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Research Paper

An aggregation function to solve multicriteria ranking problem involving several decision makers

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Abstract

Multiple Criteria Decision Aiding (MCDA) has been studied in a single decision maker framework for a long time. Nowadays, the need to take into account several conflicting opinions handled by several decisions makers arises. So, researchers are interested with multicriteria problems involving several decision makers. In this context, to solve ranking problem, we develop an aggregation model of several additive value functions. Comparisons with a derivative ELECTRE I method is done on numerical data. Clearly, it appears that the proposed aggregation function is better according to calculation complexity and computation time. Way for further research in this field is proposed.

Keywords: *Aggregation Function, Group Decision, Multicriteria Decision Aiding, Ranking Problem.*

1. Introduction

Deciding is a very complex and difficult task. Some people argue that our ability to make decisions in complex situations is the main feature that distinguishes us from animals [1]. Till nineties, when a manager should take a decision, he could consult experts, focus on similar cases or use classical operations research techniques. Indeed, these techniques are essentially concerned to optimize a so-called economic function [1, 2]. Since last three passed decades, experts focus on multicriteria paradigm which consists to guideline decision by several opposite objectives. Making decision nowadays goes beyond the strict framework of the traditional decision maker who isolates himself to make a decision [3]. The current trend involves a group of people concerned by the decision expertise. One must take into account several different points of view related to various experts or decision makers (DM) to reach a consensual view. Choice between alternatives is reached through a scalarizing process that aims to aggregate individual preferences into a collective preference. According to Jean-Luc Marichal [4], aggregation functions are generally defined and used to combine and summarize several numerical values into one so that the final result of the aggregation takes into account, in a prescribed manner, all individual values.

This paper aims at presenting an aggregation model of several additive value functions to solve a ranking problem with several DM. The multiplicity of decision makers highlights collective decision-making importance within organizations [5]. Indeed, each decision maker brings her or his judgment with respect to each action. Then, one obtains a collective ranking from which is computed a consensual result as indicated in group decision process (see Fig. 1 from [5]).

This paper is organized as follows: Section 2 presents multicriteria ranking problem with several DM, Section 3 is devoted to methods for solving ranking problem in group decision process, some concluding remarks are given in Section 4.

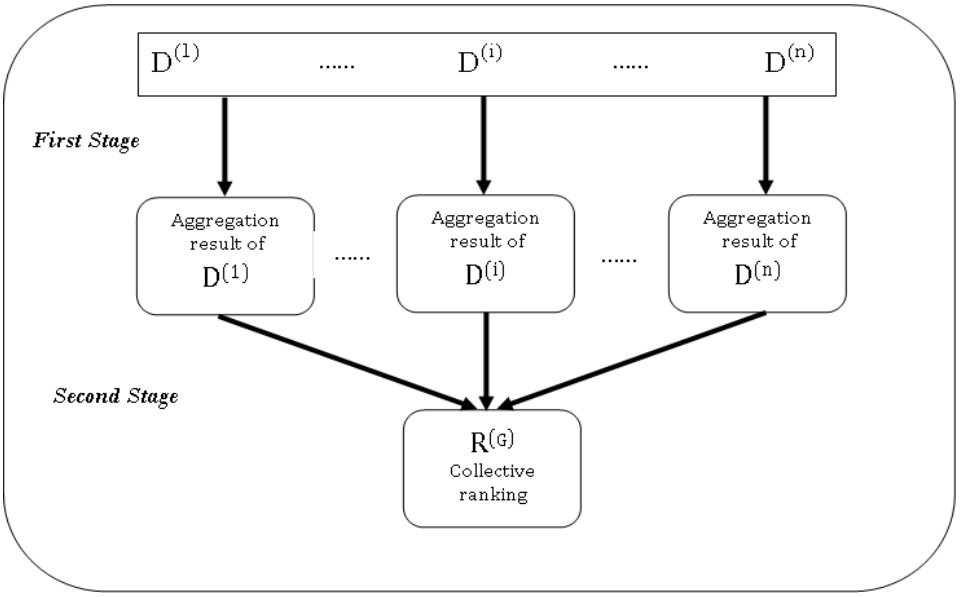


Fig. 1: Group Decision Process

2. Multicriteria ranking problem with multiple decision-makers (DM)

2.1. Multicriteria with multiple DM

Most decision-aid problems involve multiple criteria as well as DMs. Such a problem arises when:

- We have 5 sets :
- $D = \{d_1, d_2, \dots, d_s\}$ identifying all the s DMs;
- $A = \{a_1, a_2, \dots, a_n\}$ collecting the n alternatives or actions;
- $C = \{g_1, g_2, \dots, g_m\}$ determining the m criteria;
- $X = \{x_{ij} = g_j(a_i),\ i = 1, \dots, n; j = 1, \dots, m\}$ defining the performance ratings of alternative i on criterion j;
- $X^l = \{x_{ij}^l = g_j(a_i),\ i = 1, \dots, n; j = 1, \dots, m; l = 1, \dots, s\}$ defining the performance ratings of alternative i on criterion j for l^{th} DM.
- No DM is dictatorial one ;
- No alternative dominates all other ones on each criterion ;
- We need a multicriteria decision aid to found a good compromise.

2.2. Ranking problem

This problematic, denoted P_γ in literature, consists to completely or partially classify alternatives $a_i \in A, i = 1, \dots, n$ according to some preference relation by taking into account criteria $c_j \in C$. In this problem, it is mostly competition attitude which imposes itself. Selection of actions is not required. In this case, decision problem is formulated as follows: how rank competing candidates in decreasing order of merits including ties ?
So, ranking is assigning a rank position for each alternative in a subset $A'$of A; the same rank is assigned to alternatives whose data do not allow to decide between them [6], [7], [8]. Thus, to reach the ranking problem solution, classification process is required as shown in Fig. 2 from [9].

3. Solving multicriteria ranking problem in group decision

In literature, we found only one paper related to the subject: it is the adaptative fuzzy ELECTRE I method used in [10].
In this section, we present outline of the fuzzy ELECTRE I method and a new scalarizing function, the so-called CHEMATRE (**C**hoix et **E**lection des **M**eilleures **A**ctions en **T**raduisant la **RE**alité) for choosing and electing best alternatives expressing reality. Through this section we assume that data are geometric means so that one can completely rank competing alternatives.

3.1. Fuzzy ELECTRE I method

To solve multicriteria ranking problem in group decision, it is assumed that performance ratings are fuzzy numbers, especially trapezoidal one. Here is the outline of this method:

- Each DM gives the weights for all criteria;
- Aggregate the fuzzy performance ratings with report to corresponding weights;
- Convert different criteria scales into comparable one via a linear transformation;
- Found the fuzzy decision matrix;
- Define both concordance and discordance matrix for each pairwise comparison of alternatives;
- Determine boolean matrix B and H, respectively according to minimum level of concordance and discordance;
- Use Hamming distance to introduce dominance relation between each pair of alternatives;
- Conclude by a graphical representation of the binary relation ($\succ$, $\succ^{-1}$, $\approx$, ?) as presented in Fig. 3.

To achieve all these steps, one must have à high background in fuzzy sets and fuzzy logic theories. Nevertheless, it seems possible to solve the multi-Decision Maker multicriteria ranking problem simply. So we try to use some well-known scalarizing functions as weighted sum or variant of Chebychev forms. In the following subsection, we present solely the one which gives consistent results.

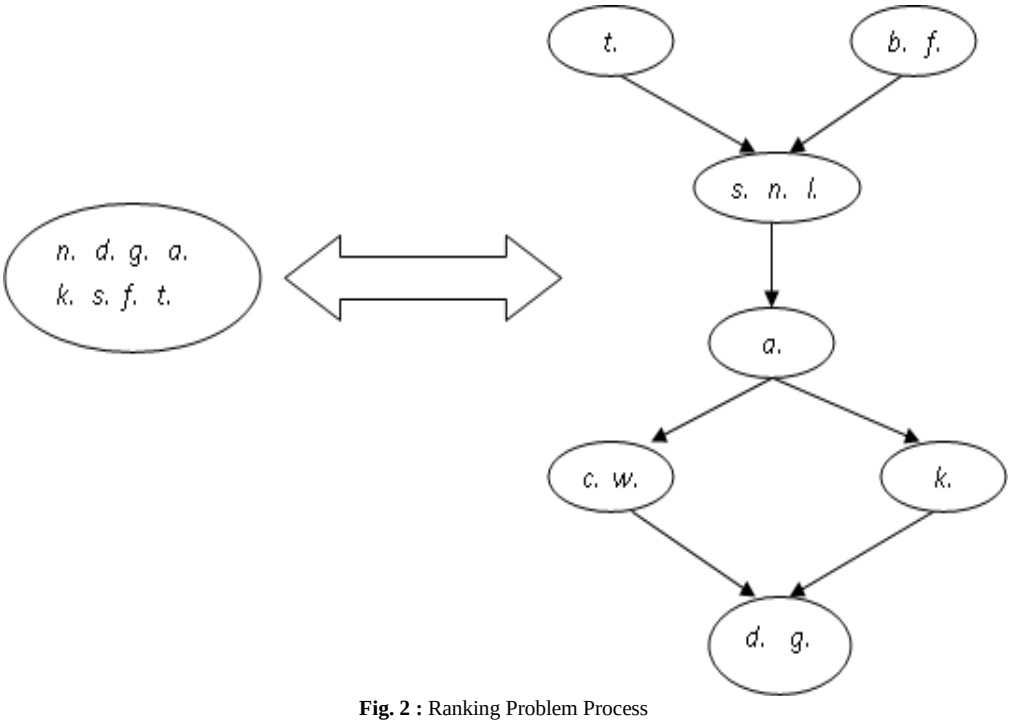


Fig. 2 : Ranking Problem Process

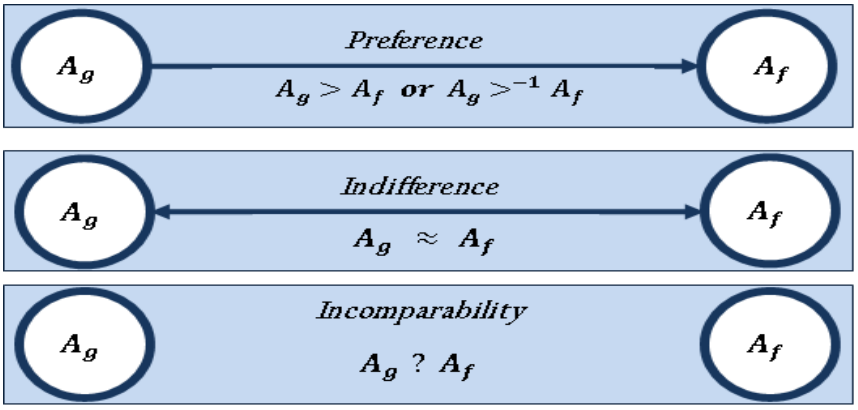


Fig. 3: Binary Relation Representation of ELECTRE I

3.2. Chematre method

This subsection is devoted to the so-called CHEMATRE method, the french for Choice and Election of Best Actions Expressing Reality. CHEMATRE belongs to methods whose principle are related to the Multi-Attribute Utility Theory (MAUT) [8]. So, in this subsection, we introduce notion on value function and aggregation of several additive value function. Before, let us define "geometric average" which is the value function here used.

3.2.1. Geometric average

Definition 1 : The geometric average of n values is obtained by the product of these n values then taking the result n^{th} root. Formally, let $r_1, r_2, \dots, r_n$ be real numbers; their geometric average is

$$M_g = \sqrt[n]{r_1 \cdot r_2 \cdot \dots \cdot r_n}$$

3.2.2. Additive value function

Let $G : A \subseteq \mathbb{R} \longrightarrow \mathbb{R}$ à function such that

$$G(a) = \sum_{j=1}^{m} \lambda_j v_j\big[g_j(a)\big]$$

Where :

- λ_j is the weight of functiong$_j$, $j = 1, \dots, n$;
- $v_j\big[g_j(a)\big]$ is a monotonically increasing (respectively decreasing) function of g_j to maximize (resp. minimize).

It is not restrictive to impose $0 \leq v_j\big[g_j\big] \leq 1$, and $\sum_{j=1}^{m} \lambda_j = 1$ and that we deal with maximization problem. Then G is an additive value function.

3.2.3. Aggregation function of several additive value function models

The general methodology for solving a multicriteria decision problem combines comparison operation and aggregation (or combination, fusion). The order in which these operations are made will determine two major types of approaches for multicriteria decision:

- "aggregate then compare" (AC) approach: MAUT is the main representative of this category,
- "compare then aggregate" (CA) approach : outranking methods are representatives methods of this group.

ELECTRE I, II, IS, Tri are goods illustrations of that approach (see [11]). Clearly, CHEMATRE method uses "AC" strategy.

3.2.4. Multi-attribute utility theory (MAUT)

Let $E = E_1 \times \dots \times E_r, r \geq 2$, à set of objects where interests are described by a set $N := 1, \dots, r$ attributes.
The objective of the multi-attribute utility theory is to numerically model preferences of a decision maker. These preferences are mathematically expressed as a binary relations that is represented by means of a global utility function $U : E \longrightarrow \mathbb{R}$ Such that $a \geq b \Leftrightarrow U(a) \geq U(b),\ \forall\, a, b \in E$
It is usually assumed that the preference relation $\geq$ is complete and transitive (see eg. [12], In the case of non-transitive or incomplete preferences). Note that the most frequently used model for the overall utility function is additive utility model (see eg. [7], [14] and [15]).
The classical approach is based on the aggregation of decision criteria into a single criterion. It is about building a single synthesis criterion while using à scalarizing function S by setting :

$$g(a) = S\left(g_1(a), g_2(a), g_3(a), \dots, g_m(a)\right).$$

In practice, the aggregation function S generally takes one of two forms:

1) Aggregation using weighted sum :

$$S\left(g_1(a), g_2(a), g_3(a), \dots, g_m(a)\right) = \sum_{j=1}^{m} \lambda_j \cdot g_j(a).$$

2) Additive aggregation :

$$S\left(g_1(a), g_2(a), g_3(a), \dots, g_m(a)\right) = \sum_{j=1}^{m} \lambda_j v_j\big[g_j(a)\big].$$

3.3. Outline of CHEMATRE method

Assume that $v_j \equiv 1_{\mathbb{R}}$ the identity function $\forall\, j = 1, \dots, m$.

- Define the matrixw_j^l, $l = 1, \dots, s$ and $j = 1, \dots, m$, criterion j intra-weight for l^{th} DM.
- Determine $G_l(a) = \sum_{j=1}^{m} w_j^l g_j^l(a),\ \forall\, a \in A$
- Compute $U(a) = \sqrt[s]{\prod_{l=1}^{s} G_l(a)}, \forall\, a \in A$
- Operate pairwise comparisons: $\forall\, a,\ b \in A$, if $U(a) \geq U(b)$ then $a \succcurlyeq b$ and if $U(a) = U(b)$ then $a = b$.
- Stop when all comparisons are made.

At the end, a complete or partial ranking is obtained in the set A of alternatives.

3.4. Didactic examples

In this section, we present two examples for performing CHEMATRE method. These examples were be solved by Rasmi Ginting [16].

3.4.1. Example 1

The problem is to find the best product between those in the set $A = \{P_1,\ P_2,\ P_3,\ P_4\}$, taking into account 5 criteria in $C = \{C_1, C_2,\ C_3, C_4, C_5\}$ where $C_1 \equiv$ Production Award (unit: Franks / liter), $C_2 \equiv$ Lifetime hinge (unit: years), $C_3 \equiv$ Paint harmfulness (nominal scale: very little, Middle harmful, very harmful), $C_4 \equiv$ Drying time,and $C_5 \equiv$ Smell of paint (nominal scale: not strong, medium, high, very high). Data are provided by 3 DMs (or assigned to the criteria) in the form of notes. Rating scales extent may differ from one DM to another, and each one of criteria can be assigned a weighting coefficient. The result is a distribution on the set A of alternatives (products), one or more outperforming others.

The principle is the following: solution that outperforms others must be accepted by the largest possible number of DMs, and should not be rejected too much, even by one of them. Each DM builds his matrix of judgments with scales whose minimum value is 0 and maximum value is 10.

For the Fuzzy ELECTRE I method used in [10], the winner is P_3.

Table 1: Judgment Matrix for DM 1

	Price	Life	Odor	Drying	Harm
Weight	6	3	2	4	3
P_1	6	5	2	4	5
P_2	5	6	3	3	4
P_3	7	5	4	6	3
P_4	6	4	5	3	6

Table 2: Judgment Matrix for DM 2

	Price	Life	Odor	Drying	Harm
Weight	7	5	3	3	4
P_1	7	6	2	3	3
P_2	6	5	2	5	3
P_3	5	7	3	6	4
P_4	5	4	4	4	3

Table 3: Judgment Matrix for DM 3

	Price	Life	Odor	Drying	Harm
Weight	6	4	2	3	3
P_1	6	5	2	4	4
P_2	7	6	3	5	3
P_3	6	5	4	3	5
P_4	5	4	3	6	4

CHEMATRE resolution.

Following Table 4 gives result of our methodology with the same set of weights where:

$$G_l(P) = \sum_{j=1}^{5} w_j^l g_j^l(P) \quad (1)$$

$$U(P) = \sqrt[3]{\prod_{j=1}^{3} G_l(P)} \qquad P \in \{P_1, P_2, P_3, P_4\} \quad (2)$$

Table 4: Result of CHEMATRE

$G_l(P)$	$G_1(P)$	$G_2(P)$	$G_3(P)$	$U(P)$
Product 1	86	106	84	91.48
Product 2	78	100	96	90.80
Product 3	98	113	88	99.14
Product 4	88	91	82	86.91

Thus, overall alternatives scores are:

Table 5: Ranking of Products

P_1	P_2	P_3	P_4
91.48	90.80	99.14	86.91
II	III	I	IV

The above mentioned table 5 shows that P_3 is the best product. It is followed by P_1 and so on.

3.4.2. Example 2

The problem is to choose a partner from the following set:

$$A = \begin{Bmatrix} Nippon\ Paint\ KK\ (P1), Courtaulds\ Coatings\ Holding\ (P2), Kansai\ Paint\ (P3), \\ International\ Paint\ (P4), US\ Sec\ of\ navy\ (\mathrm{P5}) \end{Bmatrix}$$

The set of criteria is:

$$\mathrm{C} = \begin{cases} \mathrm{C}_1 \equiv \text{Product quality (nominal scale: Good, medium, Poor)} \\ \mathrm{C}_2 \equiv \text{Technology (nominal scale : Good, medium, Poor)} \\ \mathrm{C}_3 \equiv \text{Cost (unit : franks)} \\ \mathrm{C}_4 \equiv \text{Time (unit : hour)} \end{cases}$$

A common preference scale to the four criteria was used. In practice this choice facilitates assignment of weights values associated to criteria. A partner with an average price is preferred. Indeed changing a preference scale that is associated to a criterion requires to change the weight value of this criterion in order to preserve the idea of compensation between criteria. Data are provided by DM (or assigned to the criteria by experts) as notes. The extent of rating scales may differ from one DM to another, and each criterion has a weighting factor. The result is a distribution on the alternatives set A, one or more alternatives that outperform others. Here is the principle: the alternative(s) which outperform(s) others must be accepted by a largest possible number of DMs, and should not be rejected too much, even by one of them. Each DM constructs its judgments matrix. Values scale lie from 0 to 10. For the Fuzzy ELECTRE I method used in [10], the winner is P1.

Table 6: Judgment Matrix for DM 1

	C_1	C_2	C_3	C_4
Weight	3	4	3	5
P1	6	8	9	4
P2	4	5	6	7
P3	7	6	8	4
P4	6	8	4	7
P5	5	4	7	6

Table 7: Judgment Matrix for DM 2

	C_1	C_2	C_3	C_4
Weight	4	3	2	5
P1	7	5	3	8
P2	3	6	8	4
P3	6	8	4	3
P4	5	4	6	7
P5	2	3	7	5

Table 8: Judgment Matrix for DM 3

	C_1	C_2	C_3	C_4
Weight	4	5	3	5
P1	8	3	6	7
P2	6	5	7	3
P3	5	8	4	2
P4	4	7	3	6
P5	7	6	5	8

CHEMATRE resolution.

Following Table 9 gives result of our methodology with the same set of weights:

Table 9: Result of CHEMATRE

$G_l(P)$	$G_1(P)$	$G_2(P)$	$G_3(P)$	U(P)
P1	97	89	100	95.21
P2	85	66	85	78.12
P3	89	71	82	80.31
P4	97	79	90	88.35
P5	82	56	113	80.35

Where $G_l(P)$, U(P) with $P \in \{P1, P2, P3, P4\}$ are idem to those defined in equations (1) and (2) respectively. Thus, overall alternatives scores are:

Table 10: Ranking of Products

P1	P2	P3	P4	P5
95.21	78.12	80.31	88.35	80.35
I	V	IV	II	III

The above mentioned Table 10 shows that P1 is the best product. P4 is second-best and so on.

4. Concluding remarks

Thanks to interactive systems decision aiding, approaches to assist groups seeking a common solution have been developed.

Aggregation function presented in this paper combines concepts of geometric average and weighted sum average. Indeed, given the offsetting effect of the arithmetic average, method is desired resistant to manipulability by decision makers assigning high marks for important criteria to actions they wish to be selected. The geometric average then seems to be well indicated for this problem.

As PROMETHEE methods and all ELECTRE versions, CHEMATRE method is devoted to solve ranking problem. This is done with the usual real order on overall scores generated for each alternative, by the aggregating function which avoid compensation.

Results provided by CHEMATRE and those generated by the ELECTRE I method coincide for both examples presented here. Nevertheless CHEMATRE method does not use as long calculations as concordances and discordances matrices used in ELECTRE methods. So, for its simplicity and efficiency, CHEMATRE method is a representative method when dealing with ranking problem especially in presence of several DMs and criteria.

A challenge in this field is to pursue extensive numerical experiments, to compare results with other methods and to explore some extension of outranking such as ORESTE method for solving multi-Decision Maker multicriteria ranking problems.

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