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Mhd Flow and Heat Transfer of a Non-Newtonian Power-Law Fluid Past a Stretching Sheet With Suction/Injection and Viscous Dissipation

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Abstract

This paper investigates the MHD effects on convection heat transfer of an electrically conducting, non-Newtonian power-law stretched sheet with surface heat flux by considering the viscous dissipation. The effects of suction/injection at the surface are considered. The resulting governing equations are transformed into non linear ordinary differential equations using appropriate transformation. The set of non linear ordinary differential equations are first linearized by using Quasi-linearization technique and then solved numerically by using implicit finite difference scheme. Then the system of algebraic equations is solved by using Gauss-Seidal iterative method. The solution is found to be dependent on six governing parameters including the magnetic field parameter M, the power-law fluid index n, the sheet velocity exponent p, the suction/blowing parameter f_w, Eckert number Ec and the generalized Prandtl number Pr. Numerical results are tabulated for skin friction co-efficient and the local Nusselt number. Velocity and Temperature profiles drawn for different controlling parameters reveal the tendency of the solution.

Keywords: *Magnetic field effects, Non-Newtonian power-law fluid, Power-law stretched sheet, suction/injection, Surface heat flux and Viscous dissipation.*

1 Introduction

Most of the fluids such as molten plastics, artificial fibers, drilling of petroleum, blood and polymer solutions are considered as non-Newtonian fluids. In modern technology and in industrial applications, non-Newtonian fluids play an important role. Increasing emergence of non-Newtonian fluids such as molten plastic pulp, emulsions, raw materials in a great variety of industries like petroleum and chemical processes has stimulated a considerable amount of interest in the study of the behavior of such fluids when in motion. Exact solutions of the equations of motion of non- Newtonian fluids are difficult to obtain. The difficulty arises not only due to the non-linearity but also due to increase in the order of differential equations. Many researchers have attempted to find the exact solution of non-Newtonian fluids.

The study of flow and heat transfer problems due to stretching boundary has many practical applications in technological processes, particularly in polymer processing systems involving drawing of fibers and films or thin sheets, etc. Sometimes the polymer sheet is stretched while it is extruded from a die. Usually the sheet is pulled through the viscous liquid with desired characteristics. The moving sheet may introduce a motion in the neighbouring fluid or alternatively, the fluid may have an independent forced convection motion which is parallel to that of the sheet. Sakiadis [1] was the first to investigate the flow due to sheet issuing with constant speed from a slit into a fluid at rest. Schowalter [2] has introduced the concept of the boundary layer in the theory of non-Newtonian power-law fluids. Acrivos, Shah and Petersen [3] have investigated the steady laminar flow on non-Newtonian fluids over a plate.

The interest in MHD fluid flows stems from the fact that liquid metals which occur in nature and industry are electrically conducting. Naturally, studies of these systems are mathematically interesting and physically useful but the dynamical study of such flow problems is usually complicated. However, these problems are usually investigated under various simplifying assumptions. For the non-Newtonian power-law fluids, the hydrodynamic problem of the MHD boundary layer flow over a continuously moving surface has been dealt with by several authors (e.g. Andersson et al. [4], Cortell [5] and Mahmoud and Mahmoud [6]). B. Singh and C.Thakur [7] studied the unsteady two dimensional, second grade, electrically conducting MHD non-Newtonian fluid flows.

Chiam [8] studied the boundary layer flow of a Newtonian fluid over a stretching plate in the presence of a transverse magnetic field. Pop and Na [9] performed an analysis for the MHD flow past a stretching permeable surface. Kishan and B.S. Reddy [10] studied the MHD effects on boundary layer flow of power-law fluids past a semi infinite flat plate with thermal dispersion. The well known Ostwald – de- Waele power-law model has been employed on the problem of a stretching surface by Andersson et al. [4], Kumari and Nath [11] & [12] and Liao [13].

Recently Chien-Hsin Chen [14] has studied the magneto-hydrodynamic flow and heat transfer of an electrically conducting, non-Newtonian power-law fluid past a stretching sheet in the presence of a transverse magnetic field by considering suction/injection. However in the existing convective heat transfer literature on the non-Newtonian fluids, the effect of the viscous dissipation has been generally disregarded. There are only few studies to be cited. Gebhart [15] has shown that the viscous dissipation effect plays an important role in natural convection in various devices processes on large scales(or large planets). Also, he pointed out that when the temperature is small or when the gravitational field is of high intensity, viscous dissipation heat should be taken into account. Therefore, the effect of viscous dissipation is more predominant in vigorous natural convection processes. Lawal and Mujumdar [16] studied viscous dissipation effect of heat transfer power-law fluids in arbitrary cross-sectional ducts.

The present work deals with the flow and heat transfer of electrically conducting, non-Newtonian power-law fluids past a continuously stretching sheet under the action of a transverse magnetic field with suction/injection by taking into account the effect of viscous dissipation.

2 Mathematical Formulation

Let us consider a steady two-dimensional flow of an incompressible, electrically conducting fluid obeying the power-law model past a permeable stretching sheet. The origin is located at the slit through which the sheet is drawn through the fluid medium, the x-axis is chosen along the sheet and y-axis is taken normal to it. This continuous sheet is assumed to move with a velocity according to a power-law form, i.e. = $C.x^p$, and be subject to a surface heat flux. Also, a magnetic field of strength B is applied in the positive y-direction, which produces magnetic effect in the x-direction. The magnetic Reynolds number is assumed to be small so that the induced magnetic field is negligible in comparison to the applied electric field and the Hall Effect is neglected.

Under the foregoing assumptions and invoking the usual boundary layer approximations, the problem is governed by the following equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial U}{\partial y}=\frac{K}{\rho}\frac{\partial}{\partial y}\left(\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right)-\frac{\sigma B^2 u}{\rho} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{K}{\rho C_p}\left|\frac{\partial u}{\partial y}\right|^{n+1} \tag{3}$$

Where u and ν are the velocity components, T is the temperature, B is the magnetic field strength, K is the consistency coefficient, n is the flow behavior index, ρ is the density, σ is the electrical conductivity and α is the thermal diffusivity. The appropriate boundary conditions are given by

$$u_w(x) = C.x^p, \qquad \nu = \nu_w, \qquad \frac{\partial T}{\partial y} = -\frac{q_w}{k} \qquad \text{at } y = 0,\ x > 0 \tag{4}$$

$$u \to 0, \qquad T \to T_\infty \qquad \text{as } y \to \infty \tag{5}$$

where ν_w is the surface mass flux and q_w is the surface heat flux. It should be noted that positive p indicates that the surface is accelerated while negative p implies that the surface is decelerated from the slit. Also note that positive ν_w is for fluid injection and negative for fluid suction at the sheet surface.

3 Method of Solution

We shall further transform equations (2) & (3) into a set of partial differential equations amenable to a numerical solution. For this purpose we introduce the variables

$$\eta = \left(\frac{C^{2-n}}{K/\rho}\right)^{1/(n+1)} x^{[p(2-n)-1]/(n+1)} y \tag{6}$$

$$\psi = \left(\frac{C^{1-2n}}{K/\rho}\right)^{-1/(n+1)} x^{[p(2n-1)+1]/(n+1)} f \tag{7}$$

$$\phi = \frac{(T - T_\infty)\,\mathrm{Re}_x^{1/(n+1)}}{q_w x/k} \tag{8}$$

Where the dimensionless stream function *f* satisfies the continuity equation with

$$u = \frac{\partial \psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x}.$$

Under the transformations (6), (7) and (8), the differential equations (2) and (3) reduce to

$$\left(\left|f''\right|^{n-1} f''\right)' + \frac{p(2n-1)+1}{n+1} ff'' - p(f')^2 - Mf' = 0 \tag{9}$$

$$\frac{1}{\Pr}\phi'' + \frac{p(2n-1)+1}{n+1} f\phi' + \frac{p(2-n)-1}{n+1} f'\phi + Ec\left|f''\right|^{n+1} = 0 \tag{10}$$

With the boundary conditions

$$\left.\begin{aligned} & f'(0)=1,\ f(0)=\frac{n+1}{p(2n-1)+1}\ f_w, \phi'(0)=-1 \\ & f'(\infty)=0, \phi(\infty)=0 \end{aligned}\right\} \tag{11}$$

Where M is the magnetic field parameter, f_w is the suction/injection parameter, Pr is the generalized Prandtl number for the power law fluid, Ec is the Eckert number and primes indicate the differentiation with respective to η. The parameters M, f_w, Pr and Ec are defined as

$$M=\frac{\sigma B^2 x}{\rho u_w}, \qquad f_w=-\frac{v_w}{u_w}\mathrm{Re}_x^{1/(n+1)},$$

$$\mathrm{Pr}=\frac{x u_w}{\alpha}\mathrm{Re}_x^{-2/(n+1)}, \qquad Ec=-\frac{u_w^2 k}{x q_w C_p}\mathrm{Re}_x^{1/(n+1)}$$

Where $\mathrm{Re}_x=\frac{\rho u_w^{2-n} x^n}{K}$ is the local Reynolds number. Note here that the magnetic field strength B should be proportional to x to the power (p-1)/2 to eliminate the dependence of M on x, i.e. $B(x)=B_0 x^{(p-1)/2}$ where B_0 is a constant. Quantities of main interest include the velocity components u and ν, the skin friction coefficient, viscous dissipation and the local Nusselt number. In terms of the new variables, the velocity components can be expressed as

$$u=u_w f'$$

$$v=-u_w \mathrm{Re}_x^{-1/(n+1)}\left(\frac{p(2n-1)+1}{n+1}f+\frac{p(2-n)-1}{n+1}\eta f'\right)$$

The wall shear stress is given by

$$\tau_w=\left[K\left|\frac{\partial u}{\partial y}\right|^{n-1}\frac{\partial u}{\partial y}\right]_{y=0}=\rho u_w^2 \mathrm{Re}_x^{-1/(n+1)}\left|f''(0)\right|^{n-1} f''(0)$$

To solve the system of transformed governing equations (9) and (10) with the boundary conditions (11), first equation (9) is linearized using the Quasi linearization technique[17]. Then equation (9) is changed to

$$n\{[F]^{n-1}f''' + [f'']^{n-1}F''' - [F'']^{n-1}F'''\} + \frac{p(2n-1)+1}{n+1}[Ff'' + fF'' - FF'']$$
$$-p\{2F'f' - (F')^2 - Mf' = 0 \tag{12}$$

Where F is assumed to be a known function and the above equation can be rewritten as

$$A_0 f''' + A_2 f'' - A_3 f' + A_4 f = A_5 - A_1[f'']^{n-1} \tag{13}$$

Where $A_0 = n[F'']^{n-1}$, $A_1 = nF'''$, $A_2 = \frac{p(2n-1)+1}{n+1}F$,

$A_3 = 2pF' - M$, $A_4 = \frac{p(2n-1)+1}{n+1}F''$,

$A_5 = n[F'']^{n-1}F''' + \frac{p(2n-1)+1}{n+1}FF'' - p(F')^2$,

Using implicit finite difference formulae, the equations (13) and (10) are transformed to

$$B_0[i]\, f[i+2] + B_1[i]\, f[i+1] + B_2[i]\, f[i] + B_3[i]\, f[i-1] = B_4[i] \tag{14}$$

$$D_1[i]\, \varphi[i+1] + D_2[i]\, \varphi[i] + D_3[i]\, \varphi[i-1] + D4[i] = 0 \tag{15}$$

where $B_0[i] = 2A_0[i]$, $B_1[i] = 2hA_2[i] - 6A_0[i] - h^2A_3[i]$,

$B_2[i] = 6A_0[i] - 4hA_2[i] + 2h^3A_4[i]$,

$B_3[i] = 2hA_2[i] + h^2A_3[i] - 2A_0[i]$,

$B_4[i] = 2h^3\{A_5[i] - A_1[i](F_2[i])^{n-1}\}$,

$D_1[i] = hC_1[i] + 2$, $D_2[i] = 2h^2C_2[i] - 4$, $D_3[i] = 2 - hC_1[i]$,

$$C_1[i] = \frac{p(2n-1)+1}{n+1}\Pr f$$

$$C_2[i] = \frac{p(2-n)-1}{n+1}\Pr f'$$

here 'h' represents the mesh size in η direction. The system of equations (14) & (15) are solved under the boundary conditions (11) by Gauss-Seidel iteration method and computations were carried out by using C programming. The numerical solutions of f are considered as (n+1)th order iterative solutions and F are the nth order iterative solutions. After each cycle of iteration the convergence check is performed, and the process is terminated when $|F - f| < 10^{-4}$.

4 Skin Friction

The skin friction coefficient is defined as

$$C_f = \frac{\tau_w}{\rho u_w^2 / 2} = 2\,\mathrm{Re}_x^{-1/(n+1)} \left| f''(0) \right|^{n-1} f''(0)$$

Or

$$C_f\,\mathrm{Re}_x^{1/(n+1)} = 2\left| f''(0) \right|^{n-1} f''(0) \tag{16}$$

5 Heat Transfer

The local heat transfer coefficient is

$$h = \frac{q_w}{T_w - T_\infty} = \frac{k\,\mathrm{Re}_x^{1/(n+1)}}{\phi(0)}$$

The local Nusselt number is given by

$$Nu_x = \frac{hx}{k} = \frac{\mathrm{Re}_x^{1/(n+1)}}{\phi(0)}$$

or

$$Nu_x\,\mathrm{Re}_x^{-1/(n+1)} = \frac{1}{\phi(0)} \tag{17}$$

6 Results and Discussions

The effects of various parameters on the skin friction coefficient $-f''(0)$ and the Nusselt number $1/\phi(0)$ are displayed in the tables. The values for $-f''(0)$ are

tabulated for various values of n and M in Table 1. It can be seen from the table that the value of $-f''(0)$ increases as magnetic field parameter M increases and decreases as power law fluid index n increases. Table 2 shows the results of heat transfer obtained for a Newtonian fluid in the absence of magnetic field i.e. n = 1 and M = 0. It is obvious from the table that the Nusselt number $1/\phi(0)$ increases with the increase of Prandtl number Pr and velocity exponent p. Also the effect of viscous dissipation is to reduce the value of Nusselt number $1/\phi(0)$. Table 3 lists the calculations for the flow and heat transfer characteristics, including the sheet surface temperature $\phi(0)$, the skin-friction co-efficient $C_f \operatorname{Re}_x^{1/(n+1)}$ and the Nusselt number $Nu_x \operatorname{Re}_x^{-1/(n+1)}$ for various values of n, M and f_w with Pr = 5 and p = 0.5. It is apparent from this table that the sheet surface temperature increases with increasing the magnetic field parameter M, but it decreases with increasing the suction/injection parameter f_w. With all other parameters fixed, the magnitude of skin-friction coefficient increases with increasing the magnetic field parameter due to the fact that the magnetic field retards the fluid motion and thus increases this coefficient. To impose suction is to increase the skin-friction coefficient, but fluid injection decreases it. Also, the local Nusselt number is decreased as a result of the applied magnetic field. The effect of suction ($f_w > 0$) is found to increase the Nusselt number, whereas injection has the opposite effect. More detailed discussions about the influence of various governing parameters on the local Nusselt number are presented latter by making use of figs. 14-18.

Velocity profiles presented for various values of M, n, p and f_w are shown in figs. 1-6. Figs. 1 and 2 represent the velocity profiles f for different parameters M and n. It is observed from the figures that the velocity decreases as magnetic field increases for both the cases of pseudo plastic and dilatant fluids. And it also decreases as the power-law fluid index n decreases. Velocity profiles f' are shown in figs.3–6 for different parameters n, M, f_w and p. It is observed from fig. 3 that the power-law fluid index n increases as f' increases near the wall and the reverse phenomenon is observed away from the wall. Figs.4 and 5 show that the effect of magnetic field M and suction parameter f_w decelerates the fluid motion for both the cases of pseudo plastic and dilatant fluids. It can be noticed from fig. 6 that the velocity distribution f' decreases as velocity exponent p increases for the both the cases of pseudo plastic and dilatant fluids.

Temperature distributions presented for various values of M, n, p, f_w, Pr and Ec are shown in figures 7- 13. It is clear from the fig. 7 that the effect of magnetic field increases the temperature distribution in the boundary layer. And this behavior is more noticeable for the pseudo plastic fluid (n = 0.5) when compared to dilatant fluid (n = 1.5). Figs. 8(a) and 8(b) represent the temperature profiles for various values of the power-law index n, respectively for an accelerated stretching surface (p = 1) and for a decelerated stretching surface (p = -0.3). It can be observed from the figures that for the accelerated stretching case fluid temperature decreases as the power-law index n increases, whereas an opposite

behavior exist for an decelerated stretching surface. Here, it can be seen that the influence of power-law index n on the wall temperature is more significant for an accelerated stretching surface.

The influence of sheet velocity exponent p on the temperature distributions for n = 0.5 (pseudo plastic fluid) and n = 1.5 (dilatant fluid) are shown in figs. 9(a) and 9(b) respectively. It is clear from the figures that for a pseudo plastic fluid, a considerable increase in the temperature distribution and the surface temperature are caused by increasing the value of p, but reverse to that increase in p reduces the temperature for a dilatant fluid.

The effects of the suction/injection parameter f_w on the temperature profiles are illustrated in fig.10 for a pseudo plastic fluid with n = 0.5. It is clear from the figure that the increase in suction/injection parameter decreases the temperature distribution. Moreover, surface temperature may be reduced considerably by increasing the suction/injection parameter. Fig. 11 reveals the effect of generalized Prandtl number Pr on the temperature distribution for shear thinning fluid (n = 0.5). It is obvious from the figure that an increase in Prandtl number will produce a decrease in the thermal boundary layer thickness, associated with the reduction in the temperature profiles. The effects of suction/injection parameter and Prandtl number on temperature distribution for dilatant fluids are similar to those for the pseudo plastic fluids.

The effect of viscous dissipation on the temperature distribution is shown for different values of n and p in figs. 12 and 13. It shows that an increase in Eckert number will lead to increase in temperature profiles for both dilatant fluids and pseudo plastic fluids. This effect is same for accelerated stretching surface and as well as decelerated stretching surface.

The numerical computations carried out for the discussions about the influence of all governing parameters on the local Nusselt number are shown in figs.14–18. The variation of Nusselt number in terms of $1/\phi(0)$ is presented as a function of magnetic field parameter M in fig. 14 for different values of n with Pr = 10, p = 0.5, $f_w = 0$ and Ec = 0. It is observed that the heat transfer parameter decreases as M increases monotonically, whereas the Nusselt number parameter increases as n increases. It is observed from fig.15 that the value of $1/\phi(0)$ increases with the increase in p for higher values of fluid index n (dilatant fluid) but decreases with increase in p for a lower value of fluid index n (pseudo plastic fluid). This shows that the wall temperature $\phi(0)$ will increase as p increases for a pseudo plastic fluid (n < 1) but the reverse trend is observed for a dilatants fluid (n >1). From fig.16 it is noticed that the heat transfer parameter $1/\phi(0)$ decreases with the decrease of n and increases with the increase in f_w. Fig. 17 shows that the effect of generalized Prandtl number Pr on heat transfer for various values of n and p. It can be seen that the heat transfer rate increases as n increases for an accelerated stretching surface (p = 1), whereas the opposite trend exists for a decelerated stretching surface (p=-0.5).

The heat transfer parameter $1/\phi(0)$ is plotted as a function of Ec in fig.18 for varying values of the parameters n and p. From the figure it is observed that the effect of viscous dissipation is to decrease the heat transfer parameter.

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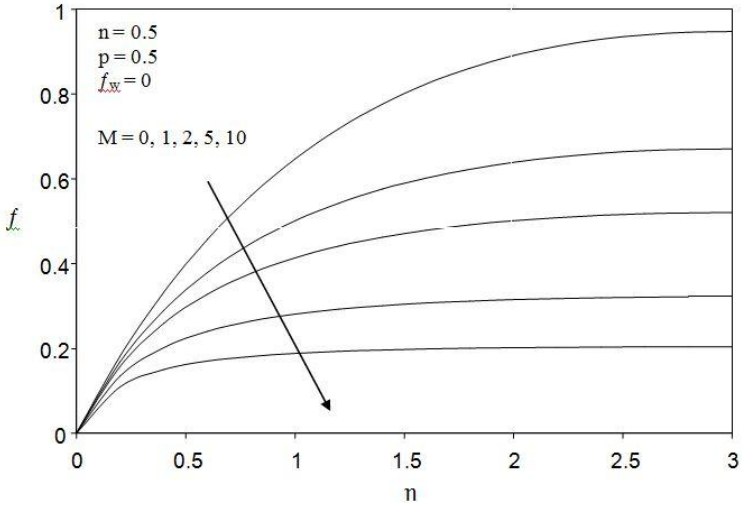
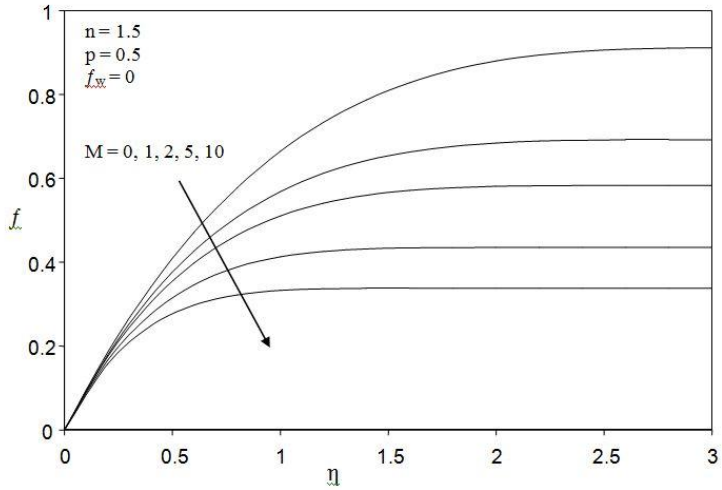


(a)



(b)

Fig.1. Velocity profiles f for different values of M with p = 0.5 and $f_w = 0$.
(a)n = 0.5; (b) n = 1.5 f'

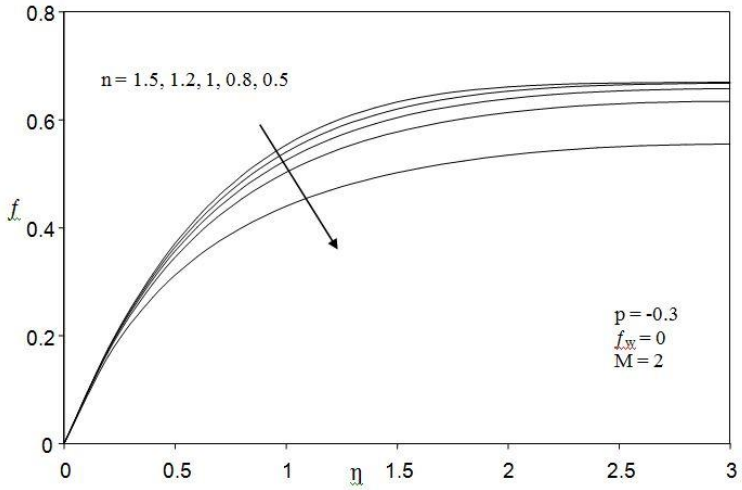


Fig.2. Velocity profiles f for different values of n with f_w = 0, p = -0.3 and M = 2

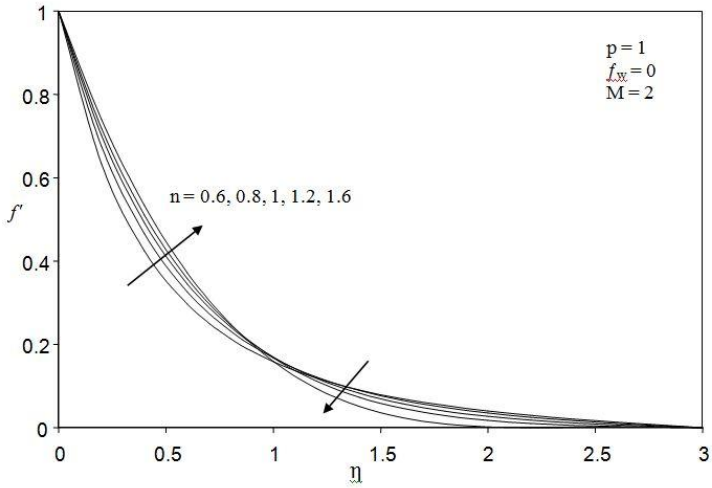
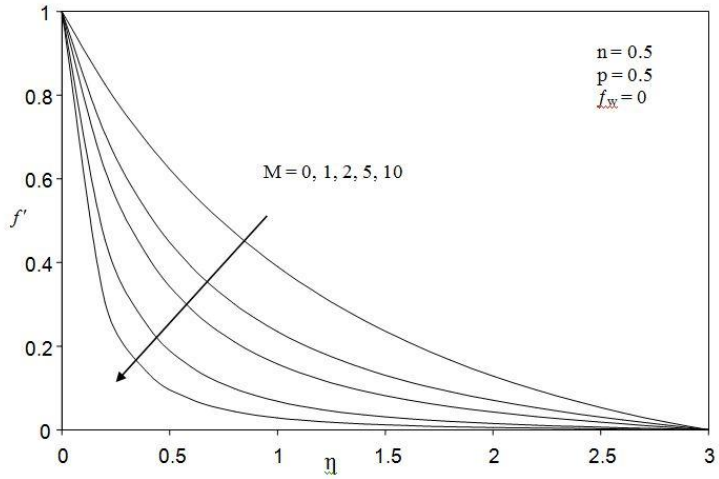


Fig.3. Velocity profiles f' for different values of n with f_w = 0, p = 1 and M = 2.



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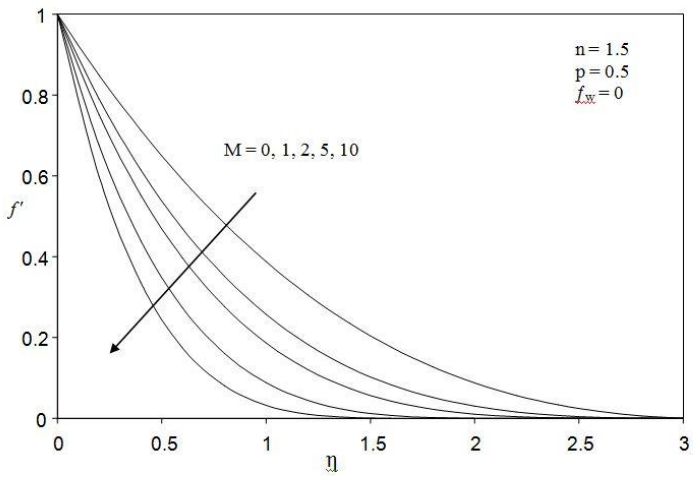
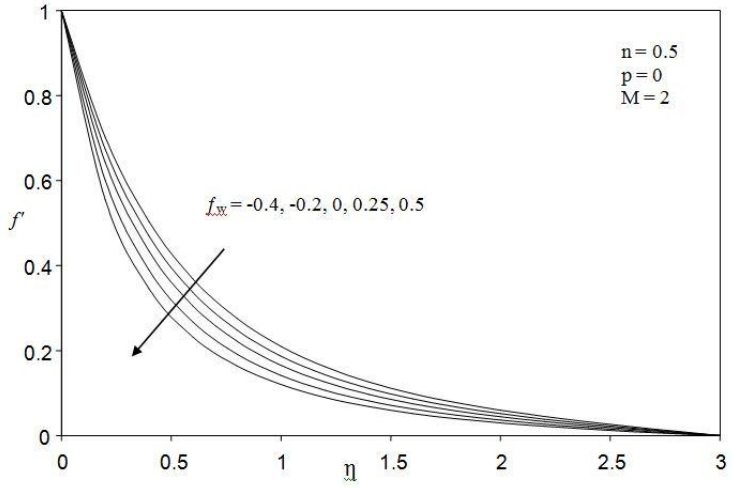
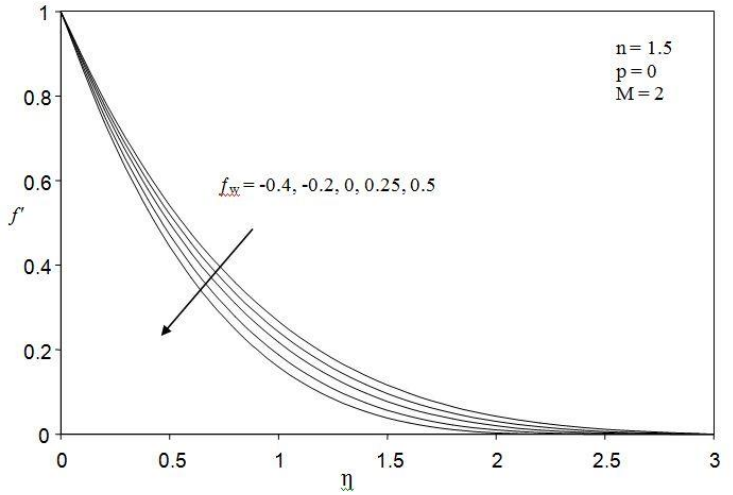


(b)

Fig.4. Velocity profiles f' for different values of M with p = 0.5 and f_w = 0.
(a) n = 0.5; (b) n = 1.5



(a)



(b)

Fig.5. Velocity profiles f' for different values of f_w with p = 0 and M = 2.
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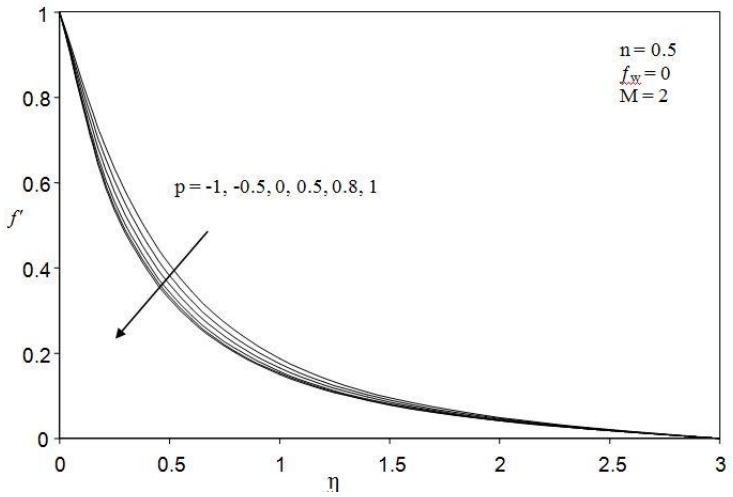
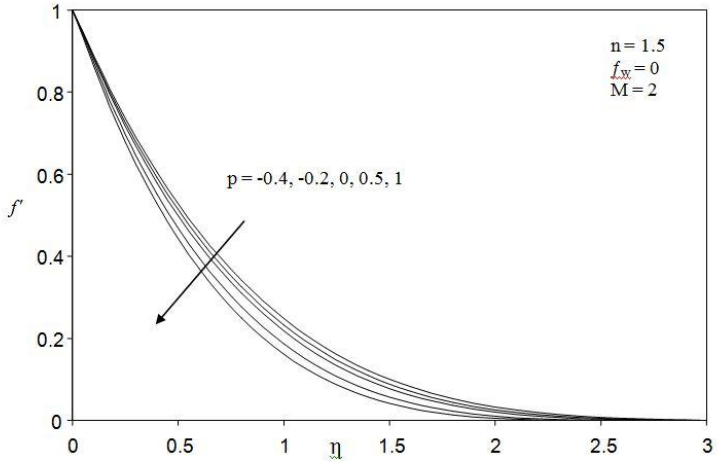
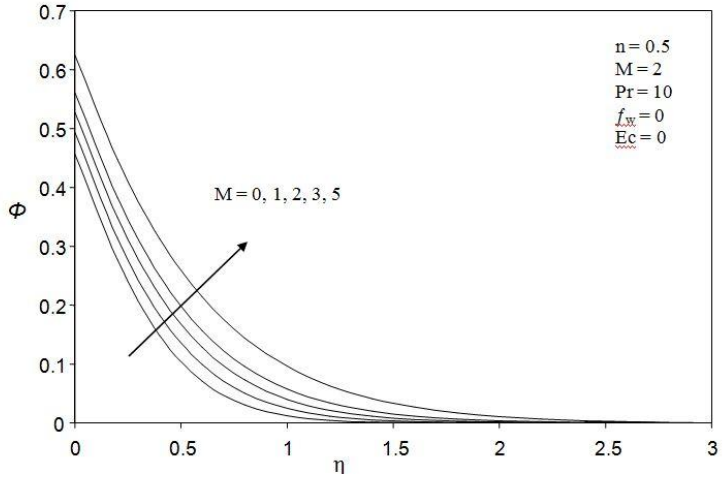


(a)

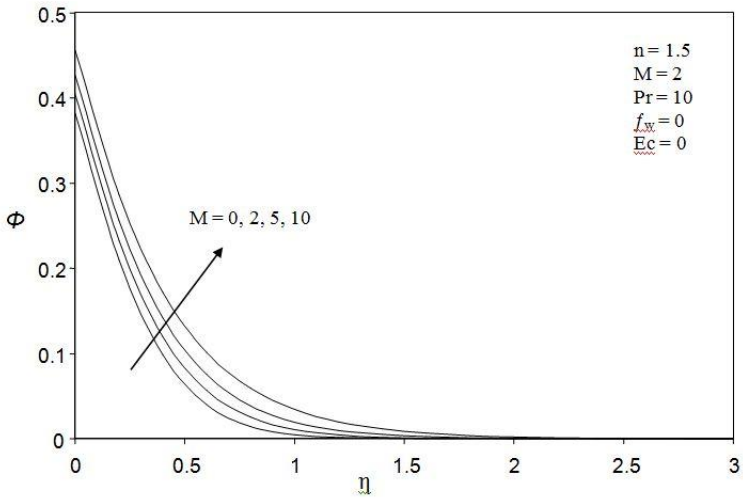


(b)

Fig.6. Velocity profiles f' for different values of p with $f_w = 0$ and M = 2.
(a) n = 0.5; (b) n = 1.5

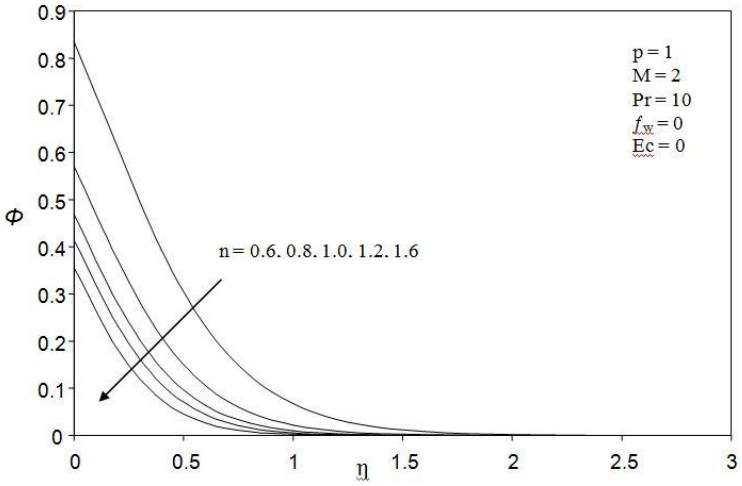


(a)

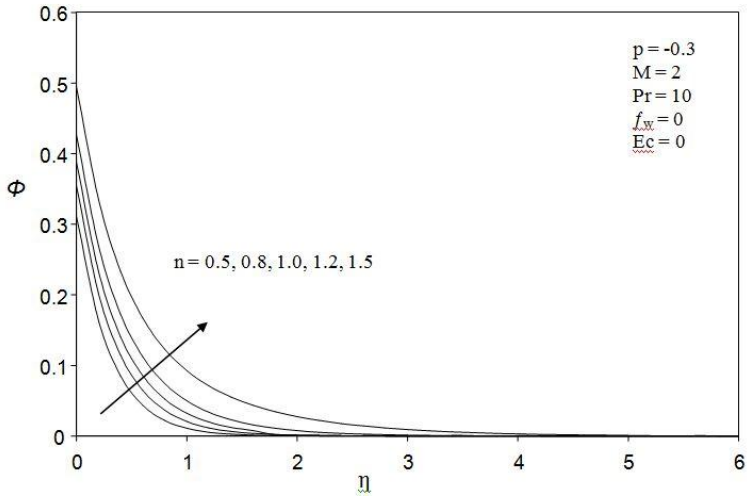


(b)

Fig.7. Temperature profiles for different values of M with p = 0, M = 2, f_w = 0 and Ec=0. (a) n = 0.5; (b) n = 1.5

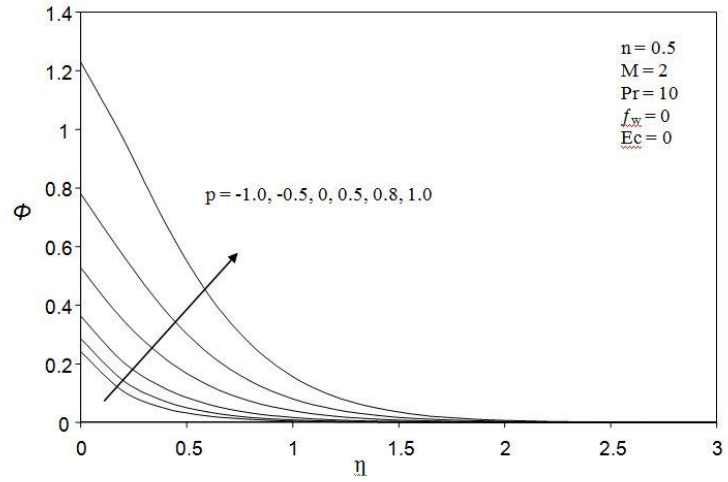


(a)

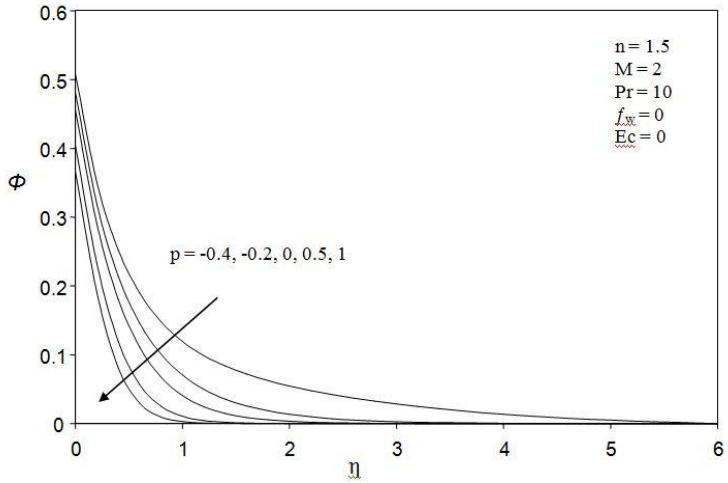


(b)

Fig.8. Temperature profiles for different values of n with M = 2, Pr = 10, f_w = 0 and Ec=0. (a) p = 1.0; (b) p = -0.3



(a)



(b)

Fig.9. Temperature profiles for different values of p with M = 2, Pr = 10, f_w=0 and Ec=0. (a) n = 0.5; (b) n = 1.5

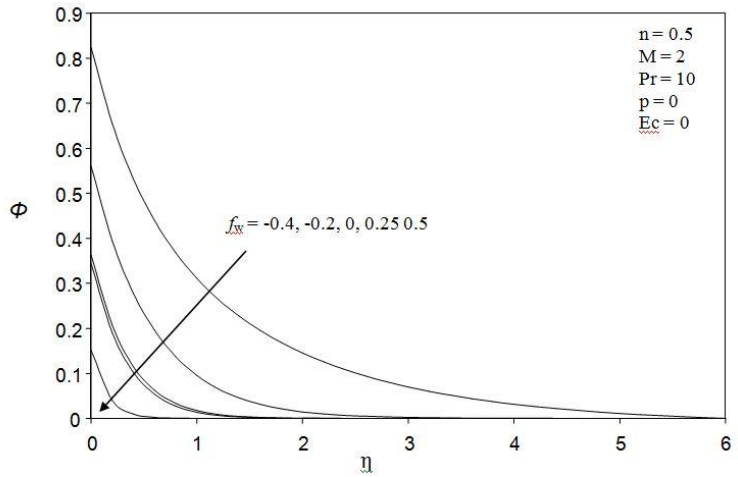


Fig.10. Temperature profiles for different values of f_w with n = 0.5, M = 2, Pr = 10, p = 0 and Ec = 0.

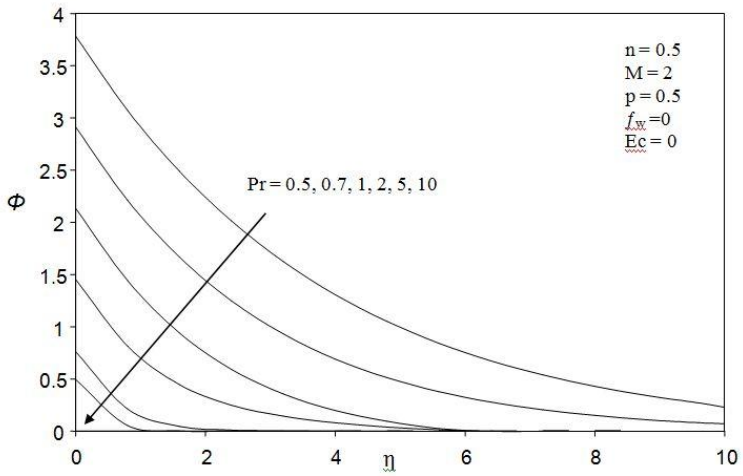
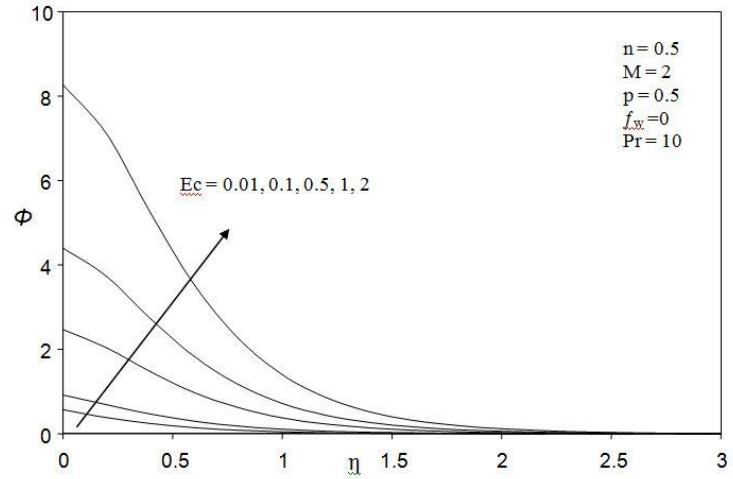
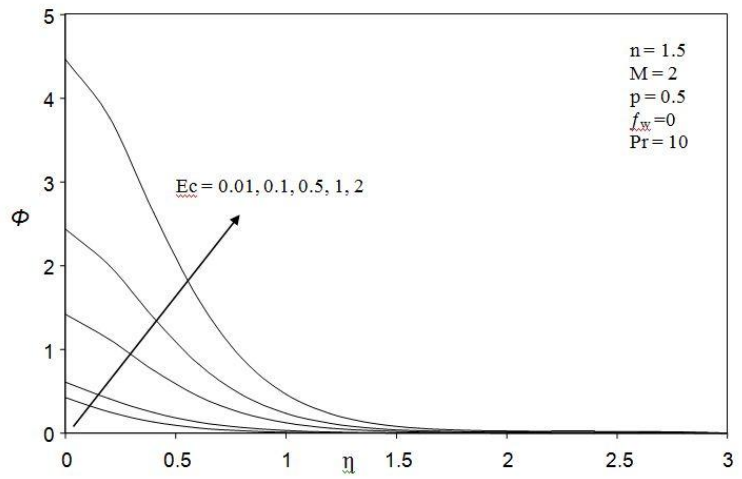


Fig.11. Temperature profiles for different values of Pr with n = 0.5, M = 2, p = 0.5, f_w =0 and Ec = 0.

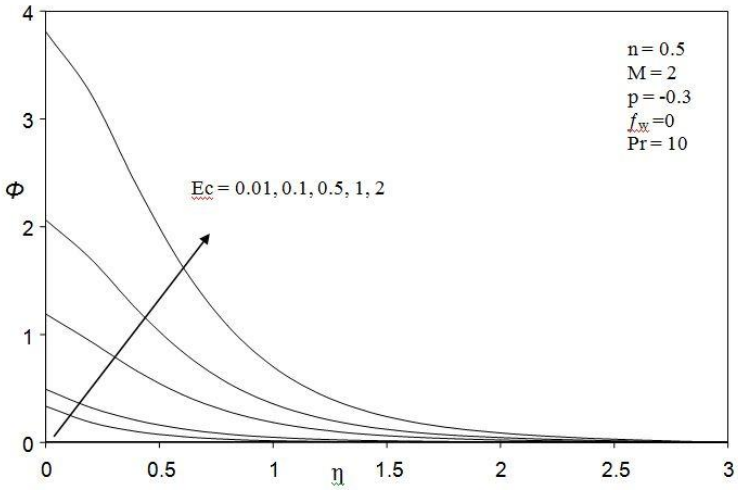


(a)

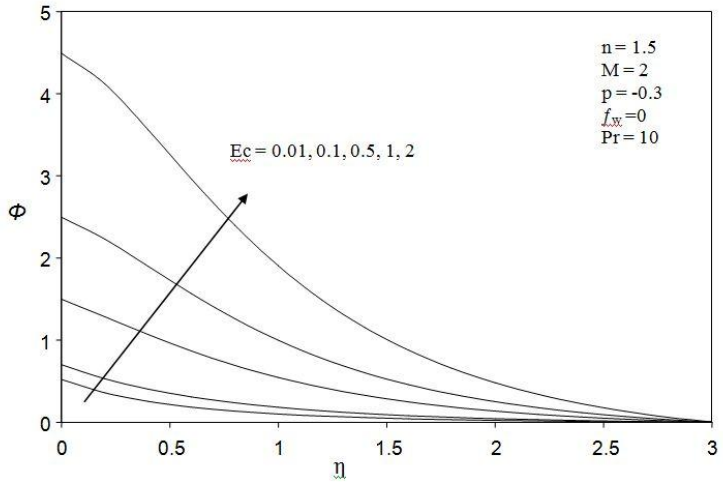


(b)

Fig.12. Temperature profiles for different values of Ec with M = 2, p = 0.5, f_w = 0 and Pr=10. (a) n = 0.5 : (b) n = 1.5.



(a)



(b)

Fig.13. Temperature profiles for different values of Ec with M = 2, p = 1, f_w = 0 and Pr=10. (a) n = 0.5 : (b) n = 1.5.

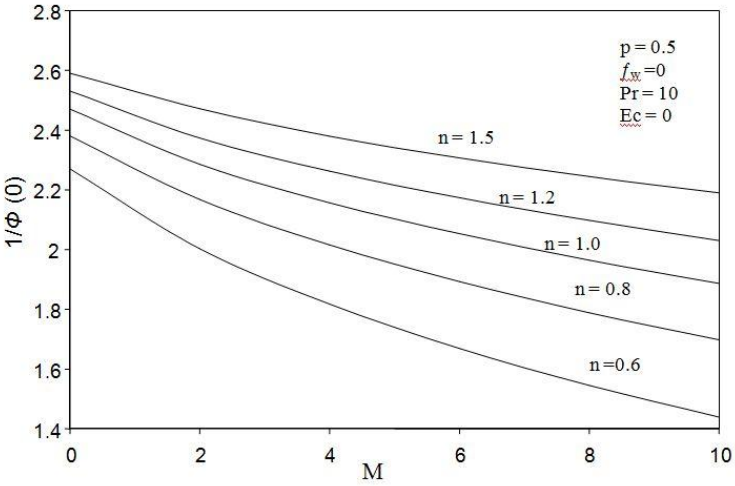


Fig.14. Variation of 1/Φ(0) as a function of M at selected values of n with p = 0.5, f_w = 0, Pr= 10 and Ec = 0.

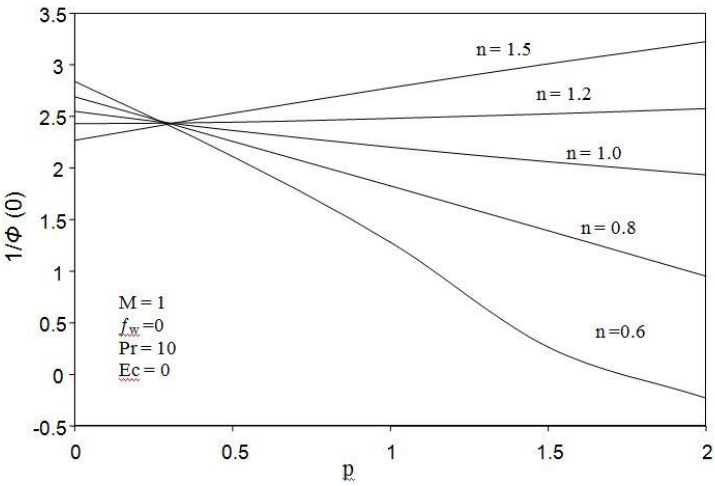


Fig.15. Variation of 1/Φ(0) as a function of p at selected values of n with M = 1, f_w = 0, Pr=10 and Ec = 0.

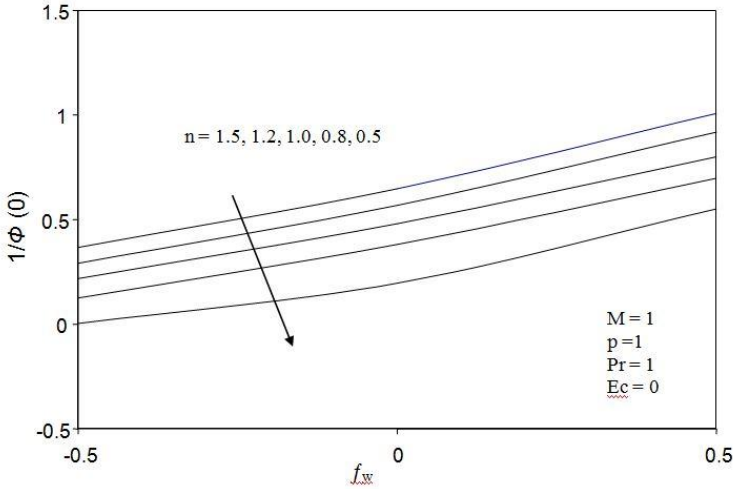


Fig.16. Variation of 1/Φ(0) as a function of f_w for various values of n with M = 1, p = 1, Pr=1 and Ec = 0.

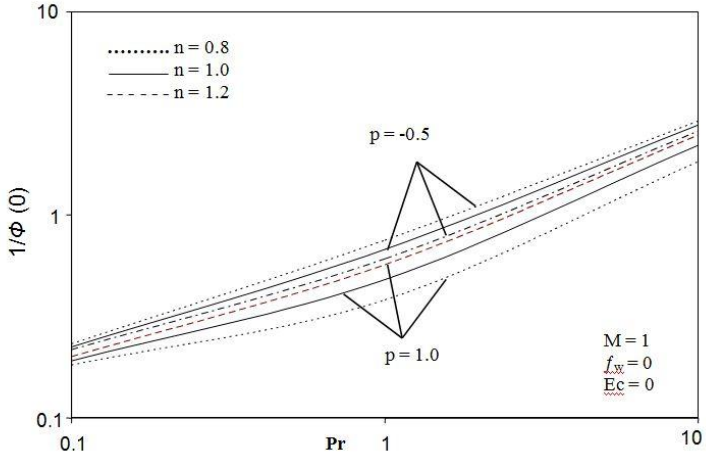


Fig.17. Variation of 1/Φ(0) as a function of Pr for various values of n and p with M = 1, f_w = 0 and Ec = 0.

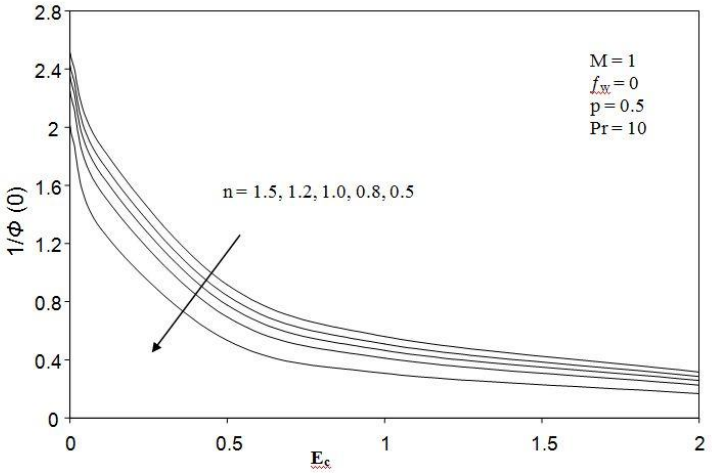


Fig.18. Variation of $1/\Phi(0)$ as a function of E_c for various values of n with M = 1, $f_w = 0$, Pr = 10 and p = 0.5