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Research Paper

Approximate solution of a model describing biological species living together using a new iterative method

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Abstract

In the present article, we implement the new iterative method proposed by Daftardar-Gejji and Jafari (NIM or DJM) [V. Daftardar-Gejji, H. Jafari, An iterative method for solving non linear functional equations, J. Math. Anal. Appl. 316 (2006) 753-763] to solve a system of two nonlinear integro-differential equations, which describes biological species living together. The results demonstrate that the method has many merits such as being derivative-free, can be easily comprehended with only a basic knowledge of Calculus. Also, the DJM overcoming the difficulty arising in calculating Adomian polynomials to handle the nonlinear terms in Adomian Decomposition Method (ADM). Does not require to calculate Lagrange multiplier as in Variational Iteration Method (VIM) and no needs to construct a homotopy as in Homotopy Perturbation Method (HPM) that implemented to solve the problem. Numerical examples are presented for several problems, to demonstrate the efficiency of the proposed method. A comparison with some existing techniques such as ADM, HPM and VIM also presented, which shows that the DJM is effective and convenient to use and overcomes the difficulties arising in existing techniques. The software used for the calculations in this study was MATHEMATICA ® 8.0.

Keywords: *Biological species living together, Integro-differential equation, Mathematical biology, New iterative method*

1. Introduction

A variety of problems in physics, chemistry and biology have their mathematical setting as integral equations. Many methods have been developed to solve integral and integro-differential equations, especially nonlinear. More details about the sources where these equations arise can be found in physics, biology and engineering applications books [1].

Finding the exact solutions for this class of equations is too complicated (or impossible). Therefore, finding either the analytical approximate or numerical solutions of such equations are of great interest.

Recently, many attempts have been made to develop analytic and approximate methods to solve a system of two nonlinear integro-differential equations, which describes biological species living together, see for examples [2, 3, 4]. Although such methods have been successfully applied but some difficulties have appeared, for examples, construct a homotopy as in HPM and solve the corresponding the algebraic equations, calculate Adomian polynomials to handle the nonlinear terms in ADM and evaluate Lagrange multiplier as in VIM, respectively.

Recently, Daftardar-Gejji and Jafari [5] have proposed a new technique for solving linear/nonlinear functional equations namely new iterative method (NIM) or (DJM). The DJM has been extensively used by many researchers for the treatment of linear and nonlinear ordinary and partial differential equations of integer and fractional order [6, 7, 8, 9]. The method converges to the exact solution if it exists through successive approximations. However, for concrete problems, a few approximations can be used for numerical purposes with high degree of accuracy. The DJM is simple to understand and easy to implement using computer packages and yields better results and does not require any restrictive assumptions for nonlinear terms as required by some existing techniques [6].

In this paper, the applications of the DJM for a system of two nonlinear integro-differential equations, which describes biological species living together will be presented to find the approximate solutions.

The paper is organized as follows. Section 2 is devoted to the description the basic idea of DJM and its convergence. In section 3 the mathematical model of biological species living together is presented and solved by DJM. In section 4 the numerical results for some examples are discussed with comparison with some existing techniques such as ADM, HPM and VIM. Finally, in section 5 the conclusion is presented.

2. The basic idea of DJM

Consider the following general functional equation [5, 6, 7, 8, 9]:

$$u = N(u) + f, \tag{1}$$

where N is a nonlinear operator from a Banach space $B \to B$ and f is a known function. We are looking for a solution u of Eq.(1) having the series form:

$$u = \sum_{i=0}^{\infty} u_i. \tag{2}$$

The nonlinear operator N can be decomposed as

$$N(\sum_{i=0}^{\infty} u_i) = N(u_0) + \sum_{i=1}^{\infty}\{N(\sum_{j=0}^{i} u_j) - N(\sum_{j=0}^{i-1} u_j)\}. \tag{3}$$

From Eqs.(2) and (3), Eq.(1) is equivalent to

$$\sum_{i=0}^{\infty} u_i = f + N(u_0) + \sum_{i=1}^{\infty}\{N(\sum_{j=0}^{i} u_j) - N(\sum_{j=0}^{i-1} u_j)\}. \tag{4}$$

We define the recurrence relation:

$$\begin{aligned} &G_0 = u_0 = f, \\ &G_1 = u_1 = N(u_0), \\ &G_m = u_{m+1} = N(u_0 + \cdots + u_m) - N(u_0 + \cdots + u_{m-1}), \quad m = 1, 2, ... \end{aligned} \tag{5}$$

Then

$$(u_1 + ... + u_{m+1}) = N(u_1 + ... + u_m), \quad m = 1, 2, ..., \tag{6}$$

and

$$u(x) = f + \sum_{i=1}^{\infty} u_i. \tag{7}$$

The m-term approximate solution of Eq.(2) is given by $u = \sum_{i=0}^{m-1} u_i$.

2.1. Convergence of the DJM

We present below the condition for convergence of the series $\sum u_i$. For more details we refer the reader to [10].

Theorem 2.1.1 :[10]

If N is $C^{(\infty)}$ in a neighbourhood of u_0 and $\| N^{(n)}(u_0) \| \leq L$, for any n and for some real $L > 0$ and $\| u_i \| \leq M < \frac{1}{e}$, $i = 1, 2, ...$, then the series $\sum_{n=0}^{\infty} G_n$ is absolutely convergent and moreover, $\| G_n \| \leq LM^n e^{n-1}(e-1), n = 1, 2, ...$

Theorem 2.1.2 : [10]

If N is $C^{(\infty)}$ and $\|N^{(n)}(u_0)\| \leq M \leq e^{-1}$, $\forall n$, then the series $\sum_{n=0}^{\infty} G_n$ is absolutely convergent.

3. The DJM for solving a system of two nonlinear integro-differential equations

3.1. Mathematical modeling of biological species living together

In this paper, the general form of a system of two nonlinear integro-differential equations is considered in the following pair of integro-differential equations [2, 3, 4]:

$$\frac{dN_1}{dt} = N_1(t)\left[m_1 - \alpha_1 N_2(t) - \int_{t-T_0}^{t} g_1(t-s)N_2(s)ds\right] + h_1(t), \quad m_1 > 0 \tag{8}$$

$$\frac{dN_2}{dt} = N_2(t)\left[-m_2 + \alpha_2 N_1(t) + \int_{t-T_0}^{t} g_2(t-s)N_1(s)ds\right] + h_2(t), \quad m_2 > 0 \tag{9}$$

with initial conditions:

$$N_1(0) = L_1, \quad N_2(0) = L_2$$

where $N_1(t)$ and $N_2(t)$ are unknown functions, and $g_1(t)$, $g_2(t)$, $h_1(t)$ and $h_2(t)$ are given functions.

It is important to mention that the system of Eqs. (8) and (9) with $h_1(t) = h_2(t) = 0$ has an important application to the biological science.

Therefore, let us consider two separate species with numbers $N_1(t)$ and $N_2(t)$ at time t where first species increases and the second decreases. If they are put together, assuming that the second species will feed on the first, there will be an increase in the rate of the second species $\frac{dN_2}{dt}$ which depends not only on the present population $N_1(t)$ but also on all previous values of the first species. When a steady-state condition is reached between these two species, it is described by the following pair of integro-differential equations [2, 3, 4]:

$$\frac{dN_1}{dt} = N_1(t)\left[m_1 - \alpha_1 N_2(t) - \int_{t-T_0}^{t} g_1(t-s)N_2(s)ds\right], \quad m_1 > 0 \tag{10}$$

$$\frac{dN_2}{dt} = N_2(t)\left[-m_2 + \alpha_2 N_1(t) + \int_{t-T_0}^{t} g_2(t-s)N_1(s)ds\right], \quad m_2 > 0 \tag{11}$$

where m_1 and m_2 represent the coefficients of increase and decease of the first and second species, respectively. The parameters α_1, g_1 and α_2, g_2 depend on the respective species. T_0 is the finite heredity duration of both species.

The detailed mathematical formulation and description of this model problem are given in [11]. Therefore, in this work both systems given in Eqs. (8-9) and (10-11) will discussed and solved.

Babolian and Biazar [2] used the Adomian decomposition method (ADM) to solve Eqs.(10-11). Shakeri and Dehghan. [3] used the variational iteration method to solve both systems given in Eqs. (8-9) and (10-11). Moreover, Roul and Meyer [4] used the homotopy perturbation method to solve both systems given in Eqs. (8-9) and (10-11).

3.2. Approximate solution of a system of two nonlinear integro-differential equations by DJM

Now, let us apply the DJM to find the approximate solution to the nonlinear integro-differential system of Eqs. (8) and (9) with the initial conditions [2, 3, 4]:

$$N_1(0) = L_1, \quad N_2(0) = L_2. \tag{12}$$

Let us consider

$$\frac{dN_1}{dt} = A_1(t) \Rightarrow N_1(t) = N_1(0) + \int_0^t A_1(s)ds, \tag{13}$$

$$\frac{dN_2}{dt} = A_2(t) \Rightarrow N_2(t) = N_2(0) + \int_0^t A_2(s)ds. \tag{14}$$

Therefore, the following system of four integral equations are achieved

$$N_1(t) = N_1(0) + \int_0^t A_1(s)ds, \tag{15}$$

$$A_1(t) = N_1(t)\left[m_1 - \alpha_1 N_2(t) - \int_{t-T_0}^t g_1(t-s)N_2(s)ds\right] + h_1(t), \tag{16}$$

$$N_2(t) = N_2(0) + \int_0^t A_2(s)ds, \tag{17}$$

$$A_2(t) = N_2(t)\left[-m_2 + \alpha_2 N_1(t) + \int_{t-T_0}^t g_2(t-s)N_1(s)ds\right] + h_2(t). \tag{18}$$

By substituting the initial conditions in (12) into (15) and (17) and substitute the new equations into (16) and (18), the following equations are obtained

$$N_1(t) = L_1 + \int_0^t A_1(s)ds, \tag{19}$$

$$A_1(t) = m_1 L_1 + m_1 \int_0^t A_1(s)ds + N_1(t)\left[-\alpha_1 N_2(t) - \int_{t-T_0}^t g_1(t-s)N_2(s)ds\right] + h_1(t), \tag{20}$$

$$N_2(t) = L_2 + \int_0^t A_2(s)ds \tag{21}$$

$$A_2(t) = -m_2L_2 - m_2 \int_0^t A_2(s)ds + N_2(t)\left[\alpha_2 N_1(t) + \int_{t-T_0}^{t} g_2(t-s)N_1(s)ds\right] + h_2(t). \tag{22}$$

A few first terms being calculated by DJM:

$$\begin{aligned} N_{10} &= L_1, \\ A_{10} &= m_1L_1, \\ N_{20} &= L_2, \\ A_{20} &= -m_2L_2. \end{aligned} \tag{23}$$

$$\begin{aligned}
&N_{11} = I_1(A_{10}) = \int_0^t A_{10}(s)ds = m_1L_1t, \\
&A_{11} = I_2(A_{10}, N_{10}, N_{20}) = m_1 \int_0^t A_{10}(s)ds + N_{10}\left[-\alpha_1 N_{20} - \int_{t-T_0}^{t} g_1(t-s)N_{20}ds\right] + h_1(t) = \\
&= L_1m_1^2t - \alpha_1L_1L_2 - \int_{t-T_0}^{t} g_1(t-s)L_1L_2ds + h_1(t), \\
&N_{21} = I_3(A_{20}) = \int_0^t A_{20}(s)ds = -m_2L_2t, \\
&A_{21} = I_4(A_{20}, N_{10}, N_{20}) = -m_2 \int_0^t A_{20}(s)ds + N_{20}\left[\alpha_2 N_{10} + \int_{t-T_0}^{t} g_2(t-s)N_{10}ds\right] + h_2(t) = \\
&= L_2m_2^2t + \alpha_2L_1L_2 + \int_{t-T_0}^{t} g_2(t-s)L_1L_2ds + h_2(t),
\end{aligned}$$

$$\begin{aligned}
&N_{12} = I_1(A_{10} + A_{11}) = \int_0^t (A_{10}(s) + A_{11}(s))\,ds - N_{11}, \\
&A_{12} = I_2(A_{10} + A_{11}, N_{10} + N_{11}, N_{20} + N_{21}) = m_1 \int_0^t (A_{10}(s) + A_{11}(s))\,ds + \\
&(N_{10} + N_{11})\left[-\alpha_1(N_{20} + N_{21}) - \int_{t-T_0}^{t} g_1(t-s)(N_{20} + N_{21})\,ds\right] + h_1(t) - A_{11}, \\
&N_{22} = I_3(A_{20} + A_{21}) = \int_0^t (A_{20}(s) + A_{21}(s))\,ds - N_{21}, \\
&A_{22} = I_4(A_{20} + A_{21}, N_{10} + N_{11}, N_{20} + N_{21}) = -m_2 \int_0^t (A_{20}(s) + A_{21}(s))\,ds + \\
&(N_{20} + N_{21})\left[\alpha_2(N_{10} + N_{11}) + \int_{t-T_0}^{t} g_2(t-s)(N_{10} + N_{11})\,ds\right] + h_2(t) - A_{21}.
\end{aligned}$$

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Continuing in this manner, the nth approximation of the exact solutions for the unknown functions $N_1(t)$ and $N_2(t)$ can be achieved once the parameters $m_1, m_2, \alpha_1, \alpha_2$ and the functions g_1, g_2, h_1, h_2 are given and solutions of unknown functions will be:

$$N_1(t) = N_{10}(t) + N_{11}(t) + N_{12}(t) + N_{13}(t) + \cdots + N_{1n}(t) = \sum_{k=0}^{n} N_{1k}(t),$$

$$N_2(t) = N_{20}(t) + N_{21}(t) + N_{22}(t) + N_{23}(t) + \cdots + N_{2n}(t) = \sum_{k=0}^{n} N_{2k}(t).$$

4. Numerical results

In this section, we shall examine some test examples to assess the performance of the DJM for obtaining the approximate solution of a system of two nonlinear integro-differential in Eqs. (8) and (9). Also, the approximate solutions to a model describing biological species living together Eqs. (10) and (11) are presented. To verify the convergence of the method, we applied the method to some test problems, for which an exact analytical solution is available. Moreover, comparisons are made with the exact solution and the approximate solutions obtained using ADM [2], the VIM [3], Pseudospectral Legendre method (PLM) [3] and HPM [4] to check the efficiency and the accuracy of the method.

Example 1. Let us first consider the system of integro-differential [4] in Eqs. (8) and (9) with

$$\begin{aligned}
&g_1(t-s) = 1, g_2(t-s) = t-1,\\
&\alpha_1 = \tfrac{1}{3}, \alpha_2 = 1,\\
&m_1 = 1, m_2 = 2,\\
&T_0 = 0.1,\\
&h_1(t) = \tfrac{-5}{2}t^3 + \tfrac{49}{12}t^2 + \tfrac{17}{12}t - \tfrac{23}{6},\\
&h_2(t) = \tfrac{15}{8}t^3 - \tfrac{1}{4}t^2 + \tfrac{3}{8}t - 1,
\end{aligned}$$

subject to the initial conditions

$$N_1(0) = L_1 = 1, \;\; N_2(0) = L_2 = 0.$$

The exact solution for this problem is in the following form

$$N_1(t) = -3t + 1, \; N_2(t) = t^2 - t. \tag{24}$$

A few first terms being calculated by DJM:

$$\begin{aligned}
&N_{10} = L_1,\\
&A_{10} = m_1 L_1,\\
&N_{20} = L_2,\\
&A_{20} = -m_2 L_2.
\end{aligned} \tag{25}$$

$$\begin{aligned}
&N_{11} = m_1 L_1 t,\\
&A_{11} = L_1 m_1^2 t - \alpha_1 L_1 L_2 - L_1 L_2 T_0 + h_1(t),\\
&N_{21} = -m_2 L_2 t,\\
&A_{21} = L_2 {m_2}^2 t + L_1 L_2(-1+t)T_0 + L_1 L_2 \alpha_2 + h_2(t),
\end{aligned} \tag{26}$$

$$\begin{aligned}
&N_{12} = \tfrac{1}{2} L_1 {m_1}^2 t^2 - L_1 L_2 t T_0 - L_1 L_2 t \alpha_1 + (-\tfrac{5}{8}t^4 + \tfrac{49}{35}t^3 + \tfrac{17}{24}t^2 - \tfrac{23}{6}t),\\
&A_{12} = \tfrac{1}{72}(276 + 2(90 + 49m_1)t^3 - 45m_1 t^4 - 36L_1 L_2 m_2 {T_0}^2 +\\
&3t^2(-98 + 12L_1 {m_1}^3 + m_1(17 + 24L_1 L_2 m_2(T_0 + \alpha_1))) -\\
&6t(17 - 12L_1 L_2 m_2(T_0 + \alpha_1) + m_1(46 + 6L_1 L_2(4T_0 + m_2 {T_0}^2 + 4\alpha_1)))),\\
&N_{22} = \tfrac{1}{2} L_2 {m_2}^2 t^2 - L_1 L_2 t T_0 + \tfrac{1}{2} L_1 L_2 t^2 T_0 + L_1 L_2 t \alpha_2 + (\tfrac{15}{32}t^4 - \tfrac{1}{12}t^3 + \tfrac{3}{16}t^2 - t),\\
&A_{22} = \tfrac{1}{96}(-45m_2 t^4 + 48(2 + L_1 L_2 m_1 {T_0}^2) - 4t^3(45 + m_2(-2 + 24L_1 L_2 m_1 T_0)) -\\
&12t(3 + 4L_1 L_2 m_1(2T_0 + {T_0}^2 - 2\alpha_2) + 4m_2(-2 + L_1 L_2(-4T_0 + m_1 {T_0}^2 + 4\alpha_2))) -\\
&6t^2(8L_2 {m_2}^3 - 4(1 + 4L_1 L_2 m_1 T_0) + m_2(3 - 8L_1 L_2(-3T_0 + 2m_1 T_0 + m_1 {T_0}^2 - 2m_1 \alpha_2)))).
\end{aligned} \tag{27}$$

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Continuing in this manner, the first three terms approximations for N_1 and N_2 are given by

$$N_1(t) = N_{10}(t) + N_{11}(t) + N_{12}(t) = L_1 + L_1 m_1 t + \frac{1}{2} L_1 {m_1}^2 t^2 - L_1 L_2 t T_0 - L_1 L_2 t \alpha_1 + (-\frac{5}{8}t^4 + \frac{49}{36}t^3 + \frac{17}{24}t^2 - \frac{23}{6}t) \quad (28)$$

and

$$N_2(t) = N_{20}(t) + N_{21}(t) + N_{22}(t) = L_2 - L_2 m_2 t + \frac{1}{2} L_2 {m_2}^2 t^2 + -L_1 L_2 t T_0 + \frac{1}{2} L_1 L_2 t^2 T_0 + L_1 L_2 t \alpha_2 + (\frac{15}{32}t^4 - \frac{1}{12}t^3 + \frac{3}{16}t^2 - t) \quad (29)$$

Table 1: The comparison between the values obtained by DJM and the HPM [4] for $N_1(t)$ in Example 1

t	Approximate Solution	Exact Solution	ϵ for the DJM	ϵ for the HPM [4]
2.0000000e-002	9.4382746e-001	9.4000000e-001	3.8274556e-003	3.8274556e-003
4.0000000e-002	8.8868551e-001	8.8000000e-001	8.6855111e-003	8.6855111e-003
6.0000000e-002	8.3463590e-001	8.2000000e-001	1.4635900e-002	1.4635900e-002
8.0000000e-002	7.8173796e-001	7.6000000e-001	2.1737956e-002	2.1737956e-002
1.0000000e-001	7.3004861e-001	7.0000000e-001	3.0048611e-002	3.0048611e-002
1.2000000e-001	6.7962240e-001	6.4000000e-001	3.9622400e-002	3.9622400e-002
1.4000000e-001	6.3051146e-001	5.8000000e-001	5.0511456e-002	5.0511456e-002
1.6000000e-001	5.8276551e-001	5.2000000e-001	6.2765511e-002	6.2765511e-002
1.8000000e-001	5.3643190e-001	4.6000000e-001	7.6431900e-002	7.6431900e-002
2.0000000e-001	4.9155556e-001	4.0000000e-001	9.1555556e-002	9.1555556e-002

Table 2: The comparison between the values obtained by DJM and the HPM [4] for $N_2(t)$ in Example 1

t	Approximate Solution	Exact Solution	ϵ for the DJM	ϵ for the HPM [4]
2.0000000e-002	-1.9925592e-002	-1.9600000e-002	3.2559167e-004	3.2559167e-004
4.0000000e-002	-3.9704133e-002	-3.8400000e-002	1.3041333e-003	1.3041333e-003
6.0000000e-002	-5.9336925e-002	-5.6400000e-002	2.9369250e-003	2.9369250e-003
8.0000000e-002	-7.8823467e-002	-7.3600000e-002	5.2234667e-003	5.2234667e-003
1.0000000e-001	-9.8161458e-002	-9.0000000e-002	8.1614583e-003	8.1614583e-003
1.2000000e-001	-1.1734680e-001	-1.0560000e-001	1.1746800e-002	1.1746800e-002
1.4000000e-001	-1.3637359e-001	-1.2040000e-001	1.5973592e-002	1.5973592e-002
1.6000000e-001	-1.5523413e-001	-1.3440000e-001	2.0834133e-002	2.0834133e-002
1.8000000e-001	-1.7391893e-001	-1.4760000e-001	2.6318925e-002	2.6318925e-002
2.0000000e-001	-1.9241667e-001	-1.6000000e-001	3.2416667e-002	3.2416667e-002

We note that the same approximate solutions are obtained in [4] employing the HPM for $N_1(t)$. However, for $N_2(t)$ it is noticed that all the terms are the same except one term which is equal to $\frac{1}{2}L_1 L_2 t^2 T_0$ in DJM and $\frac{1}{2}L_1 L_2 t T_0^2$ in HPM.

The comparison between the approximate analytic solutions obtained in Eq.(28) and Eq.(29) with the exact solutions in Eq.(24) and the results obtained by HPM in [4] are illustrated in Tables 1 and 2 for $N_1(t)$ and $N_2(t)$, respectively. Moreover, the results of the corresponding absolute errors $\epsilon = |\text{exact solution} - \text{approximate solution}|$ are presented in the same tables, using only the third order term of the approximate solutions. It can be seen from the Tables 1 and 2 that, our approximate solutions are in very good agreement with the exact solutions and HPM, and the absolute errors for DJM and HPM is completely the same for both $N_1(t)$ and $N_2(t)$. The main advantage of DJM are simple, easy to implement (straightforward) and does not required to construct a homotopy as in HPM with some knowledge of deformation from Topology and solve the corresponding the algebraic equations.

Example 2. Let us first consider the system of integro-differential [3] in Eqs. (8) and (9) with

$g_1(t-s)=t, g_2(t-s)=t+1,$
$\alpha_1=\frac{1}{2},\ \alpha_2=3,$
$m_1=m_2=1,$
$T_0=\frac{1}{4},$
$h_1=2t-1-(t^2-t)(1+\frac{11}{18}e^{-3t}-\frac{1}{36}e^{\frac{3}{4}-3t}),$

$h_2=\frac{1}{3072}e^{-3t}(10080t^2-10304t+6275),$

subject to the initial conditions

$N_1(0)=L_1=0,\ \ N_2(0)=L_2=-1.$

The exact solution for this problem is in the following form

$$N_1(t)=t^2-t,\ N_2(t)=-e^{-3t}. \tag{30}$$

Table 3: The comparison of the values obtained by DJM, the VIM [3] and the PLM [3] for $N_1(t)$ in Example 2

t	Approximate Solution	Exact Solution	ϵ for the DJM	ϵ for the VIM [3]	ϵ for the PLM [3]
1.0000000e-001	-8.9858679e-002	-9.0000000e-002	1.4132072e-004	3.5986134e-007	7.9989597e-004
2.0000000e-001	-1.5978291e-001	- 1.6000000e-001	2.1709353e-004	2.6616874e-007	1.4803154e-003
3.0000000e-001	-2.1002770e-001	-2.1000000e-001	2.7696278e-005	4.6652296e-007	2.0523337e-003
4.0000000e-001	-2.4055679e-001	-2.4000000e-001	5.5678587e-004	1.6419084e-005	2.5270256e-003
5.0000000e-001	-2.5112530e-001	-2.5000000e-001	1.1252999e-003	6.9311921e-005	2.9154666e-003
6.0000000e-001	-2.4135220e-001	-2.4000000e-001	1.3521969e-003	1.7360199e-004	3.2287316e-003
7.0000000e-001	-2.1077607e-001	-2.1000000e-001	7.7606656e-004	3.2376196e-004	3.4778957e-003
8.0000000e-001	-1.5889338e-001	-1.6000000e-001	1.1066152e-003	4.9263156e-004	3.6740342e-003
9.0000000e-001	-8.5181920e-002	-9.0000000e-002	4.8180799e-003	6.4142403e-004	3.8282222e-003
1.00000000000	1.0885780e-002	0	1.0885780e-002	7.3721658e-004	3.9515348e-003

Table 4: The comparison of the values obtained by DJM, the VIM [3] and the PLM [3] for $N_2(t)$ in Example 2

t	Approximate Solution	Exact Solution	ϵ for the DJM	ϵ for the VIM [3]	ϵ forthe PLM [3]
1.0000000e-001	-7.4075400e-001	-7.4081822e-001	6.4217832e-005	1.0937008e-005	3.6138715e-002
2.0000000e-001	-5.4876134e-001	-5.4881164e-001	5.0294288e-005	1.5527075e-005	4.5324876e-002
3.0000000e-001	-4.0537135e-001	-4.0656966e-001	1.1983089e-003	8.2270759e-006	4.0630294e-002
4.0000000e-001	-2.9706288e-001	-3.0119421e-001	4.1313305e-003	9.2645680e-005	3.0614270e-002
5.0000000e-001	-2.1454824e-001	-2.2313016e-001	8.5819221e-003	3.9523206e-004	2.0493159e-002
6.0000000e-001	-1.5171682e-001	-1.6529889e-001	1.3582069e-002	9.3721042e-004	1.3006800e-002
7.0000000e-001	-1.0474306e-001	-1.2245643e-001	1.7713370e-002	1.6279583e-003	9.0603832e-003
8.0000000e-001	-7.1419310e-002	-9.0717953e-002	1.9298643e-002	2.2679647e-003	8.1999576e-003
9.0000000e-001	-5.0672708e-002	-6.7205513e-002	1.6532804e-002	2.6314333e-003	8.9646965e-003
1.00000000000	-4.2196235e-002	-4.9787068e-002	7.5908330e-003	2.5745924e-003	9.1478607e-003

It is useful to mention that the number of iterations in the DJM is 4 for both $N_1(t)$ and $N_2(t)$, the comparison between the approximate solutions obtained by DJM with the exact solutions in Eq.(30) and the results obtained by VIM in [3] are illustrated in Tables 3 and 4. It can be seen clearly from Tables 3 and 4 that more iterations require for DJM to achieve the same accuracy as in VIM.

Example 3. Consider the following a system of two integro-differential equations describing the biological species living together: [2, 3, 4], Eqs. (10) and (11) with:

$$g_1(t-s) = g_2(t-s) = e^{-(t-s)},$$

with the initial conditions

$$N_1(0) = L_1, \;\; N_2(0) = L_2.$$

As indicated before this system of two nonlinear integro-differential equations is a special case of the system presented in (8) and (9). Therefore, by following the same procedure as in Eqs. (13-22), a few first terms being calculated by DJM:

$$\begin{aligned} &N_{10} = L_1, \\ &A_{10} = m_1 L_1, \\ &N_{20} = L_2, \\ &A_{20} = -m_2 L_2. \end{aligned} \tag{31}$$

$$N_{11} = I_1(A_{10}) = \int_0^t A_{10}(s)ds = m_1 L_1 t,$$

$$A_{11} = I_2(A_{10}, N_{10}, N_{20}) = m_1 \int_0^t A_{10}(s)ds + N_{10}\left[-\alpha_1 N_{20} - \int_{t-T_0}^t e^{-(t-s)} N_{20} ds\right] =$$
$$= -(1-e^{-T_0})L_1L_2 + L_1 m_1^2 t - L_1 L_2 \alpha_1,$$

$$N_{21} = I_3(A_{20}) = \int_0^t A_{20}(s)ds = -m_2 L_2 t,$$

$$A_{21} = I_4(A_{20}, N_{10}, N_{20}) = -m_2 \int_0^t A_{20}(s)ds + N_{20}\left[\alpha_2 N_{10} + \int_{t-T_0}^t e^{-(t-s)} N_{10} ds\right] =$$
$$= (1-e^{-T_0})L_1L_2 + L_2 {m_2}^2 t + L_1 L_2 \alpha_2,$$

$$N_{12} = I_1(A_{10}+A_{11}) = \int_0^t \left(A_{10}(s) + A_{11}\,(s)\right) ds - N_{11},$$
$$= -L_1L_2t + e^{-T_0}L_1L_2t + \tfrac{1}{2}L_1{m_1}^2t^2 - L_1L_2t\alpha_1$$

$$A_{12} = I_2(A_{10}+A_{11}, N_{10}+N_{11}, N_{20}+N_{21}) = m_1 \int_0^t \left(A_{10}(s) + A_{11}\,(s)\right) ds +$$
$$(N_{10} + N_{11})\left[-\alpha_1 \left(N_{20} + N_{21}\right) - \int_{t-T_0}^t e^{-(t-s)} \left(N_{20} + N_{21}\right) ds\right] - A_{11},$$

$$= \tfrac{1}{2}\mathrm{e}^{-\mathrm{T}_0} L_1(e^{T_0}{m_1}^3t^2 - L_2(2 - 2m_1t + e^{T_0}(-2 + 2T_0 - 2m_2tT_0 + m_2{T_0}^2 - 2m_2t\alpha_1 +$$
$$m_1t(2 + 2T_0 - 2m_2tT_0 + m_2{T_0}^2 + 4\alpha_1 - 2m_2t\alpha_1))))$$

$N_{22} = I_3(A_{20} + A_{21}) = \int\limits_0^t (A_{20}(s) + A_{21}(s))\,ds - N_{21} =$
$L_1 L_2 t - e^{-T_0} L_1 L_2 t + \frac{1}{2} L_2 m_2{}^2 t^2 + L_1 L_2 t \alpha_2$

$A_{22} = I_4(A_{20} + A_{21}, N_{10} + N_{11}, N_{20} + N_{21}) = -m_2 \int\limits_0^t (A_{20}(s) + A_{21}(s))\,ds +$
$(N_{20} + N_{21}) \left[\alpha_2 (N_{10} + N_{11}) + \int\limits_{t-T_0}^t e^{-(t-s)} (N_{10} + N_{11})\,ds \right] - A_{21} =$

$= -\frac{1}{2} e^{-T_0} L_2 (e^{T_0} m_2{}^3 t^2 + L_1(-2(1 + m_2 t) + e^{T_0}(2 - 2(-1 + t)(1 + m_1 t) T_0 + m_1(-1 + t) T_0^2 -$

$2 m_1 t \alpha_2 + m_2 t(2 + 2(-1 + t)(1 + m_1 t) T_0 + (m_1 - m_1 t) T_0^2 + 4\alpha_2 + 2 m_1 t \alpha_2))))$

Therefore, the first three terms of DJM approximate solutions for the system can be given by

$$N_1(t) = N_{10}(t) + N_{11}(t) + N_{12}(t) = L_1 - L_1 L_2 t + e^{-T_0} L_1 L_2 t + L_1 m_1 t + \frac{1}{2} L_1 m_1{}^2 t^2 - L_1 L_2 t \alpha_1 \quad (32)$$

$$N_2(t) = N_{20}(t) + N_{21}(t) + N_{22}(t) = L_2 + L_1 L_2 t - e^{-T_0} L_1 L_2 t - L_2 m_2 t + \frac{1}{2} L_2 m_2{}^2 t^2 + L_1 L_2 t \alpha_2 \quad (33)$$

We note that the same approximate solutions for the system of integro-differential equations describing the biological species living together are obtained in [3, 4]. However, it is worth to mention here the minus sign is missing in calculating A_{20}, A_{21} and A_{22} in [2] by using the Adomian decomposition method which made the the approximate solution of $N_2(t)$ is inaccurate.

5. Conclusion

In this paper, the a system of two nonlinear integro-differential equations describing biological species living together is successfully solved using so-called new iterative method or DJM. The main advantages of proposed method are being derivative-free, overcome the difficulty of some existing techniques, simple to understand and easy to implement. It is economical in terms of computer power/memory and does not involve tedious calculations. There is less computation needed in comparison with the Adomian decomposition method, the homotopy perturbation method and the variational iteration method. Moreover, by solving some examples, it is seems that the DJM appears to be accurate to employ with reliable results.

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