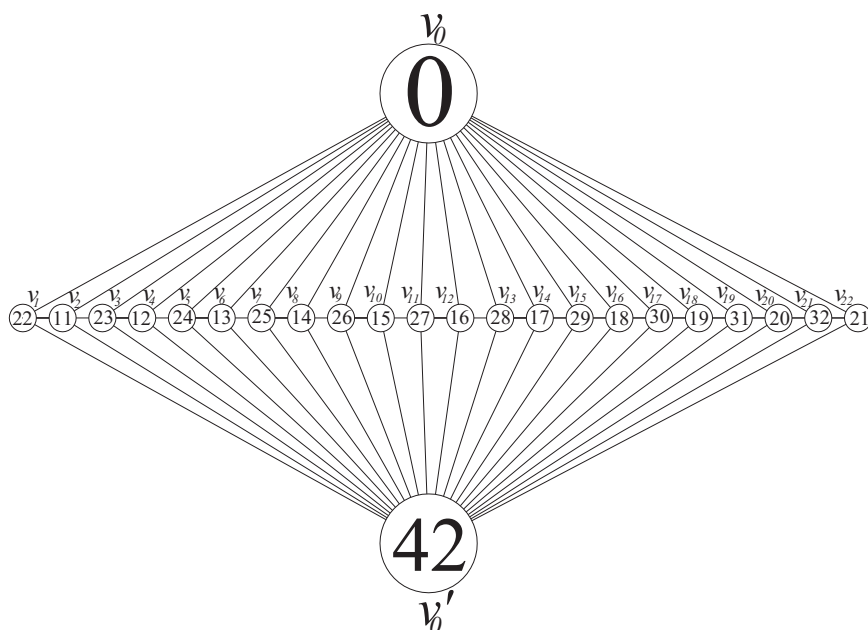


Fig 4 43-Cordial labeling of double fan Df_{22} .

3. Concluding Remarks

Here we have contributed general result for fan and a particular result for double fan to the theory of k -cordial labeling. To derive general results for other graph families is an open problem.

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