



Mathematical modeling process of liquid filtration taking into account reverse influence of process characteristics on medium characteristics

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Abstract

The article presents and solves the questions of accounting for reverse influence of process characteristics (the contamination concentration of liquid and sediment) on medium characteristics (the coefficients of porosity, filtration, diffusion, mass-transfer and others) by the example of liquid cleaning in magnetic and sorption filters. The algorithm of numerical-asymptotic approximation to the solution of the relevant model task which is described by the system of nonlinear singular perturbative differential equations of the type «convection-diffusion-mass-transfer». The proper correlations (formulas) are effective for conducting theoretical researches which are aimed at the «productivity» (in particular, optimization) of the parameters of filtration process (namely: time of protective action of load, sizes of filter, and others) in cases of predominance of convection and sorption components of the proper process above diffusive and desorption components, that takes place in large majority of filtration installations. The computer experiment was conducted on this basis. These ones results show the advantages of the offered model in comparing to classic.

Keywords: Filtration; Reverse Influence; Multicomponent Concentration; Magnetic Filter; Model of the Magnetic Sedimentation; Sorption Treatment; Asymptotic Upshots; Nonlinear Tasks.

1. Introduction

The analysis of researches results which was conducted in [1-17] testifies about the presence of difficult structure of the interrelations of different factors, which determined the processes of filtration and filtering through porous mediums, which was not taken into account in the “traditional” (classic, phenomenological) models of such systems. Taking into account the different interdependences, and also different additional factors which are inserting in a “initial” (base) model with the purpose of more deep study of process, often directs researchers to the necessity of construction of bulky and ineffective (in terms of numeral realization and practical using,) mathematical models. However in many practically important cases during researching of such processes it is possible to come in terms of modeling of different kind of perturbations of the known (idealizing, averaging, base) backgrounds.

In accordance with the researches, which were considered earlier, the article presents the questions of account of reverse influence of process characteristics (the contamination concentrations of liquid and sediment) on medium characteristics (the coefficients of porosity, filtration, diffusion, mass-transfer and others) on the example of liquid cleaning in magnetic and sorption filters.

2. Setting a task

Consider the one-dimensional process of cleaning liquid by filtration in the filter layer with thickness L , which is identified with the cut $[0, L]$ axis Ox . This layer is placed that abscissa axis is perpendicular to its surface, and origin of coordinates is on its upper boundary. The particles of contamination of admixture substance can pass from one state in other (processes of capture-tearing away, sorption-desorption) at same time the contamination concentrations are influenced on the considered layer. A concentration of contamination is multicomponent. The proper process of filtration with the account of reverse influence of characteristics of process (concentrations of liquid and sediment contamination) on medium characteristics (coefficients of porosity, filtration, diffusion, mass-transfer and others) is described the following system of interconnected differential equations:

$$\begin{cases} \frac{\partial(\sigma(\rho)c_i)}{\partial t} + \frac{\partial\rho}{\partial t} + \frac{\partial(vc_i)}{\partial x} = D_i \frac{\partial^2 c_i}{\partial x^2}, \\ \frac{\partial\rho}{\partial t} = \beta(\rho) \left(\sum_{i=1}^m k_i c_i \right) - \alpha(\rho)\rho + D^* \frac{\partial^2 \rho}{\partial x^2}, \end{cases} \quad (1)$$

$$c_i|_{x=0} = c_i^*(t), \quad \rho|_{x=0} = \rho^*(t), \quad \frac{\partial c_i}{\partial x} \Big|_{x=L} = 0, \quad \frac{\partial \rho}{\partial x} \Big|_{x=L} = 0, \quad c_i|_{t=0} = c_i^*(x), \quad \rho|_{t=0} = \rho^*(x), \quad (2)$$

$$v = \kappa(\rho) \cdot \text{grad } P, \quad (3)$$

where $c_i(x, t)$ – concentrations of admixtures in the liquid environment, which is filtered; $\rho(x, t)$ – concentrations of admixtures, which are sedimentation in the filter attachment; $\beta(\rho)$ – coefficient, which characterizes the mass volumes of admixture particles sedimentation for time unit; $(\beta(\rho) = \beta_0 - \varepsilon\beta^*\rho(x, t))$, $\alpha(\rho)$ – coefficient, which characterizes the mass volumes of torn-off for that time from the granules of filing of admixture particles, $(\alpha(\rho) = \alpha_0 + \varepsilon\alpha^*\rho(x, t))$, v – speed of filtration, $c_i^*(t)$ – concentrations of admixture particles at the input of the filter, $\sigma(\rho)$ – porosity of filter attachment (σ_0 – the initial porosity of attachment, $\sigma(\rho) = \sigma_0 - \varepsilon\sigma^*\rho(x, t)$); $D_i = b_i\varepsilon$, $D^* = b^*\varepsilon$, $\beta_0, \beta^*, \alpha_0, \alpha^*, \sigma^*, b_i, k_i, \varepsilon$ – hard parameters (they characterize the proper coefficients), $\beta(\rho), \alpha(\rho), \sigma(\rho)$ – soft parameters and they founded an experimental method), ε – small parameter, $i = \overline{1, m}$, P – pressure.

3. Algorithm (asymptotic) of the solution

Solution of system (1) in the terms (2) was founded in the kind of the asymptotic rows [9] – [17]:

$$\begin{aligned} c_i(x, t) &= c_{i,0}(x, t) + \sum_{j=1}^n \varepsilon^j c_{i,j}(x, t) + \sum_{j=0}^{n+1} \varepsilon^j U_{i,j}(\xi, t) + \sum_{j=0}^{n+1} \varepsilon^j \tilde{U}_{i,j}(\tilde{\xi}, t) + R_{c_i}(x, t, \varepsilon), \\ \rho(x, t) &= \rho_0(x, t) + \sum_{j=1}^n \varepsilon^j \rho_j(x, t) + \sum_{j=0}^{2n+1} \varepsilon^{j/2} P_j(\mu, t) + \sum_{j=0}^{2n+1} \varepsilon^{j/2} \tilde{P}_j(\tilde{\mu}, t) + R_\rho(x, t, \varepsilon), \end{aligned} \quad (4)$$

where R_{c_j}, R_ρ – the remaining members, $c_{i,j}(x, t)$, $\rho_j(x, t)$ ($i = \overline{1, m}; j = \overline{0, n}$) – the members of regular parts of asymptote, $U_{i,j}(\xi, t)$, $\tilde{U}_{i,j}(\tilde{\xi}, t)$ ($i = \overline{1, m}; j = \overline{0, n+1}$), $P_j(\mu, t)$, $\tilde{P}_j(\tilde{\mu}, t)$ ($j = \overline{0, 2n+1}$) – the functions of type of boundary layer (accordingly corrections at the input and at the output of filtration flow), $\xi = x \cdot \varepsilon^{-1}$, $\tilde{\mu} = x \cdot \varepsilon^{-1/2}$, $\tilde{\xi} = (L-x) \cdot \varepsilon^{-1}$, $\tilde{\mu} = (L-x) \cdot \varepsilon^{-1/2}$ – the proper regulating transformations.

Like to [17], after a substitution (4) in (1) and application of standard “procedure of equation”, for finding of functions $c_{i,j}$ and ρ_j ($j = \overline{0, n}$) we come to such tasks:

$$\begin{cases} \sigma_0 \frac{\partial c_{i,0}}{\partial t} + v \frac{\partial c_{i,0}}{\partial x} + k_i c_i = 0, \quad \frac{\partial \rho_0}{\partial t} = \beta_0 \left(\sum_{i=1}^m k_i c_{i,0} \right) - \alpha_0 \rho_0, \\ c_{i,0}|_{x=0} = c_i^*(t), \quad \rho_0|_{x=0} = \rho^*(t), \quad c_{i,0}|_{t=0} = c_i^*(x), \quad \rho_0|_{t=0} = \rho^*(x), \end{cases} \quad (5)$$

$$\begin{cases} -\sigma^* \rho_{i-1} \frac{\partial c_{i,j}}{\partial t} + v \frac{\partial c_{i,j}}{\partial x} - k_i \sigma^* \frac{\partial \rho_{j-1}}{\partial t} c_{i,j} = g_{i,j}, \\ \frac{\partial \rho_j}{\partial t} = -\beta^* \rho_{j-1} \left(\sum_{i=1}^m k_i c_{i,j} \right) - \alpha^* \rho_{j-1} \rho_j, \\ c_{1,i} \Big|_{x=0} = 0, c_{2,i} \Big|_{x=0} = 0, \rho_i \Big|_{x=0} = 0, c_{1,i} \Big|_{t=0} = 0, c_{2,i} \Big|_{t=0} = 0, \rho_i \Big|_{t=0} = 0, i = \overline{1,m}, j = \overline{1,n}, \end{cases} \quad (6)$$

As a result of their solving we have:

$$c_{i,0}(x,t) = \begin{cases} c_i^* \left(t - \frac{\sigma_0 x}{v} \right) \cdot e^{-\frac{k_i x}{v}}, & t \geq \frac{\sigma_0 x}{v}, \\ c_i^* \left(x - \frac{vt}{\sigma_0} \right) \cdot e^{-k_1 t}, & t < \frac{\sigma_0 x}{v}, \end{cases}$$

$$c_{i,j}(x,t) = \begin{cases} \frac{-\int_0^x \lambda_j(\tilde{x}, f(\tilde{x})+t-f(x)) d\tilde{x}}{v} \cdot \frac{\int_0^x g_{i,j}(\tilde{x}, f(\tilde{x})+t-f(x)) e^{-\int_0^{\tilde{x}} \lambda_j(\tilde{x}, f(\tilde{x})+t-f(x)) d\tilde{x}}}{\rho_{j-1}(\tilde{x}, f(\tilde{x})+t-f(x))} d\tilde{x}, & t \geq f(x), \\ \frac{-\int_0^t \lambda_j(f^{-1}(\tilde{t}+f(x)-t), \tilde{t}) d\tilde{t}}{e^{-\int_0^t \lambda_j(f^{-1}(\tilde{t}+f(x)-t), \tilde{t}) d\tilde{t}}} \cdot \frac{-\int_0^{\tilde{t}} \lambda_j(f^{-1}(\tilde{t}+f(x)-t), \tilde{t}) d\tilde{t}}{e^{-\int_0^{\tilde{t}} \lambda_j(f^{-1}(\tilde{t}+f(x)-t), \tilde{t}) d\tilde{t}}} g_{i,j}(f^{-1}(\tilde{t}+f(x)-t), \tilde{t}) d\tilde{t}, & t < f(x), \end{cases}$$

$$\rho_j(x,t) = -\beta^* e^{-\alpha^* \int_0^t \rho_{j-1}(x, \tilde{t}) d\tilde{t}} \int_0^t \rho_{j-1}(x, \tilde{t}) \left(\sum_{i=1}^m c_{i,j}(x, \tilde{t}) \right) e^{\alpha^* \int_0^{\tilde{t}} \rho_{j-1}(x, \tilde{t}) d\tilde{t}} d\tilde{t},$$

Where $g_{i,j}(x,t) = b_i \frac{\partial^2 c_{i,j-1}}{\partial x^2} + k_1 \rho_{j-1}$, $\lambda_j(x,t) = -k_i \sigma^* \frac{\partial \rho_{j-1}}{\partial t}$. The approximate values of functions $f_j(x)$ are founded by way of interpolation of array, (x_i, t_i) , $i = \overline{1,n}$, where $x_i = \Delta x \cdot i$, $t_{i+1} = t_i + \frac{\Delta x}{v} \sigma^* \rho_{j-1}(x_i, t_i)$.

The functions $U_i = \sum_{j=0}^{n+1} U_{i,j} \varepsilon^j$, $\tilde{U}_i = \sum_{j=0}^{n+1} \tilde{U}_{i,j} \varepsilon^j$, ($i = 1, 2; j = \overline{0, n+1}$), $P = \sum_{j=0}^{2n+1} P_j \varepsilon^{j/2}$, $\tilde{P} = \sum_{j=0}^{2n+1} \tilde{P}_j \varepsilon^{j/2}$ ($j = \overline{0, 2n+1}$) which were assigned for the removal of inconsistencies, which were brought by the built regular parts, $c_i(x,t) = \sum_{j=0}^n c_{i,j} \varepsilon^j$, $\rho(x,t) = \sum_{j=0}^n \rho_j \varepsilon^j$ in areas around the points with some accuracy $x = 0$, $x = L$ (input and output of filtration flow), that is providing implementation of terms: $\frac{\partial}{\partial x}(c + U_i) = O(\varepsilon^{n+1})$, $\frac{\partial}{\partial x}(c + \tilde{U}_i) = O(\varepsilon^{n+1})$, $\frac{\partial}{\partial x}(\rho + P) = O(\varepsilon^{n+1})$, $\frac{\partial}{\partial x}(\rho + \tilde{P}) = O(\varepsilon^{n+1})$. These functions are founded like to [17]. We are have proper task analogical to [9] for the estimation of remaining members.

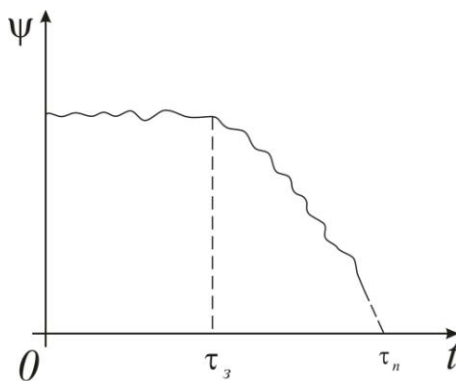


Fig. 1: The Efficiency of Treatment Process

4. Numerical calculations

4.1. Magnetic filter

Let us look at the process of cleaning of liquid mediums from ferromagnetic admixtures in magnetized porous nozzles that is one of main tasks of exception of corrosion products admixtures as a result of continuous corrosion of technological equipment. The admixture particles of mediums at working of magnetic power factor $F_c = H \cdot \text{grad}H$ settling in points of the contact of nozzles granules, where value F_c can arrive the size at the value in order $2 \cdot 10^{15} \text{ A}^2/\text{m}^3$ (H – magnetic field intensity). In initial moment of time ($t=0$) porous nozzle is relatively “clean”, that is unsaturated admixture particles, its porosity – σ_0 . In the process of settling of admixtures the size of porosity σ is gradually diminishing, the coefficient of hydraulic resistance is increasing and accordingly in the case of reserve of the system, the size of overfall of pressure ΔP in the porous nozzle. The Efficiency of cleaning process of medium remains at enough high level during definite time (time of filtercycle, time of protective action of filter). At the accumulation of critical mass of admixtures in the volume of porous nozzle which is characterized by the size of working capacity of absorption, efficiency of cleaning process which equals the relation of difference of concentrations of admixtures at input and output of filter to the concentration at input, is diminishing and the treatment regime passes to the non-stationary stage (Fig. 1). As known from [17], at $t > \tau_3$, certain amount of admixtures settled in the pores layers of nozzle yet. Greater their part “breaks away” and darts out with medium which is cleaning. Gradually, barns on length of porous nozzle are maximally saturated admixtures and are self-switching-off at achievement of sometime τ_n efficiency of cleaning is diminishing to the zero.

The process of magnetic settling of admixtures, which is realized in magnetic filter ($0 \leq x \leq L$) with homogeneous granular filter nozzle, is realized by operation of laws, the prototype of which is a classic model of filtration [15], taking into account reverse influence of the besieged particles on porosity σ and coefficient α , and on the coefficient of filtration also [17].

$$\begin{cases} \frac{\partial(\sigma(\rho)c(x,t))}{\partial t} + \frac{\partial\rho(x,t)}{\partial t} + v \frac{\partial c(x,t)}{\partial x} = 0, \\ \frac{\partial\rho(x,t)}{\partial t} = \beta c(x,t) - \varepsilon\alpha(\rho)\rho(x,t), \end{cases} \quad (7)$$

$$c|_{x=0} = c^*(t), \quad c|_{t=0} = 0, \quad \rho|_{x=0} = 0, \quad \rho|_{t=0} = 0, \quad \frac{\partial c}{\partial x}|_{x=L} = 0, \quad \frac{\partial\rho}{\partial x}|_{x=L} = 0, \quad (8)$$

$$v = \kappa(\rho) \cdot \text{grad} P, \quad (9)$$

Where $\alpha(\rho)$ – coefficient which characterizes the mass volumes which were torn-off during that time from the granules of nozzle of admixture particles;

$$\alpha(\rho) = \alpha_0 + \varepsilon\alpha^*\rho(x,t) \quad (10)$$

v – Speed of filtration ($v = \text{const}$, which characterizes locking of technological process), $\sigma(x,t)$ – the porosity of filter nozzle (σ_0 – the initial porosity of nozzle),

$$\sigma(x,t) = \sigma_0 - \varepsilon\sigma^*\rho(x,t) \quad (11)$$

$\kappa(\rho)$ – Coefficient of filtration, ρ_2 – limit of filling by sediment,

$$\kappa(\rho) = \begin{cases} \kappa_0 - \varepsilon\gamma\rho(x,t), & \rho < \rho_2 \quad (t < \tau_3), \\ \kappa^0, & \rho = \rho_2 \quad (t \geq \tau_3), \end{cases} \quad (12)$$

$\kappa^0 = \kappa_0 - \varepsilon\gamma\rho_2$, $\alpha_0, \alpha^*, \sigma^*, \kappa_0, \gamma, \varepsilon$ – hard parameters (they are characterized the proper coefficients), $\alpha(\rho), \sigma(x,t), \kappa(x,t)$ – soft parameters and they founded an experimental method), P – pressure.

Such character of change of porosity and coefficient of the torned-off particles is explained that at the increase of admixture particles in nozzle, the proper parameters of filtration change. As a system is reserved, the change of coefficient of filtration causes the change of size of overfall of pressure $\Delta P = P(L,t) - P(0,t)$ in porous nozzle.

The solution of system (7) in the terms (8) is founded similar to (1)-(2) in the form of asymptotic series (4) (see [9], [17]):

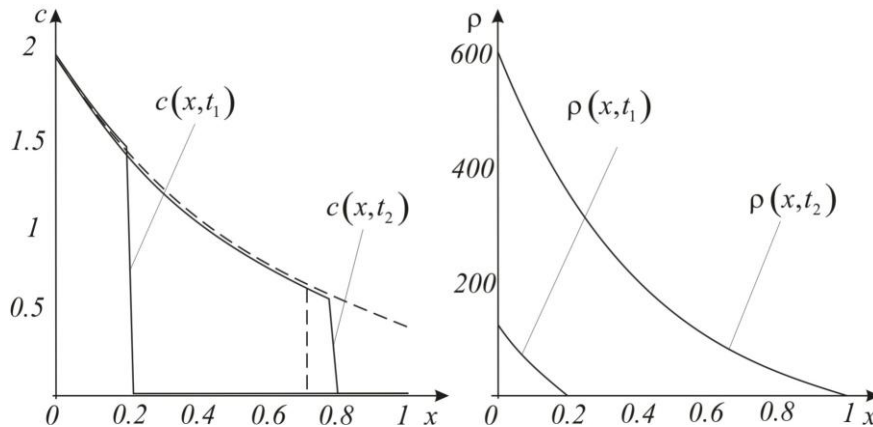


Fig. 2: The Distribution of Concentrations of Admixtures in the Liquid and Sediment Along the Filter at the Time Moment $t_1 = 20$ Hours, $t_2 = 80$ Hours.

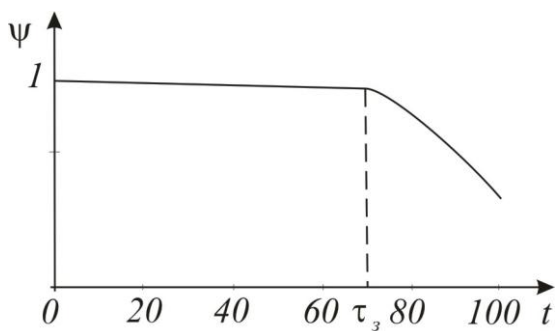


Fig. 3: The Variation $grad P$ On Output Of Filter With Time

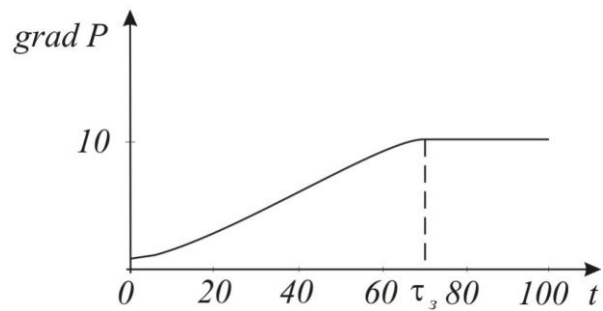


Fig. 4: The Distribution Of Filter Efficiency ($\psi = (c^* - c) / c^*$)

According to [4] coefficients of captured admixture particles and the torned-off sediment particles are calculated using the formula: $\beta = \frac{\beta_0 H^{0.75}}{vd^2}$ [13], where β_0 - free parameter, H - magnetic field intensity, v - speed of filtration, d - diameter of granular filter nozzle.

The results of calculations by the formulas look like (3) at $c^*(t) = 2 \text{ mg/l}$, $v = 200 \text{ m / zod}$, $L = 1 \text{ m}$, $\beta_0 = 0.7 \cdot 10^{-9} \text{ s}^{-1}$, $\alpha_0 = 0.35 \text{ s}^{-1}$, $H = 60 \text{ kA/m}$, $d = 2.4 \text{ mm}$, $\alpha^* = 1$.

Figure 2 shows the distribution of admixtures concentration in liquid and sediment in the certain time moment. Thence, giving at the output filter (when $L = 1$) the permissible concentration $c = c_{kp} = 0.59 \text{ mg / l}$, we find time its protective action: $t = \tau_3 = 71$ hours, that on 4 hours is differed from data which conducted by the test method [13]. At this filter will accumulate sediment by weight 240 g.

We emphasize that in the process of calculation we took $v = const$, though the coefficient of filtration (and porosity also) decreases by sticking to the walls (filling) solid particles. This enables to find in each cross-section filter (each point x, $0 \leq x \leq L$) pressure gradient, especially using the formula $grad P = \frac{v}{\kappa(\rho)}$ we can find the time of passage more

than critical value gradient and to solve proper "decisions of automatization". The change $grad P$ is shown in Figure 3 with time.

As we can see (Fig. 4), if the case $c^*(t) = c^* = const$ the filter efficiency is unchanged practically until the time moment τ_3 , which confirms the known fact of filter efficiency distribution with time.

4.2. Sorption filters

The process of filtering in sorption filters does not require closed system. So speed of filtering is not a constant and the speed is changing along the filter over time usually. For simplification calculations, we assume that the concentration of pollution is one-component. Also we must consider the reverse effect on the porosity and coefficients which is characterizing the settling of particles of dirt and sediment particles tearing-off [17] and longitudinal diffusion. Coming from the above facts system (1) - (2) can be rewritten as:

$$\begin{cases} \frac{\partial(\sigma(x,t)c(x,t))}{\partial t} + \frac{\partial\rho(x,t)}{\partial t} + \frac{\partial(v(x,t)c(x,t))}{\partial x} = D \frac{\partial^2 c}{\partial x^2}, \\ \frac{\partial\rho(x,t)}{\partial t} = \beta(\rho)c(x,t) - \varepsilon\alpha(\rho)\rho(x,t) + D^* \frac{\partial^2 \rho}{\partial x^2}, \end{cases} \quad (13)$$

$$c|_{x=0} = c^*(t), \quad c|_{t=0} = 0, \quad \rho|_{x=0} = 0, \quad \rho|_{t=0} = 0, \quad \frac{\partial c}{\partial x}|_{x=L} = 0, \quad \frac{\partial \rho}{\partial x}|_{x=L} = 0, \quad (14)$$

The solution of system (13) at the terms (14) we are founding similar to the general problem in the form of asymptotic series (see [9], [17]).

The results of calculations by formulas (4) when $c^*(t) = 170 \text{ mg/l}$, $L = 0.8 \text{ m}$, $\beta_0 = 0.3 \text{ s}^{-1}$, $\alpha_0 = 0.0056 \text{ s}^{-1}$, $\sigma_0 = 0.5$, are $\alpha^* = 1$, $\beta^* = 1$, $\sigma^* = 1$, $b = b^* = 1$, $\varepsilon = 0.001$.

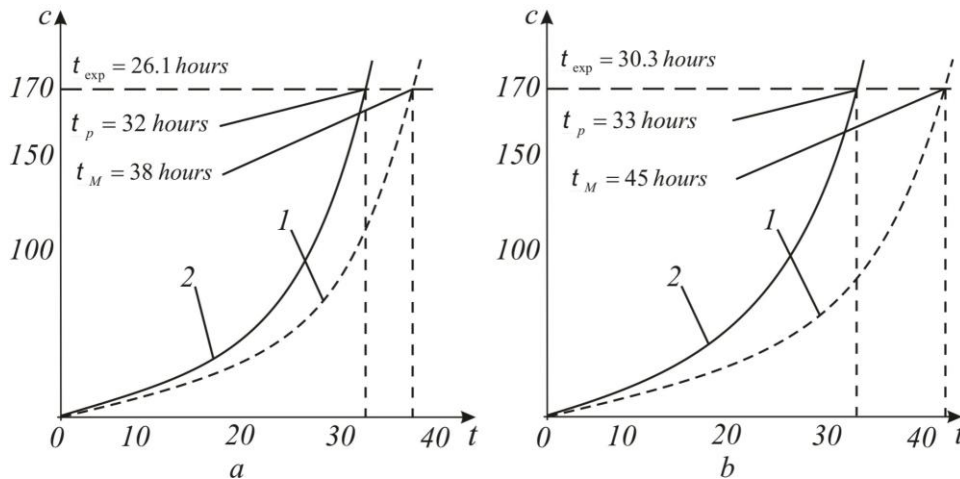


Fig. 5: A). The Distribution of Admixtures Concentrations at the Output Filter During the Time Of Protective Action: 1 -According To Model of Minz; 2 - According To Formulas (4), at $D = 0.78 \text{ Mm}$, $V = 10 \text{ M/Hour}$.
 B). The Distribution of Admixtures Concentrations at the Output Filter During the Time of Protective Action: 1 -According To Model of Minz; 2 - According To Formulas (4), at Mm , M/Hour .

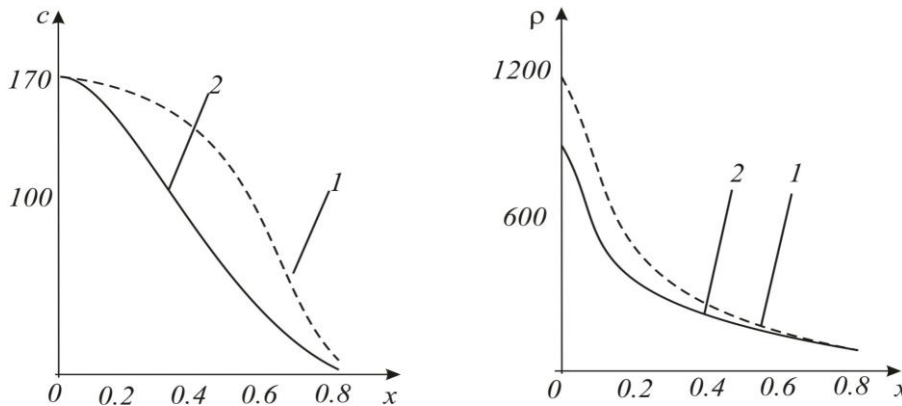


Fig. 6: A) The Distribution of Admixtures Concentrations Along the Filter During at the Time Moment $T = 26 \text{ Hours}$: 1 -According To Model of Minz; 2 - Founded by Formulas (4), At $D = 0.78 \text{ Mm}$, $V = 10 \text{ M/Hour}$.
 B) The Distribution of Sediment Concentrations along the Filter during At the Time Moment $T = 26 \text{ Hours}$: 1 -According To Model of Minz; 2 - Founded By Formulas (4), At $D = 0.78$

In the figures 5-6 were illustrated the comparative characteristics of the test data obtained and calculated by the classical model of Minz [14] and calculated by formulas (4). So the results of calculations by formulas (7) are providing greater accuracy in comparison with the classical model calculation formulas of Minz. Also the obtained results allow calculating the dynamics of promoting concentration of contamination and sediment along the filter (Fig. 7- 8).

5. Conclusion

In the work, the mathematical model was built ,which taking into account reverse influence of process characteristics (the contamination concentration of liquid and sediment) on medium characteristics (the coefficients of porosity, filtration, diffusion, mass-transfer and others) on the example of liquid cleaning in magnetic and sorption filters, namely:

Mathematical model is the built, transferred on the process that describes the regularities of magnetic settling of admixtures in porous filtering nozzle, the regularities of accumulation (“skidding”) admixtures in the nozzle, and also takes into account the reverse influence of sediment concentration on coefficients of porosity, filtration and mass-transfer. The proposed algorithm for solving the proper problem, in particular, includes: determining the time τ_3 , protective action of filtering nozzle, determining of the limit overfall of pressure ΔP and value $grad P$ at change $x \in [0, L]$ and $t \in [0, \tau_3]$. The results of calculations of concentration distribution and mass amount of admixtures in height filtering porous nozzle for different time moments are given, values of coefficient filtering for different values of length L of nozzle, which corresponds to time of protective action (filtercycle) of nozzle. In this model provides the possibility of automatic control of process of efficient settling of admixtures in magnetic filtering nozzle depending on the initial data of the water medium which is cleaned;

The offered mathematical model is transferred on the process of sewage treatment in sorption filters with taking into account reverse influence of sediment concentration on the medium characteristics and variable speed of filtering. The results of calculations of concentration distribution and mass amount of admixtures in height filtering porous nozzle for different time moments, values of filtering coefficient for different values of filtering speed, and characteristics of filling of filter are given. There were conducted comparative characteristics of the data which were obtained through research and calculated on base of classic model of Minza and formulas obtained by us (including, according to data presented in fig. 5-6, we see that the accuracy of calculations by formulas proposed by us is more higher in compared to estimates obtained by the classical Minz model).

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