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Research paper

On the Number of Paths of Length 6 in a Graph

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Abstract

In this paper, we obtain an explicit formula for the total number of paths of length 6 in a simple graph G, in terms of the adjacency matrix and with the help of combinatorics.
Keywords: *Adjacency Matrix, Cycle, Graph Theory, Path, Subgraph, Walk .*

1. Introduction

In a simple graph G, a walk is a sequence of vertices and edges of the form $v_0, e_1, v_1, ..., e_k, v_k$ such that the edge e_i has ends v_{i-1} and v_i. A walk is called closed if $v_0 = v_k$. If the vertices of a walk are distinct then the walk is called a path. A cycle is a non-trivial closed walk in which all the vertices are distinct except the end vertices.
It is known that if a graph G has adjacency matrix A=$[a_{ij}]$, then for k = 0, 1, ... , the ij-entry of A^k is the number of $v_i - v_j$ walks of length k in G. It is also known that tr(A^n) is the sum of the diagonal entries of A^n and d_i is the degree of the vertex v_i.
In 1971, Frank Harary and Bennet Manvel [3], gave formulae for the number of cycles of lengths 3 and 4 in simple graphs as given by the following theorems:

Theorem 1.1 [3] *If G is a simple graph with adjacency matrix A, then the number of 3-cycles in G is* $\frac{1}{6}$ *tr*(A^3)*.*
(It is known that $tr(A^3) = \sum_{i=1}^{n} a_{ii}^{(3)} = \sum_{j\neq i} a_{ij}^{(2)} a_{ij}$*).*

Theorem 1.2 [3] *If G is a simple graph with adjacency matrix A, then the number of* $4-$*cycles in G is* $\frac{1}{8}[\ tr(A^4) - 2q - 2\sum_{j\neq i} a_{ij}^{(2)}]$*, where q is the number of edges in G.*
(It is obvious that the above formula is also equal to $\frac{1}{8}\ [trA^4 -\ trA^2 - 2\sum_{j\neq i} a_{ij}^{(2)}]$ *)*

Theorem 1.3 [3] *If G is a simple graph with n vertices and the adjacency matrix A=* $[a_{ij}]$*, then the number of* $5-$*cycles in G is* $\frac{1}{10}[tr(A^5) + 5\ tr(A^3) - 5\sum_{i=1}^{n} d_i a_{ii}^{(3)}]$

They also gave a formula for the number of 5-cycles in a simple graph. Their proofs are based on the following fact: The number of n-cycles (n$= 3, 4, 5$) in a graph G is equal to $\frac{1}{2n}(\mathrm{tr}(\mathrm{A}^n) - x\ )$ where x is the number of closed walks of length n, which are not n-cycles.

In 1986, Tomescu [5], gave some formulae for the number of paths of length s, having k edges in common with a fixed s-path of a complete graph. In 1994, Bax [6], gave an algorithm to count number of all paths and $v_i - v_j$ paths in a graph. His algorithm cannot count number of paths of a specific size.
In 1996, Eric Bax and Joel Franklin [8], gave an algorithm to count paths and cycles of a given length in a directed graph. In [7, 9, 10, 11, 13, 14, 16], we have also some bounds to estimate the total time complexity for finding or counting paths and cycles in a graph.
In the previous works there is no formula to count the exact number of paths of a specific size in a graph.
In our recent works [1, 2], we obtained some formulae and propositions to find the exact number of paths of lengths 3, 4 and 5, in a simple graph G, given below:

Proposition 1.4 [1] *In a simple graph G with n vertices and the adjacency matrix A$= [a_{ij}]$, the number of paths of length n is* $\sum_{j\neq i} a_{ij}^{(n)} - x$, *where x is the number of non-closed walks of length n in G, which are not paths.*

Proposition 1.5 [1] *In a simple graph G with n vertices and the adjacency matrix A$= [a_{ij}]$, the number of paths of length n, each of which begins with a specific vertex v_i is* $\sum_{j=1, j\neq i}^{n} a_{ij}^{(n)} - x$, *where x is the number of non-closed walks of length n in G, starting from the vertex v_i, which are not paths.*

Proposition 1.6 [1] *In a simple graph G with n vertices and the adjacency matrix A$= [a_{ij}]$, the number of $v_i - v_j$ $(j \neq i)$ paths of length n is* $a_{ij}^{(n)} - x$, *where x is the number of non-closed $v_i - v_j$ walks of length n in G, which are not paths.*

Theorem 1.7 [1] *Let G be a simple graph with n vertices and the adjacency matrix A$= [a_{ij}]$. The number of paths of length 3 in G is* $\sum_{j\neq i} a_{ij}^{(2)}(d_j - a_{ij} - 1)$.

Theorem 1.8 [1] *Let G be a simple graph with n vertices and the adjacency matrix A$= [a_{ij}]$. The number of paths of length 4 in G is* $\sum_{j\neq i}[a_{ij}^{(4)} - 2a_{ij}^{(2)}(d_j - a_{ij})] - \sum_{i=1}^{n}[(2d_i - 1)a_{ii}^{(3)} + 6\begin{pmatrix} d_i \\ 3 \end{pmatrix}]$.

Theorem 1.9 [1] *Let G be a simple graph with n vertices and the adjacency matrix A$= [a_{ij}]$. The number of paths of length 3 in G, each of which starts from a specific vertex v_i is* $\sum_{j=1, j\neq i}^{n} a_{ij}^{(2)}(d_j - a_{ij} - 1)$.

Theorem 1.10 [1] *Let G be a simple graph with n vertices and the adjacency matrix A$= [a_{ij}]$. The number of paths of length 4 in G, each of which starts from a specific vertex v_i is* $\sum_{j=1, j\neq i}^{n} [a_{ij}^{(4)} - (d_i + d_j - 3a_{ij})a_{ij}^{(2)} - (a_{ii}^{(3)} + a_{jj}^{(3)} + 2\begin{pmatrix} d_j - 1 \\ 2 \end{pmatrix})a_{ij}]$.

Theorem 1.11 [1] *Let G be a simple graph with n vertices and the adjacency matrix A$= [a_{ij}]$. The number of $v_i - v_j$ $(j \neq i)$ paths of length* 3 *in G is* $\sum_{k=1, k\neq i,j}^{n} (a_{ik}^{(2)} - a_{ij})a_{jk}$.

Theorem 1.12 [2] *Let G be a simple graph with n vertices and the adjacency matrix A$= [a_{ij}]$. The number of paths of length 5 in G is* $\sum_{j\neq i} a_{ij}^{(5)} - 2\sum_{j\neq i} a_{ij}^{(4)} + 2\sum_{i=1}^{n} a_{ii}^{(3)}(d_i - 2) + 4\sum_{j\neq i} a_{ij}^{(2)} - 2\sum_{j\neq i} a_{ij}^{(2)}(d_j - a_{ij} - 1) - 4\sum_{j\neq i} a_{ij}^{(2)}\begin{pmatrix} d_i - a_{ij} - 1 \\ 2 \end{pmatrix} + 6\sum_{j\neq i} a_{ij}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix} - 2\sum_{j\neq i} a_{ii}^{(3)} a_{ij}^{(2)} - 2\sum_{i=1}^{n} a_{ii}^{(3)}\begin{pmatrix} d_i - 2 \\ 2 \end{pmatrix} - 2\sum_{i=1}^{n}(a_{ii}^{(4)} - a_{ii}^{(2)} - 2\begin{pmatrix} d_i \\ 2 \end{pmatrix} - \sum_{j=1, j\neq i}^{n} a_{ij}^{(2)})(d_i - 2) - \sum_{j\neq i} a_{ij} -$ *3 tr* $A^4 +$ *6 tr* $A^3 +$ *3 tr* A^2.

Theorem 1.13 [2] *If G is a simple graph with n vertices and the adjacency matrix* $A=[a_{ij}]$*, then the number of* $4-$*cycles each of which contains a specific vertex* v_i *of G is* $\frac{1}{2}\ [a_{ii}^{(4)} - a_{ii}^{(2)} - 2\begin{pmatrix} d_i \\ 2 \end{pmatrix} - \sum_{j=1, j\neq i}^{n} a_{ij}^{(2)}]$.

In this paper we give a formula to count the exact number of paths of length 6 in a simple graph G, in terms of the adjacency matrix of G and with the help of combinatorics.

2. Number of Paths of Length 6

In this section, we give formulae to count the number of paths of length 6 in a simple graph G. We first give a result below which is useful to prove our main theorem. In [4], we can see a formula for the number of 5-cycles that pass trough the vertex v_i of a graph G but their formula has some problems in coefficients. Here we have written the correct formula with its proof.

Theorem 2.1 *If G is a simple graph with n vertices and the adjacency matrix* $A=[a_{ij}]$*, then the number of* $5-$*cycles each of which contains a specific vertex* v_i *of G is* $\frac{1}{2}\ [a_{ii}^{(5)} - 5a_{ii}^{(3)} - 2(d_i-2)a_{ii}^{(3)} - 2\sum_{j=1, j\neq i}^{n} a_{ij}^{(2)} a_{ij}(d_j-2) - 2\sum_{j=1, j\neq i}^{n} a_{ij}(\frac{1}{2}a_{jj}^{(3)} - a_{ij}a_{ij}^{(2)})]$.

Proof: The number of $5-$cycles each of which contains a specific vertex v_i of the graph G is equal to $\frac{1}{2}\ (a_{ii}^{(5)} - x)$, where x is the number of closed walks of length 5 from the vertex v_i to v_i that are not $5-$cycles. To find x, we have 4 cases as considered below; the cases are based on the configurations-(subgraphs) that generate $v_i - v_i$ walks of length 5 that are not cycles. In each case, N denotes the number of walks of length 5 from v_i to v_i that are not cycles in the corresponding subgraph, M denotes the number of subgraphs of G of the same configuration and F denotes the total number of $v_i - v_i$ walks of length 5 that are not cycles in all possible subgraphs of G of the same configuration. It is clear that F is equal to N$\times$ M. To find N in each case, we have to include in any walk, all the edges and the vertices of the corresponding subgraphs at least once.

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Case 1: For the configuration of Figure 1, N= 10, M= $\frac{1}{2}\ a_{ii}^{(3)}$ and F= $5a_{ii}^{(3)}$.

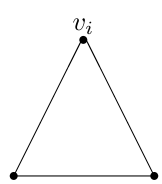


Fig 1

Case 2: For the configuration of Figure 2, N= 4, M= $\frac{1}{2}(d_i-2)a_{ii}^{(3)}$ and F= $2(d_i-2)a_{ii}^{(3)}$.

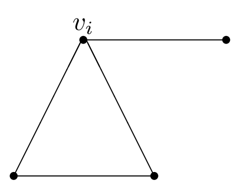


Figure 2

Case 3: For the configuration of Figure 3, N= 2, M= $\sum_{j=1, j\neq i}^{n} a_{ij}^{(2)} a_{ij}(d_j-2)$ and F= $2\sum_{j=1, j\neq i}^{n} a_{ij}^{(2)} a_{ij}(d_j-2)$.

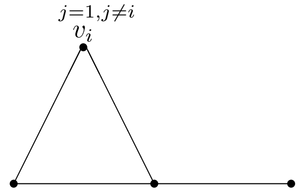


Figure 3

Case 4: For the configuration as shown in Figure 4, N= 2, M= $\sum_{j=1,j\neq i}^{n} a_{ij}(\frac{1}{2}a_{jj}^{(3)} - a_{ij}a_{ij}^{(2)})$ and F= 2 $\sum_{j=1,j\neq i}^{n} a_{ij}(\frac{1}{2}a_{jj}^{(3)} - a_{ij}a_{ij}^{(2)})$.

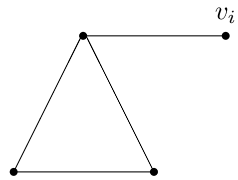


Figure 4

Consequently, $x = 5a_{ii}^{(3)} + 2(d_i - 2)a_{ii}^{(3)} + 2 \sum_{j=1,j\neq i}^{n} a_{ij}^{(2)}a_{ij}(d_j - 2) + 2 \sum_{j=1,j\neq i}^{n} a_{ij}(\frac{1}{2}a_{jj}^{(3)} - a_{ij}a_{ij}^{(2)})$ and we get the required result. □

Example 2.2 *In Figure* 5, $a_{11}^{(5)} = 68$, $5a_{11}^{(3)} = 20$, $2(d_1 - 2)a_{11}^{(3)} = 8$, $2\sum_{j=2}^{7} a_{1j}^{(2)}a_{1j}(d_j - 2) = 20$, $2\sum_{j=2}^{7} a_{1j}(\frac{1}{2}a_{jj}^{(3)} - a_{1j}a_{1j}^{(2)})) = 12$, *So by Theorem* 2.1, *the number of* 5*-cycles each of which contains the vertex* v_1 *in the graph of fig* 5 *is* 4.

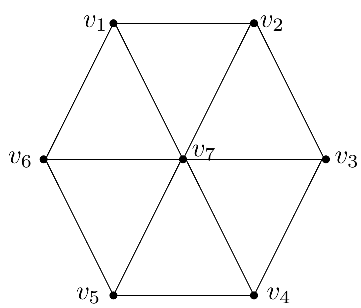


Figure 5

Theorem 2.3 *Let G be a simple graph with n vertices and the adjacency matrix A=* $[a_{ij}]$. *The number of paths of length 6 in G is* $\sum_{j\neq i} a_{ij}^{(6)} - x$, *where* x *is the summation of F in the cases which are considered below.*

Proof: By Proposition 1.4, the number of paths of length 6 in a graph G is equal to $\sum_{j\neq i} a_{ij}^{(6)} - x$, where x is the number of non-closed walks of length 6, that are not paths. To find x, we have 26 cases as considered below; the cases are based on the configurations-(subgraphs) that generate all non-closed walks of length 6, that are not paths. In each case, N denotes the number of non-closed walks of length 6, that are not paths in the corresponding subgraph, M denotes the number of subgraphs of G of the same configuration and F denotes the total number of non-closed walks of length 6, that are not paths in all possible subgraphs of G of the same configuration. However, in the cases with more than one Fig (cases 9, 10, 12, 16, 19, 20, 21, 23, 24, 25, 26), N, M and F are based on the first graph of the respective figures and P_1, P_2,... denotes the number of subgraphs of G which do not have the same configuration as the first graph but are counted in M. It is clear that F is equal to N × (M− P_1−P_2−...). To find N in each case, we have to include in any walk, all the edges and the vertices of the corresponding subgraphs at least once.

Case 1: For the configuration of Fig 6, N= 8, M= $\frac{1}{2} \sum_{j\neq i} a_{ij}^{(2)}$ and F= 4 $\sum_{j\neq i} a_{ij}^{(2)}$.

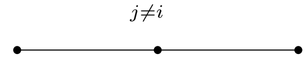


Fig 6

Case 2: For the configuration of Fig 7, N= 16, M= $\frac{1}{2} \sum_{j\neq i} a_{ij}^{(2)}(d_j - a_{ij} - 1)$ and F= 8 $\sum_{j\neq i} a_{ij}^{(2)}(d_j - a_{ij} - 1)$. (See Theorem 1.7)

Fig 7

Case 3: For the configuration of Fig 8, N= 14, M= $\frac{1}{2}\,[\sum\limits_{j\neq i}[a_{ij}^{(4)}-2a_{ij}^{(2)}(d_j-a_{ij})]-\sum\limits_{i=1}^{n}[(2d_i-1)a_{ii}^{(3)}+6\begin{pmatrix} d_i \\ 3 \end{pmatrix}]]$ and F= $7\sum\limits_{j\neq i}[a_{ij}^{(4)}-2a_{ij}^{(2)}(d_j-a_{ij})]-7\sum\limits_{i=1}^{n}[(2d_i-1)a_{ii}^{(3)}+6\begin{pmatrix} d_i \\ 3 \end{pmatrix}]$. (See Theorem 1.8)

Fig 8

Case 4: For the configuration of Fig 9, N= 4, M= $\frac{1}{2}\,[\sum\limits_{j\neq i}a_{ij}^{(5)}-2\sum\limits_{j\neq i}a_{ij}^{(4)}+2\sum\limits_{i=1}^{n}a_{ii}^{(3)}(d_i-2)+4\sum\limits_{j\neq i}a_{ij}^{(2)}-2\sum\limits_{j\neq i}a_{ij}^{(2)}(d_j-a_{ij}-1)-4\sum\limits_{j\neq i}a_{ij}^{(2)}\begin{pmatrix} d_i-a_{ij}-1 \\ 2 \end{pmatrix}+6\sum\limits_{j\neq i}a_{ij}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}-2\sum\limits_{j\neq i}a_{ii}^{(3)}a_{ij}^{(2)}-2\sum\limits_{i=1}^{n}a_{ii}^{(3)}\begin{pmatrix} d_i-2 \\ 2 \end{pmatrix}-2\sum\limits_{i=1}^{n}(a_{ii}^{(4)}-a_{ii}^{(2)}-2\begin{pmatrix} d_i \\ 2 \end{pmatrix}-\sum\limits_{j=1,j\neq i}^{n}a_{ij}^{(2)})(d_i-2)-\sum\limits_{j\neq i}a_{ij}-3\text{ tr }A^4+6\text{ tr }A^3+3\text{ tr }A^2]$ and F= $2\,[\sum\limits_{j\neq i}a_{ij}^{(5)}-2\sum\limits_{j\neq i}a_{ij}^{(4)}+2\sum\limits_{i=1}^{n}a_{ii}^{(3)}(d_i-2)+4\sum\limits_{j\neq i}a_{ij}^{(2)}-2\sum\limits_{j\neq i}a_{ij}^{(2)}(d_j-a_{ij}-1)-4\sum\limits_{j\neq i}a_{ij}^{(2)}\begin{pmatrix} d_i-a_{ij}-1 \\ 2 \end{pmatrix}+6\sum\limits_{j\neq i}a_{ij}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}-2\sum\limits_{j\neq i}a_{ii}^{(3)}a_{ij}^{(2)}-2\sum\limits_{i=1}^{n}a_{ii}^{(3)}\begin{pmatrix} d_i-2 \\ 2 \end{pmatrix}-2\sum\limits_{i=1}^{n}(a_{ii}^{(4)}-a_{ii}^{(2)}-2\begin{pmatrix} d_i \\ 2 \end{pmatrix}-\sum\limits_{j=1,j\neq i}^{n}a_{ij}^{(2)})(d_i-2)-\sum\limits_{j\neq i}a_{ij}-3\text{ tr }A^4+6\text{ tr }A^3+3\text{ tr }A^2]$. (See Theorem 1.12)

Fig 9

Case 5: For the configuration of Fig 10, N= 102, M= $\frac{1}{6}\,\text{tr}A^3$ and F= 17 $\text{tr}A^3$. (See Theorem 1.1)

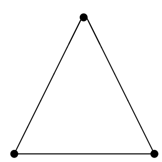


Fig 10

Case 6: For the configuration of Fig 11, N= 74, M= $\frac{1}{2}\sum\limits_{i=1}^{n}a_{ii}^{(3)}(d_i-2)$ and F= $37\sum\limits_{i=1}^{n}a_{ii}^{(3)}(d_i-2)$.

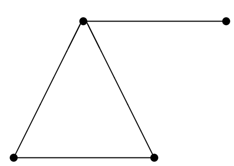


Fig 11

Case 7: For the configuration of Fig 12, N= 10, M= $\sum\limits_{j\neq i}a_{ij}^{(2)}\begin{pmatrix} d_i-a_{ij}-1 \\ 2 \end{pmatrix}$ and F= $10\sum\limits_{j\neq i}a_{ij}^{(2)}\begin{pmatrix} d_i-a_{ij}-1 \\ 2 \end{pmatrix}$.

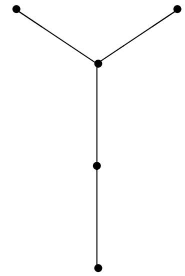


Fig 12

Case 8: For the configuration of Fig 13, N= 30, M= $\sum_{i=1}^{n}\binom{d_i}{3}$ and F= 30 $\sum_{i=1}^{n}\binom{d_i}{3}$.

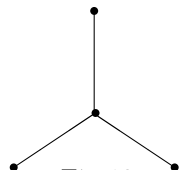
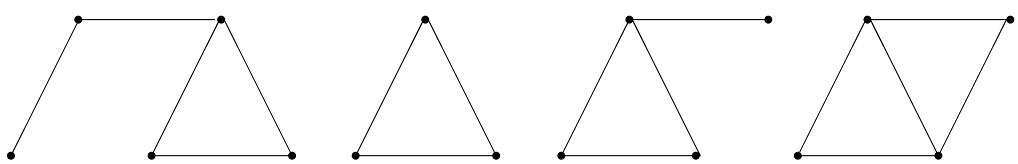


Fig 13

Case 9: For the configuration of Fig 14(a), N= 20, M= $\frac{1}{2}\sum_{j\neq i} a_{ii}^{(3)}a_{ij}^{(2)}$. Let P_1 denotes the number of all subgraphs of G that have the same configuration as the graph of Fig 14(b) and are counted in M. Thus $P_1= 6\times\frac{1}{6}\times \text{tr}A^3$, where $\frac{1}{6}\times\text{tr}A^3$ is the number of subgraphs of G that have the same configuration as the graph of Fig 14(b) (See Theorem 1.1) and 6 is the number of times that this subgraph is counted in M. Let P_2 denotes the number of all subgraphs of G that have the same configuration as the graph of Fig 14(c) and are counted in M. Thus $P_2= 2\times\frac{1}{2}\times\sum_{i=1}^{n} a_{ii}^{(3)}(d_i-2)$, where $\frac{1}{2}\times\sum_{i=1}^{n} a_{ii}^{(3)}(d_i-2)$ is the number of subgraphs of G that have the same configuration as the graph of Fig 14(c) and 2 is the number of times that this subgraph is counted in M. Let P_3 denotes the number of all subgraphs of G that have the same configuration as the graph of Fig 14(d) and are counted in M. Thus $P_3= 4\times\frac{1}{2}\times\sum_{j\neq i}\binom{a_{ij}^{(2)}}{2}a_{ij}$, where $\frac{1}{2}\times\sum_{j\neq i}\binom{a_{ij}^{(2)}}{2}a_{ij}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 14(d) and 4 is the number of times that this subgraph is counted in M.

Consequently, F= $10\sum_{j\neq i} a_{ii}^{(3)}a_{ij}^{(2)} - 20\ \text{tr}A^3 - 20\sum_{i=1}^{n} a_{ii}^{(3)}(d_i-2) - 40\sum_{j\neq i}\binom{a_{ij}^{(2)}}{2}a_{ij}$.



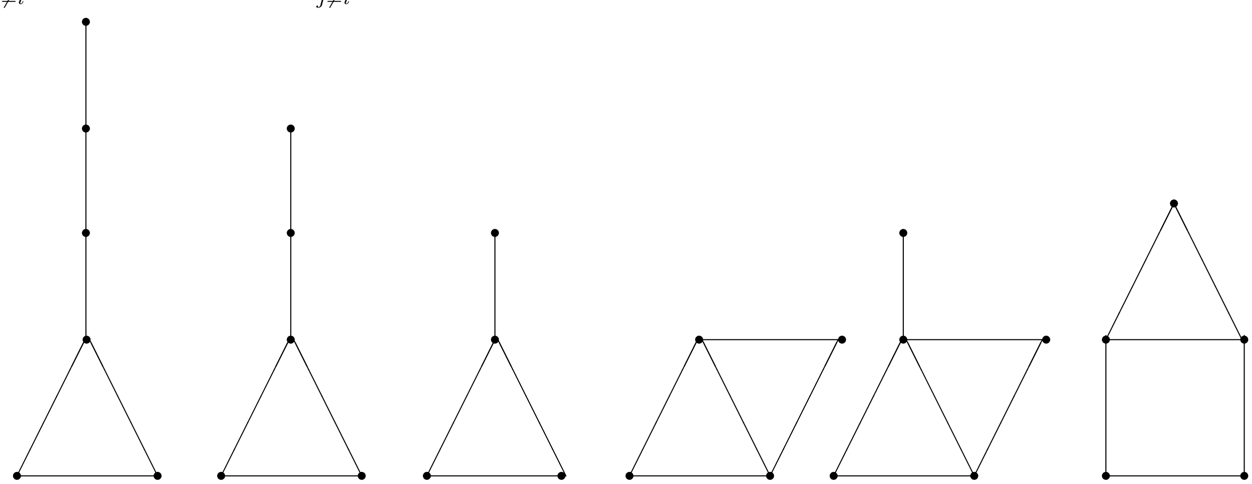
(a) (b) (c) (d)

Fig 14

Case 10: For the configuration of Fig 15(a), N= 4, M= $\frac{1}{2}\sum_{j\neq i} a_{ii}^{(3)}a_{ij}^{(2)}(d_j-a_{ij}-1)$ (See theorem 1.7). Let P_1 denotes the number of all subgraphs of G that have the same configuration as the graph of Fig 15(b) and are counted in M. Thus $P_1 = 2\times[\frac{1}{2}\sum_{j\neq i} a_{ii}^{(3)}a_{ij}^{(2)} - \text{tr}A^3 - \sum_{i=1}^{n} a_{ii}^{(3)}(d_i-2) - 2\sum_{j\neq i}\binom{a_{ij}^{(2)}}{2}a_{ij}]$ (See case 9), where $\frac{1}{2}\sum_{j\neq i} a_{ii}^{(3)}a_{ij}^{(2)} - \text{tr}A^3 - \sum_{i=1}^{n} a_{ii}^{(3)}(d_i-2) - 2\sum_{j\neq i}\binom{a_{ij}^{(2)}}{2}a_{ij}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 15(b) and 2 is the number of times that this subgraph is counted in M. Let P_2 denotes the number of all subgraphs of G that have the same configuration as the graph of Fig 15(c) and are counted in M. Thus $P_2 =$

$2 \times \frac{1}{2}\sum_{i=1}^{n} a_{ii}^{(3)}(d_i - 2)$, where $\frac{1}{2}\sum_{i=1}^{n} a_{ii}^{(3)}(d_i - 2)$ is the number of subgraphs of G that have the same configuration as the graph of Fig 15(c) and 2 is the number of times that this subgraph is counted in M. Let P_3 denotes the number of all subgraphs of G that have the same configuration as the graph of Fig 15(d) and are counted in M. Thus $P_3 = 8 \times \frac{1}{2}\sum_{j\neq i}\binom{a_{ij}^{(2)}}{2} a_{ij}$, where $\frac{1}{2}\sum_{j\neq i}\binom{a_{ij}^{(2)}}{2} a_{ij}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 15(d) and 8 is the number of times that this subgraph is counted in M. Let P_4 denotes the number of all subgraphs of G that have the same configuration as the graph of Fig 15(e) and are counted in M. Thus $P_4 = 2 \times \sum_{j\neq i}\binom{a_{ij}^{(2)}}{2} a_{ij}(d_j - 3)$, where $\sum_{j\neq i}\binom{a_{ij}^{(2)}}{2} a_{ij}(d_j - 3)$ is the number of subgraphs of G that have the same configuration as the graph of Fig 15(e) and 2 is the number of times that this subgraph is counted in M. Let P_5 denotes the number of all subgraphs of G that have the same configuration as the graph of Fig 15(f) and are counted in M. Thus $P_5 = 2 \times [\frac{1}{2}\sum_{j\neq i} a_{ij}^{(2)}(d_j - a_{ij} - 1)(a_{ij}a_{ij}^{(2)}) - 2\sum_{j\neq i} a_{ij}\binom{a_{ij}^{(2)}}{2}]$ (See case 12), where $\frac{1}{2}\sum_{j\neq i} a_{ij}^{(2)}(d_j - a_{ij} - 1)(a_{ij}a_{ij}^{(2)}) - 2\sum_{j\neq i} a_{ij}\binom{a_{ij}^{(2)}}{2}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 15(f) and 2 is the number of times that this subgraph is counted in M.

Consequently, $F = 2\sum_{j\neq i} a_{ii}^{(3)} a_{ij}^{(2)}(d_j - a_{ij} - 1) - 4\sum_{j\neq i} a_{ii}^{(3)} a_{ij}^{(2)} + 8\ \mathrm{tr}A^3 + 4\sum_{i=1}^{n} a_{ii}^{(3)}(d_i - 2) + 16\sum_{j\neq i}\binom{a_{ij}^{(2)}}{2} a_{ij} - 8\sum_{j\neq i}\binom{a_{ij}^{(2)}}{2} a_{ij}(d_j - 3) - 4\sum_{j\neq i} a_{ij}^{(2)}(d_j - a_{ij} - 1)(a_{ij}a_{ij}^{(2)})$.



(a) (b) (c) (d) (e) (f)

Fig 15

Case 11: For the configuration of Fig 16, N= 64, M= $\frac{1}{2}\sum_{j\neq i}\binom{a_{ij}^{(2)}}{2} a_{ij}$ and F= $32\sum_{j\neq i}\binom{a_{ij}^{(2)}}{2} a_{ij}$.

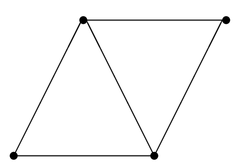
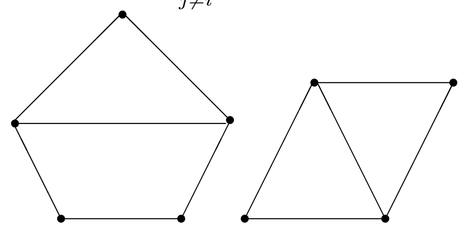


Fig 16

Case 12: For the configuration of Fig 17(a), N= 12, M=$\frac{1}{2}\sum_{j\neq i} a_{ij}^{(2)}(d_j - a_{ij} - 1)(a_{ij}a_{ij}^{(2)})$ (See theorem 1.7). Let P_1 denotes the number of walks in all subgraphs of G that have the same configuration as in Figure 17(b) and are counted in M. Thus $P_1 = 4 \times \frac{1}{2} \times \sum_{j \neq i} a_{ij} \begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}$, where $\frac{1}{2} \times \sum_{j \neq i} a_{ij} \begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}$ is the number of subgraphs of G that have the same configuration as in Figure 17(b) and 4 is the number of times that this Fig is counted in M. Consequently, $F = 6 \sum_{j \neq i} a_{ij}^{(2)} (d_j - a_{ij} - 1)(a_{ij} a_{ij}^{(2)}) - 24 \sum_{j \neq i} a_{ij} \begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}$.



(a) (b)

Fig 17

Case 13: For the configuration of Fig 18, N= 16, $M = \frac{1}{2} \sum_{i=1}^{n} a_{ii}^{(3)} \begin{pmatrix} d_i - 2 \\ 2 \end{pmatrix}$ and $F = 8 \sum_{i=1}^{n} a_{ii}^{(3)} \begin{pmatrix} d_i - 2 \\ 2 \end{pmatrix}$.

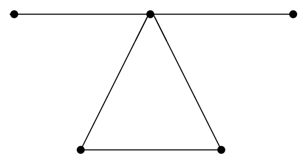


Fig 18

Case 14: For the configuration of Fig 19, N= 32, $M = \frac{1}{8}$ ($\mathrm{tr}A^4 - \mathrm{tr}A^2 - 2 \sum_{j \neq i} a_{ij}^{(2)}$) and $F = 4$ ($\mathrm{tr}A^4 - \mathrm{tr}A^2 - 2 \sum_{j \neq i} a_{ij}^{(2)}$) (See Theorem 1.2) .

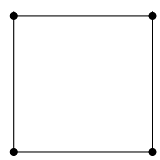


Fig 19

Case 15: For the configuration of Figure 20, N= 30, $M = \frac{1}{10}[\mathrm{tr}(A^5) + 5\ \mathrm{tr}(A^3) - 5 \sum_{i=1}^{n} d_i a_{ii}^{(3)}]$ (See Theorem 1.3) and $F = 3\mathrm{tr}(A^5) + 15\mathrm{tr}(A^3) - 15 \sum_{i=1}^{n} d_i a_{ii}^{(3)}$.

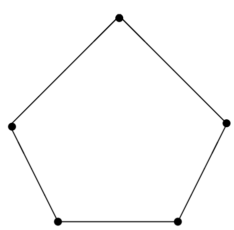
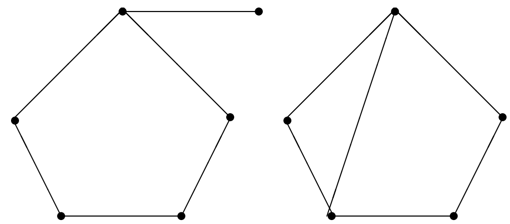


Fig 20

Case 16: For the configuration of Figure 21(a), N= 4, $M = \frac{1}{2} [\sum_{i=1}^{n} [(a_{ii}^{(5)} - 5a_{ii}^{(3)} - 2(d_i - 2)a_{ii}^{(3)})(d_i - 2) - 2 \sum_{j=1, j \neq i}^{n} a_{ij}^{(2)} a_{ij} (d_j - 2)(d_i - 2) - 2 \sum_{j=1, j \neq i}^{n} a_{ij}(d_i - 2)(\frac{1}{2} a_{jj}^{(3)} - a_{ij} a_{ij}^{(2)})]]$ (See Theorem 2.1). Let P_1 denotes the number of all subgraphs of G that have the same configuration as the graph of Fig 21(b) and are counted in M. Thus $P_1 = 2 \times [\frac{1}{2} \sum_{j \neq i} a_{ij}^{(2)} (d_j - a_{ij} - 1)(a_{ij} a_{ij}^{(2)}) - 2 \sum_{j \neq i} a_{ij} \begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}]$, where $\frac{1}{2} \sum_{j \neq i} a_{ij}^{(2)} (d_j - a_{ij} - 1)(a_{ij} a_{ij}^{(2)}) - 2 \sum_{j \neq i} a_{ij} \begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 21(b) (See case 12) and 2 is the number of times that this subgraph is counted in M. Consequently, F$= 2\sum_{i=1}^{n}(a_{ii}^{(5)} - 5a_{ii}^{(3)} - 2(d_i-2)a_{ii}^{(3)})(d_i - 2) - 4\sum_{j\neq i} a_{ij}^{(2)} a_{ij}(d_j-2)(d_i-2) - 4\sum_{j\neq i} a_{ij}(d_i-2)(\frac{1}{2}a_{jj}^{(3)} - a_{ij}a_{ij}^{(2)}) - 4\sum_{j\neq i} a_{ij}^{(2)}(d_j - a_{ij} - 1)(a_{ij}a_{ij}^{(2)}) + 16\sum_{j\neq i} a_{ij}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}$.



(a) (b)

Fig 21

Case 17: For the configuration of Figure 22, N= 24, M$= \sum_{i=1}^{n}\begin{pmatrix} d_i \\ 4 \end{pmatrix}$ and F$= 24\sum_{i=1}^{n}\begin{pmatrix} d_i \\ 4 \end{pmatrix}$.

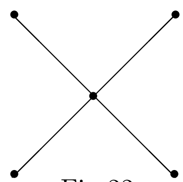


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Fig 22

Case 18: For the configuration of Fig 23, N= 12, M$= \sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}(d_i-3)a_{ij}$ and F$= 12\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}(d_i-3)a_{ij}$.

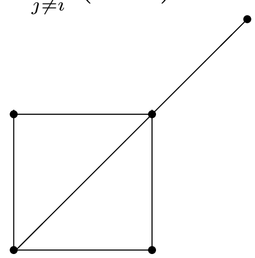
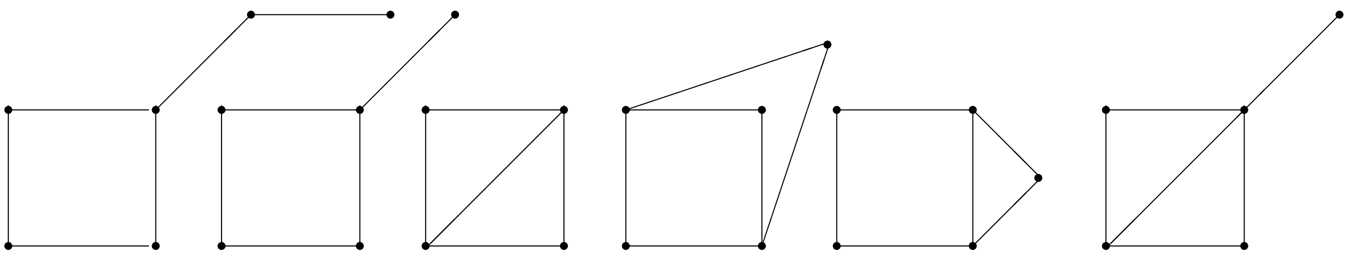


Fig 23

Case 19: For the configuration of Fig 24(a), N= 4, M$= \frac{1}{2}\sum_{i=1}^{n}[(a_{ii}^{(4)} - a_{ii}^{(2)} - 2\begin{pmatrix} d_i \\ 2 \end{pmatrix} - \sum_{j=1,j\neq i}^{n} a_{ij}^{(2)})(\sum_{j=1,j\neq i}^{n}(a_{ij}^{(2)}) - 2)]$ (See Theorem 1.13). Let P_1 denotes the number of all subgraphs of G that have the same configuration as the graph of Fig 24(b) and are counted in M. Thus $P_1 = 2\times[\frac{1}{2}\sum_{i=1}^{n}(a_{ii}^{(4)} - a_{ii}^{(2)} - 2\begin{pmatrix} d_i \\ 2 \end{pmatrix} - \sum_{j=1,j\neq i}^{n} a_{ij}^{(2)})(d_i-2) - \sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}a_{ij}]$, where $\frac{1}{2}\sum_{i=1}^{n}(a_{ii}^{(4)} - a_{ii}^{(2)} - 2\begin{pmatrix} d_i \\ 2 \end{pmatrix} - \sum_{j=1,j\neq i}^{n} a_{ij}^{(2)})(d_i-2) - \sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}a_{ij}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 24(b) (See case 21) and 2 is the number of times that this subgraph is counted in M. Let P_2 denotes the number of all subgraphs of G that have the same configuration as the graph of Fig 24(c) and are counted in M. Thus $P_2 = 8\times\frac{1}{2}\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}a_{ij}$, where $\frac{1}{2}\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}a_{ij}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 24(c) (See case 11) and 8 is the number of times that this subgraph is counted in M. Let P_3 denotes the number of all subgraphs of G that have the same configuration as the graph of Fig 24(d) and are counted in M. Thus $P_3 = 6\times\frac{1}{2}\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 3 \end{pmatrix}$, where $\frac{1}{2}\sum_{j\neq i}\binom{a_{ij}^{(2)}}{3}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 24(d) (See case 22) and 6 is the number of times that this subgraph is counted in M. Let P_4 denotes the number of all subgraphs of G that have the same configuration as the graph of Fig 24(e) and are counted in M. Thus $P_4 = 2\times[\frac{1}{2}\sum_{j\neq i} a_{ij}^{(2)}(d_j - a_{ij} - 1)(a_{ij}a_{ij}^{(2)}) - 2\sum_{j\neq i} a_{ij}\binom{a_{ij}^{(2)}}{2}]$, where $\frac{1}{2}\sum_{j\neq i} a_{ij}^{(2)}(d_j - a_{ij} - 1)(a_{ij}a_{ij}^{(2)}) - 2\sum_{j\neq i} a_{ij}\binom{a_{ij}^{(2)}}{2}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 24(e) (See case 12) and 2 is the number of times that this subgraph is counted in M. Let P_5 denotes the number of all subgraphs of G that have the same configuration as the graph of Fig 24(f) and are counted in M. Thus $P_5 = 1\times\sum_{j\neq i}\binom{a_{ij}^{(2)}}{2}(d_i - 3)a_{ij}$, where $\sum_{j\neq i}\binom{a_{ij}^{(2)}}{2}(d_i - 3)a_{ij}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 24(f) (See case 18) and 1 is the number of times that this subgraph is counted in M. Consequently, $F = 2\sum_{i=1}^{n}[(a_{ii}^{(4)} - a_{ii}^{(2)} - 2\binom{d_i}{2} - \sum_{j=1,j\neq i}^{n} a_{ij}^{(2)})(\sum_{j=1,j\neq i}^{n}(a_{ij}^{(2)}) - 2)] - 4\sum_{i=1}^{n}(a_{ii}^{(4)} - a_{ii}^{(2)} - 2\binom{d_i}{2} - \sum_{j=1,j\neq i}^{n} a_{ij}^{(2)})(d_i - 2) + 8\sum_{j\neq i}\binom{a_{ij}^{(2)}}{2}a_{ij} - 4\sum_{j\neq i} a_{ij}^{(2)}(d_j - a_{ij} - 1)(a_{ij}a_{ij}^{(2)}) - 12\sum_{j\neq i}\binom{a_{ij}^{(2)}}{3} - 4\sum_{j\neq i}\binom{a_{ij}^{(2)}}{2}(d_i - 3)a_{ij}$.

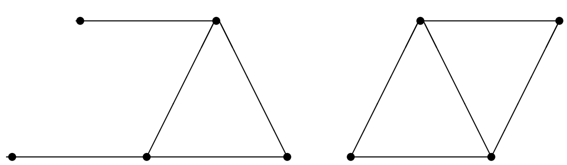


(a) (b) (c) (d) (e) (f)

Fig 24

Case 20: For the configuration of Figure 25(a), N=14, $M = \frac{1}{2}\sum_{j\neq i} a_{ij}^{(2)}a_{ij}(d_j - 2)(d_i - 2)$. Let P_1 denotes the number of all subgraphs of G that have the same configuration as in Figure 25(b) and are counted in M. Thus $P_1 = 2\times\frac{1}{2}\sum_{j\neq i} a_{ij}\binom{a_{ij}^{(2)}}{2}$, where $\frac{1}{2}\sum_{j\neq i} a_{ij}\binom{a_{ij}^{(2)}}{2}$ is the number of subgraphs of G that have the same configuration as in Figure 25(b) and 2 is the number of times that this subgraph is counted in M.

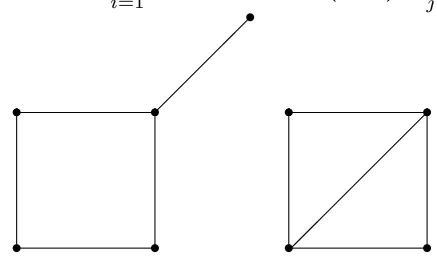
Consequently, $F = 7\sum_{j\neq i} a_{ij}^{(2)}a_{ij}(d_j - 2)(d_i - 2) - 14\sum_{j\neq i} a_{ij}\binom{a_{ij}^{(2)}}{2}$.



(a) (b)

Fig 25

Case 21: For the configuration of Fig 26(a), N= 12, $M = \frac{1}{2}\sum_{i=1}^{n}(a_{ii}^{(4)} - a_{ii}^{(2)} - 2\binom{d_i}{2} - \sum_{j=1,j\neq i}^{n} a_{ij}^{(2)})(d_i - 2)$ (See Theorem 1.13). Let P_1 denotes the number of all subgraphs of G that have the same configuration as the graph of Fig 26(b) and are counted in M. Thus $P_1 = 2\times\frac{1}{2}\sum_{j\neq i}\binom{a_{ij}^{(2)}}{2}a_{ij}$, where $\frac{1}{2}\sum_{j\neq i}\binom{a_{ij}^{(2)}}{2}a_{ij}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 26(b) and 2 is the number of times that this subgraph is counted in M. Consequently, F= $6\sum_{i=1}^{n}(a_{ii}^{(4)}-a_{ii}^{(2)}-2\begin{pmatrix} d_i \\ 2 \end{pmatrix}-\sum_{j=1,j\neq i}^{n}a_{ij}^{(2)})(d_i-2)-12\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}a_{ij}$.



(a) (b)

Fig 26

Case 22: For the configuration of Figure 27, N=12, M= $\frac{1}{2}\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 3 \end{pmatrix}$, F= $6\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 3 \end{pmatrix}$.

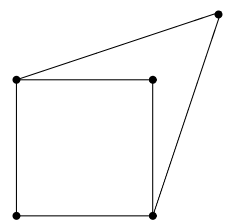
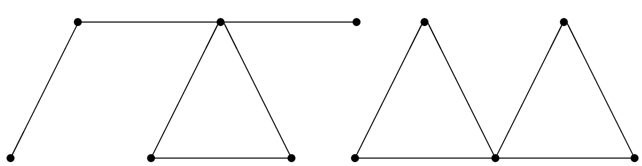


Fig 27

Case 23: For the configuration of Fig 28(a), N= 4, M= $\frac{1}{2}\sum_{j\neq i}a_{ii}^{(3)}a_{ij}^{(2)}(d_i-3)-\sum_{j\neq i}a_{ij}^{(2)}a_{ij}(d_i-3)-\sum_{i=1}^{n}a_{ii}^{(3)}(d_i-2)(d_i-3)-2\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}(d_i-3)a_{ij}$ (See case 9). Let P_1 denotes the number of all subgraphs of G that have the same configuration as the graph of Fig 28(b) and are counted in M. Thus $P_1= 4\times[\sum_{i=1}^{n}\begin{pmatrix} \frac{1}{2}a_{ii}^{(3)} \\ 2 \end{pmatrix}-\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}a_{ij}]$, where $\sum_{i=1}^{n}\begin{pmatrix} \frac{1}{2}a_{ii}^{(3)} \\ 2 \end{pmatrix}-\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}a_{ij}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 28(b) and 4 is the number of times that this subgraph is counted in M. Consequently, F= $2\sum_{j\neq i}a_{ii}^{(3)}a_{ij}^{(2)}(d_i-3)-4\sum_{j\neq i}a_{ij}^{(2)}a_{ij}(d_i-3)-4\sum_{i=1}^{n}a_{ii}^{(3)}(d_i-2)(d_i-3)-8\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}(d_i-3)a_{ij}-16\sum_{i=1}^{n}\begin{pmatrix} \frac{1}{2}a_{ii}^{(3)} \\ 2 \end{pmatrix}+16\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}a_{ij}$.

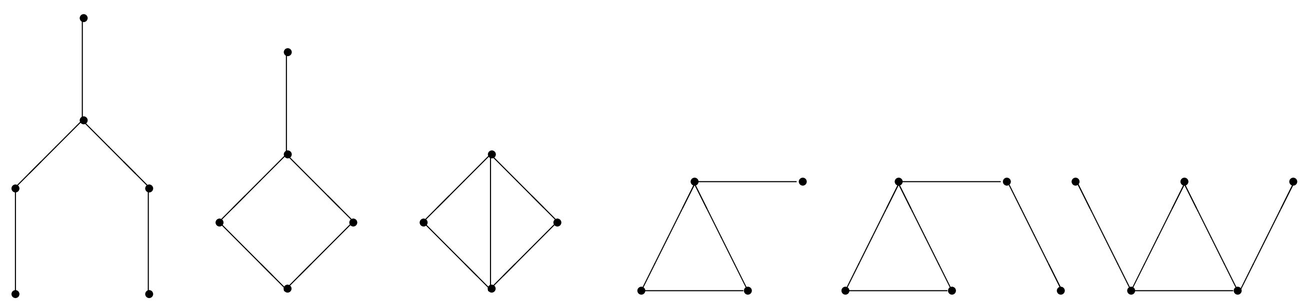


(a) (b)

Fig 28

Case 24: For the configuration of Fig 29(a), N= 2, M= $\sum_{i=1}^{n}[\begin{pmatrix} \sum_{j=1,j\neq i}^{n}a_{ij}^{(2)} \\ 2 \end{pmatrix}-\sum_{j\neq i}\begin{pmatrix} d_j-a_{ij} \\ 2 \end{pmatrix}a_{ij}](d_i-2)$. Let P_1 denotes the number of all subgraphs of G that have the same configuration as the graph of Fig 29(b) and are counted in M. Thus $P_1 = 1\times[\frac{1}{2}\sum_{i=1}^{n}(a_{ii}^{(4)}-a_{ii}^{(2)}-2\begin{pmatrix} d_i \\ 2 \end{pmatrix}-\sum_{j=1,j\neq i}^{n}a_{ij}^{(2)})(d_i-2)-\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}a_{ij}]$, where

$\frac{1}{2}\sum_{i=1}^{n}(a_{ii}^{(4)} - a_{ii}^{(2)} - 2\begin{pmatrix} d_i \\ 2 \end{pmatrix} - \sum_{j=1,j\neq i}^{n} a_{ij}^{(2)})(d_i-2) - \sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix} a_{ij}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 29(b) (See case 21) and 1 is the number of times that this subgraph is counted in M. Let P_2 denotes the number of all subgraphs of G that have the same configuration as the graph of Fig 29(c) and are counted in M. Thus $P_2 = 6 \times \frac{1}{2}\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix} a_{ij}$, where $\frac{1}{2}\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix} a_{ij}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 29(c) (See case 11) and 6 is the number of times that this subgraph is counted in M. Let P_3 denotes the number of all subgraphs of G that have the same configuration as the graph of Fig 29(d) and are counted in M. Thus $P_3 = 1 \times \frac{1}{2}\sum_{i=1}^{n} a_{ii}^{(3)}(d_i-2)$, where $\frac{1}{2}\sum_{i=1}^{n} a_{ii}^{(3)}(d_i-2)$ is the number of subgraphs of G that have the same configuration as the graph of Fig 29(d) and 1 is the number of times that this subgraph is counted in M. Let P_4 denotes the number of all subgraphs of G that have the same configuration as the graph of Fig 29(e) and are counted in M. Thus $P_4 = 2 \times [\frac{1}{2}\sum_{j\neq i} a_{ii}^{(3)} a_{ij}^{(2)} - \sum_{i=1}^{n} a_{ii}^{(3)} - \sum_{i=1}^{n} a_{ii}^{(3)}(d_i-2) - 2\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix} a_{ij}]$, where $\frac{1}{2}\sum_{j\neq i} a_{ii}^{(3)} a_{ij}^{(2)} - \sum_{i=1}^{n} a_{ii}^{(3)} - \sum_{i=1}^{n} a_{ii}^{(3)}(d_i-2) - 2\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix} a_{ij}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 29(e) (See case 9) and 2 is the number of times that this subgraph is counted in M. Let P_5 denotes the number of all subgraphs of G that have the same configuration as the graph of Fig 29(f) and are counted in M. Thus $P_5 = 2 \times [\frac{1}{2}\sum_{j\neq i} a_{ij}^{(2)} a_{ij}(d_j-2)(d_i-2) - \sum_{j\neq i} a_{ij}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}]$, where $\frac{1}{2}\sum_{j\neq i} a_{ij}^{(2)} a_{ij}(d_j-2)(d_i-2) - \sum_{j\neq i} a_{ij}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 29(f) (See case 20) and 2 is the number of times that this subgraph is counted in M. Consequently, F= $2\sum_{i=1}^{n}[\begin{pmatrix} \sum_{j=1,j\neq i}^{n} a_{ij}^{(2)} \\ 2 \end{pmatrix} - \sum_{j\neq i}\begin{pmatrix} d_j - a_{ij} \\ 2 \end{pmatrix} a_{ij}](d_i-2) - \sum_{i=1}^{n}(a_{ii}^{(4)} - a_{ii}^{(2)} - 2\begin{pmatrix} d_i \\ 2 \end{pmatrix} - 2\sum_{j=1,j\neq i}^{n} a_{ij}^{(2)})(d_i - 2) - 2\sum_{j\neq i} a_{ii}^{(3)} a_{ij}^{(2)} + 4\sum_{i=1}^{n} a_{ii}^{(3)} + 3\sum_{i=1}^{n} a_{ii}^{(3)}(d_i-2) + 8\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix} a_{ij} - 2\sum_{j\neq i} a_{ij}^{(2)} a_{ij}(d_j-2)(d_i-2)$



(a) (b) (c) (d) (e) (f)

Fig 29

Case 25: For the configuration of Fig 30(a), N= 4, M= $\sum_{j\neq i} a_{ij}^{(2)}(d_j - a_{ij} - 1)\begin{pmatrix} d_i - a_{ij} - 1 \\ 2 \end{pmatrix}$ (See case 2). Let P_1 denotes the number of all subgraphs of G that have the same configuration as the graph of Fig 30(b) and are counted in M. Thus P_1= 2× $[\frac{1}{2}\sum_{j\neq i} a_{ij}^{(2)} a_{ij}(d_j-2)(d_i-2) - \sum_{j\neq i} a_{ij}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}]$, where $\frac{1}{2}\sum_{j\neq i} a_{ij}^{(2)} a_{ij}(d_j-2)(d_i-2) - \sum_{j\neq i} a_{ij}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 30(b) (See case 20) and 2 is the number of times that this subgraph is counted in M.

Consequently, F= $4\sum_{j\neq i} a_{ij}^{(2)}(d_j - a_{ij} - 1)\begin{pmatrix} d_i - a_{ij} - 1 \\ 2 \end{pmatrix} - 4\sum_{j\neq i} a_{ij}^{(2)} a_{ij}(d_j - 2)(d_i - 2) + 8\sum_{j\neq i} a_{ij}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}$.

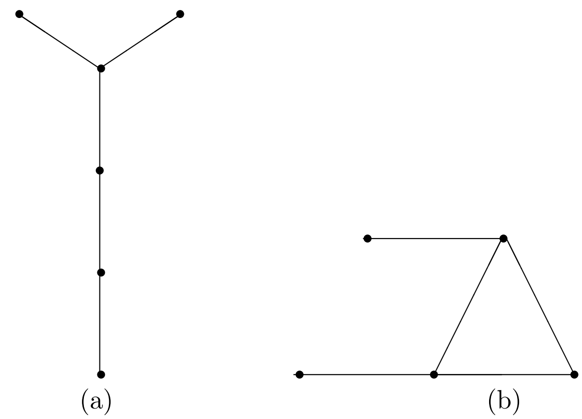


Fig 30

Case 26: For the configuration of Fig 31(a), N= 4, M= $\frac{1}{2}\sum_{i=1}^{n}(a_{ii}^{(4)} - a_{ii}^{(2)} - 2\begin{pmatrix} d_i \\ 2 \end{pmatrix} - \sum_{j=1,j\neq i}^{n} a_{ij}^{(2)})\begin{pmatrix} d_i - 2 \\ 2 \end{pmatrix}$ (See Theorem 1.13). Let P_1 denotes the number of all subgraphs of G that have the same configuration as the graph of Fig 31(b) and are counted in M. Thus $P_1 = 1\times\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}(d_i - 3)a_{ij}$ (See case 18), where $\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}(d_i - 3)a_{ij}$ is the number of subgraphs of G that have the same configuration as the graph of Fig 31(b) and 1 is the number of times that this subgraph is counted in M. Consequently, F= $2\sum_{i=1}^{n}(a_{ii}^{(4)} - a_{ii}^{(2)} - 2\begin{pmatrix} d_i \\ 2 \end{pmatrix} - \sum_{j=1,j\neq i}^{n} a_{ij}^{(2)})\begin{pmatrix} d_i - 2 \\ 2 \end{pmatrix} - 4\sum_{j\neq i}\begin{pmatrix} a_{ij}^{(2)} \\ 2 \end{pmatrix}(d_i - 3)a_{ij}$.

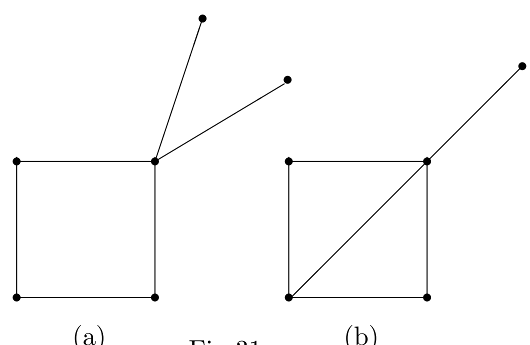


Fig 31

Now we add the values of F arising from the above cases and determine x. By putting the value of x in $\sum_{j\neq i} a_{ij}^{(6)} - x$ and simplifying, we get the desired result. □

Example 2.4 *In K_7 we have, Case $1 = 840$, Case $2 = 6720$, Case $3 = 17640$, Case $4 = 10080$, Case $5 = 3570$, Case $6 = 31080$, Case $7 = 12600$, Case $8 = 4200$, Case $9 = 25200$, Case $10 = 10080$, Case $11 = 13440$, Case $12 = 15120$, Case $13 = 10080$, Case $14 = 3360$, Case $15 = 7560$, Case $16 = 10080$, Case $17 = 2520$, Case $18 = 15120$, Case $19 = 10080$, Case $20 = 17640$, Case $21 = 15120$, Case $22 = 2520$, Case $23 = 10080$, Case $24 = 5040$, Case $25 = 10080$, Case $26 = 5040$. So, we have $x = 274890$ and $\sum_{j\neq i} a_{ij}^{(6)} = 279930$.*

Consequently, by theorem 2.3, the number of paths of length 6 in K_7 is 5040.

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