International Journal of Applied Mathematical Research, 3 (4) (2014) 380-389
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www.sciencepubco.com/index.php/IJAMR
doi: 10.14419/ijamr.v3i4.3273
Research Paper

The trend of dog rabies with vaccination using seir model case study: Bolgatanga district, Ghana

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Abstract

Many models to study the trend of infectious diseases like smallpox, rubella, measles, chicken pox etc in populations have been analyzed mathematically. Our study uses standard $SEIR$ model with vaccination to predict the spread of dog rabies in the Bolgatanga District of Ghana. Data from Ghana Veterinary Medical Association Report [9] was collected and analyzed. We estimated the non-endemic basic reproductive ratio (R_0) of 0.3755 and determine that 24.63% of the population are to be vaccinated using the Herd immunity threshold. The stability analysis of the disease-free equilibrium point was $(37.5085, 0, 0, 48.6233)$ and the stability analysis of endemic equilibrium point is asymptotically stable. We perform sensitivity analysis and realize that a transmission coefficient of $\beta \geq 8.102 \times 10^{-3}$ will lead to $R_0 \geq 1.0897$ which is endemic.

Keywords: *Endemic, Exposed, Infectious, Rabies, Recovered, Susceptible*.

1. Introduction

Bolgatanga, one of the districts of Ghana is a regional capital of the Upper East Region. Geographically, the total area is about 729 km^2 (Wikipedia, [4]) and located north of Tamale. The predominant occupation is subsistence farming along with some handicraft like straw baskets, smocks, leather bags among others. The inhabitants are Muslims and enjoy dog's meat as a sumptuous meal.

There is a growing concern about the rise of rabies in this district together with Talensi-Nandam and Bongo district. Rabies is an acute fatal disease caused by a virus and the single mode of spread is through the bite of an infected animal. It is characterized by disturbed consciousness, increased nerve sensitivity and consequent symptoms of paralysis.

2. Related literature

Rabies is a Latin word which means madness or rage. It is sometimes called hydrophobia because of the inability by the infected individual to swallow water. The disease caused by a virus of the family of Rhabdoviridea. Rabies causes acute neurological syndrome dominated by forms of hyperactivity or paralysis following a bite from a suspected rabid dog but occasionally by other form of contact. The disease is natural to dogs, raccoons, cats, bats, foxes, wolves and other warm blooded organisms (Johnson, [1]). Rabies is well established with population vaccination of 4.7% in Bongo, 0.9% in Talensi-Nandam and 5.6% in Bolgatanga in the Upper East Region (Polkuu et al., [5])

Yorke and London [6] included latent period of the infectious agent for ascertainment of SEIR model with seasonal force to describe irregular and biennial oscillatory of measles incidence. Grais [7] used variation of SEIR epidemic model to estimate the transmission of measles in Niamey, Niger. Macdonald [10] uses descriptive study to determine the ecological factors that could influence the spatial propagation of virus such as habit quality or fox densities as fox rabies continues to advance southwesternly into France and Switzerland.

Zhang et al. [3] developed a model to explore effective control and preventive measures as well as a deterministic model to study the transmission dynamics of rabies in China. The findings of their study estimate the basic reproductive number $R_0 = 2$ for rabies transmission in china and predicts that the number of human rabies is decreasing but may reach a peak in 2030. The models above is situational and do not apply to Ghana. Our study is to formulate a time-dependent model that will mimic the behaviuor of the spread and simulate it, determine the mode of transmission of rabies with vaccination disease in the Bolgatanga district of Ghana.

3. Mathematical formulation of seir model

Using the SEIR model in epidemiology for the analysis. The preys include men, children and women and will be mainly restricted to Bolgatanga districts. We shall consider the case with vaccination.

3.1. Seir model of rabies with vaccination

In our SEIR model, the population is divided into four different sections. These are the susceptible class (S), the class of exposed (E) victims, then there are the infected class (I) and the removed or recovered class (R).
Assumptions

1) Here, the R compartment constitutes dogs which have recovered from the infection upon administration of the rabies vaccine.
2) A portion αS of the susceptible go to the recovered class (R) directly due to pre-exposure vaccination.
3) A portion αE of the exposed go to the recovered class (R) directly due to post-exposure vaccination.
4) A portion kR of the recovered go to the susceptible class(S) directly due to the wanning immunity of the rabies vaccine.
5) All infective dogs die so there is no chance that they will progress to the recovered class (R).
6) The birth rate of dogs is equal to their death rate so the population under consideration is closed. Figure 3.1 shows the SEIR model with vaccination.

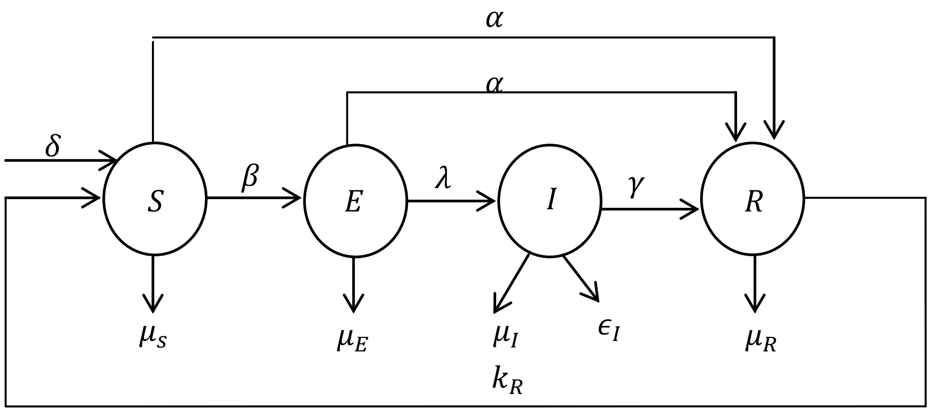

Fig. 3.1: Flow Chart Showing SEIR Model with Vaccination

With the model equations defined as

$$\frac{dS}{dt} = \delta + kR - \beta SI - \alpha S - \mu S \quad (3.1)$$

$$\frac{dE}{dt} = \beta SI - \alpha E - \mu E - \nu E \quad (3.2)$$

$$\frac{dI}{dt} = \nu E - \mu I - \epsilon I \quad (3.3)$$

$$\frac{dR}{dt} = \alpha S + \alpha E - \mu R - \kappa R \quad (3.4)$$

With $N(t) = S(t) + E(t) + I(t) + R(t)$ and the parameters for the model are defined as
$\delta =$ Birth rate of dogs, $\beta =$ transmission co-efficient between dogs, $\lambda =$ latency rate in dogs, $\mu =$ death rate in dogs, $\alpha =$ vaccination rate co-efficient, $\epsilon =$ and disease induced mortality of dogs, $\kappa =$ wanning immunity in dogs.

4. Basic reproduction ratio (r_o) rabies transmission with vaccination

Using the Next Generation matrix Approach, we re-arrange the model equation to take the form E, I, S, R for the model equations and linearize to obtain.

$$\frac{dE}{dt} = \beta\left(\frac{S(t)}{N(t)}\right)I - (\alpha + \mu + \nu)E \qquad U(E, I, S, R)$$

$\frac{dI}{dt} = \nu E - (\mu + \epsilon)I$ $\qquad X(E, I, S, R)$ (3.5)
$\frac{dS}{dt} = \delta + \kappa R - \beta\left(\frac{S(t)}{N(t)}\right)I - (\alpha + \mu)S$ $\qquad Y(E, I, S, R)$
$\frac{dR}{dt} = \alpha(S + E) - (\mu + \kappa)R$ $\qquad Z(E, I, S, R)$

We therefore compute R_0 using the Next Generation Matrix Approah which comprise the following:
F − matrix form of the new infections that will arise.
V − matrix form of the transfer of infections from one compartment to another
R_0 − the dorminant eigen value of the matrix $G = FV^{-1}$
Linearizing the disease free-equilibrium.

$$G = \begin{bmatrix} F-V & 0 \\ J_1 & J_2 \end{bmatrix} = \begin{bmatrix} \frac{\partial U}{\partial E} & \frac{\partial U}{\partial I} & \frac{\partial U}{\partial S} & \frac{\partial U}{\partial R} \\ \frac{\partial X}{\partial E} & \frac{\partial X}{\partial I} & \frac{\partial X}{\partial S} & \frac{\partial X}{\partial R} \\ \frac{\partial Y}{\partial E} & \frac{\partial Y}{\partial I} & \frac{\partial Y}{\partial S} & \frac{\partial Y}{\partial R} \\ \frac{\partial Z}{\partial E} & \frac{\partial Z}{\partial I} & \frac{\partial Z}{\partial S} & \frac{\partial Z}{\partial R} \end{bmatrix} = \begin{bmatrix} -\alpha-\mu-\nu & \beta & 0 & 0 \\ \nu & -\mu-\epsilon & 0 & 0 \\ 0 & -\beta & -\alpha-\mu & \kappa \\ \alpha & 0 & \alpha & -\mu-\kappa \end{bmatrix}$$

We then partition the G matrix into a 2x2 square matrix into the following where

$$F - V = \begin{bmatrix} -\alpha-\mu-\nu & \beta \\ \nu & -\mu-\epsilon \end{bmatrix} = \begin{bmatrix} 0 & \beta \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} \alpha+\mu+\nu & 0 \\ -\nu & \mu+\epsilon \end{bmatrix}$$

$$V^{-1} = \frac{1}{(\alpha+\mu+\nu)(\mu+\epsilon)}\begin{bmatrix} \mu+\epsilon & 0 \\ \nu & \alpha+\mu+\nu \end{bmatrix} = \begin{bmatrix} \frac{1}{(\alpha+\mu+\nu)} & 0 \\ \frac{\nu}{(\alpha+\mu+\nu)(\mu+\epsilon)} & \frac{1}{(\mu+\epsilon)} \end{bmatrix}$$

$$FV^{-1} = \begin{bmatrix} 0 & \beta \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \frac{1}{(\alpha+\mu+\nu)} & 0 \\ \frac{\nu}{(\alpha+\mu+\nu)(\mu+\epsilon)} & \frac{1}{(\mu+\epsilon)} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\beta\nu}{(\alpha+\mu+\nu)(\mu+\epsilon)} & \frac{\beta}{(\mu+\epsilon)} \\ 0 & 0 \end{bmatrix}$$

$$R_0 = \frac{\beta\nu}{(\alpha+\mu+\nu)(\mu+\epsilon)} \tag{3.6}$$

4.1. Equilibrium points of rabies model with vaccination

The models stability is determine by first evaluating the steady states of the differential equations (3.1), (3.2), (3.3) and (3.4). At the steady state $\frac{dS}{dt} = 0, \frac{dE}{dt} = 0, \frac{dI}{dt} = 0, \frac{dR}{dt} = 0$ to obtain

$$\delta + kR - \beta SI - (\alpha + \mu)S = 0 \tag{3.7}$$
$$\beta SI - \alpha E - (\mu + \nu)E = 0 \tag{3.8}$$
$$\nu E - \mu I - \epsilon I = 0 \tag{3.9}$$
$$\alpha(S + E) - \mu R - \kappa R = 0 \tag{3.10}$$

We solve for S, E, I and R to obtain

$$S = \frac{(\alpha+\mu+\nu)(\mu+\epsilon)}{\beta\nu} \tag{3.11}$$
$$E = \frac{\delta\beta\nu - [\alpha(k-1)+\mu)](\alpha+\mu+\nu)(\mu+\epsilon)}{\beta\nu(\alpha+\mu+\nu)(\mu+\epsilon) - \alpha\beta\nu} \tag{3.12}$$
$$I = \frac{\delta\beta\nu^2 - [\alpha(k-1)+\mu)](\alpha+\mu+\nu)(\mu+\epsilon)}{(\mu+\epsilon)[\beta\nu(\alpha+\mu+\nu)(\mu+\epsilon) - \alpha\beta\nu]} \tag{3.13}$$
$$R = \frac{1}{\mu+\kappa}\left[\frac{\alpha(\mu+\epsilon)(\alpha+\mu+\nu) - \mu}{\beta\nu(\mu+\kappa)} + \frac{\alpha\delta}{(\mu+\kappa)(\alpha+\mu+\nu) - \alpha}\right] \tag{3.14}$$

The equilibrium point is then given by

$$(S^*, E^*, I^*, R^*) = \begin{pmatrix} \frac{(\alpha+\mu+\nu)(\mu+\epsilon)}{\beta\nu}, \frac{\delta\beta\nu - [\alpha(k-1)+\mu)](\alpha+\mu+\nu)(\mu+\epsilon)}{\beta\nu(\alpha+\mu+\nu)(\mu+\epsilon) - \alpha\beta\nu}, \frac{\delta\beta\nu^2 - [\alpha(k-1)+\mu)](\alpha+\mu+\nu)(\mu+\epsilon)}{(\mu+\epsilon)[\beta\nu(\alpha+\mu+\nu)(\mu+\epsilon) - \alpha\beta\nu]}, \\ \frac{1}{\beta\nu(\mu+\kappa)}\left[\alpha(\alpha+\mu+\nu)(\mu+\epsilon) + \frac{\alpha\delta\beta\nu}{(\mu+\kappa)(\alpha+\mu+\nu) - \alpha} - \frac{\mu\alpha(\mu+\epsilon)(\alpha+\mu+\nu)}{(\mu+k)(\alpha+\mu+\nu)}\right] \end{pmatrix}$$

4.2. The disease- free equilibrium point of rabies with vaccination

At the disease-free equilibrium we consider a situation where there is no infection. In this case $E = I = 0.$ Putting $\frac{dS}{dt} = 0$ and $\frac{dR}{dt} = 0$ we can now solve for the values of S and R.

$$\frac{dS}{dt} = 0, E = I = 0, \Longrightarrow S = \frac{kR+\delta}{(\alpha+\mu)} \tag{3.15}$$

Also, $\frac{dR}{dt} = 0, E = I = 0, \quad \Longrightarrow S = \frac{(\mu+\kappa)R}{\alpha}$ (3.16)

Solving (3.15/19) and (3.16/20), we have

$\frac{kR+\delta}{(\alpha+\mu)} = \frac{(\mu+\kappa)R}{\alpha}$ Where

$$R = \frac{\alpha\delta}{\mu^2+(\alpha+\kappa)\mu} \quad (3.17)$$

From (3.16)

$$S = \frac{\delta(\mu+\kappa)}{\mu^2+(\alpha+\kappa)\mu} \quad (3.18)$$

Thus at the disease-free equilibrium, $(S,E,I,R) = \left(\frac{\delta(\mu+\kappa)}{\mu^2+(\alpha+\kappa)\mu}, 0, 0, \frac{\alpha\delta}{\mu^2+(\alpha+\kappa)\mu}\right)$ the system behaves in a damped oscillatory manner with a certain period determined by the parameters. Given the model parameters, the period of the oscillation plays a role for us to predict the further behavior of the infection.

4.3. Stability analysis of disease -free equilibrium point of rabies transmission with vaccination

We determine the stability of the system at the disease-free equilibrium by linearizing the system of equations about the equilibrium point. The Jacobian Matrix for

$$\dot{S} = f(S,E,I,R) = \frac{dS}{dt} = \delta + \kappa R - \beta\left(\frac{S}{N}\right)I - (\alpha+\mu)S \quad (3.19)$$

$$\dot{E} = g(S,E,I,R) = \frac{dE}{dt} = \beta\left(\frac{S}{N}\right)I - (\alpha+\mu+\nu)E \quad (3.20)$$

$$\dot{I} = h(S,E,I,R) = \frac{dI}{dt} = \nu E - (\mu+\epsilon)I \quad (3.21)$$

$$\dot{R} = k(S,E,I,R) = \frac{dR}{dt} = \alpha S + \alpha E - (\mu+\kappa)R \quad (3.22)$$

Is

$$J(S,E,I,R) = \begin{bmatrix} \frac{\partial f}{\partial S} & \frac{\partial f}{\partial E} & \frac{\partial f}{\partial I} & \frac{\partial f}{\partial R} \\ \frac{\partial g}{\partial S} & \frac{\partial g}{\partial E} & \frac{\partial g}{\partial I} & \frac{\partial g}{\partial R} \\ \frac{\partial h}{\partial S} & \frac{\partial h}{\partial E} & \frac{\partial h}{\partial I} & \frac{\partial h}{\partial R} \\ \frac{\partial I}{\partial S} & \frac{\partial k}{\partial E} & \frac{\partial k}{\partial I} & \frac{\partial k}{\partial R} \end{bmatrix} = \begin{bmatrix} -\alpha-\mu-\beta I & 0 & -\beta S & \kappa \\ \beta I & -\alpha-\mu-\nu & \beta S & 0 \\ 0 & \nu & -\mu-\epsilon & 0 \\ \alpha & \alpha & 0 & -\mu-\kappa \end{bmatrix}$$

Introducing $S = 1$ and $I = 0$ the characteristic equation becomes

$$\begin{vmatrix} -\alpha-\mu-\lambda & 0 & -\beta S & \kappa \\ 0 & -\alpha-\mu-\nu-\lambda & \beta S & 0 \\ 0 & \nu & -\mu-\epsilon-\lambda & 0 \\ \alpha & \alpha\, 0 & & -\mu-\kappa-\lambda \end{vmatrix} = 0$$

$$(-\mu-\epsilon-\lambda)\big(\lambda^3 + (3\mu+2\alpha+\beta\nu++\kappa)\lambda^2 + (3\mu^2+(4\alpha+3\kappa+2\nu-\beta\nu)\mu+2\alpha\kappa+\alpha^2+\alpha\nu+\alpha\beta\nu)\lambda + (\mu^3+(2\alpha+\beta\nu+\kappa+\nu)\mu^2+(\alpha^2+\alpha\nu+\alpha\beta\nu+\alpha\kappa+\nu\kappa+\beta\nu\kappa)\mu)\big) = 0 \quad (3.23)$$

Let the cubic equation be represented by

$p = (3\mu+2\alpha+\beta\nu+\nu+\kappa)$

$q = (3\mu^2+(4\alpha+3\kappa+2\nu-\beta\nu)\mu+2\alpha\kappa+\alpha^2+\alpha\nu+\alpha\beta\nu+\beta\nu\kappa)$

$r = (\mu^3+(2\alpha+\beta\nu+\kappa+\nu)\mu^2+(\alpha^2+\alpha\nu+\alpha\beta\nu+\alpha\kappa+\nu\kappa+\beta\nu\kappa)\mu)$

Putting the expression for p, q and r into equation (3.23), we have

$$\lambda^3 + p\lambda^2 + q\lambda + r = 0 \quad (3.24)$$

The solution of (3.24) gives us three eigen values λ_1, λ_2 and λ_3. Equation (3.24) has the discriminant

$\Delta = p^2q^2 - 4q^3 - 4p^3r - 27r^2 + 18pqr$

With the following cases as roots

a) If $\Delta > 0$, then the equation has three distinct real roots

b) If $\Delta = 0$, then the equation has a multiple real and equal roots

c) If $\Delta < 0$, then the equation has one real root and two complex conjugate roots.

To find the roots of the cubic equation, let the coefficient of λ^3 be represented by a; i.e$a = 1$. The equation (3.24) becomes

$a\,\lambda^3 + p\lambda^2 + q\lambda + d = 0$ (3.25)

If equation (3.28) with integer co-efficients will have rational root, which can be obtain using the rational roots tests. If r is the roots of equation (3.25), then we may have a factor out $(\lambda_1 - r)$ using polynomial long division to obtain

$(\lambda - r)(a\lambda^2 + (p+a\lambda)\lambda + q + pr + ar^2) = a\lambda^3 + p\lambda^2 + q\lambda + d.$ Hence any known single roots will help to obtain other roots by the quadratic formula to obtain

$$\lambda_{2,3} = \frac{-(p+ra) \pm \sqrt{p^2 - 4aq - 2apr - 3a^2r^2}}{2a}$$

As the other two roots.

4.4. Herd immunity ratio for rabies transmission with vaccination

A percentage of the population needs to be immuned in order to control transmission of the disease. Diekmann and Heesterbeek [8] proposed a threshold for Herd Immunity as

$H_1 = 1 - \frac{1}{H_0}$

$$H_1 = \frac{\beta\nu - (\alpha+\mu+\nu)(\mu+\epsilon)}{\beta\nu} \quad (3.26)$$

5. Model analysis of results

After the end of the free anti-rabies immunization in 1998, currently Bolgatanga district has
Experience an increase of positive cases of rabies. In this paper, standard results for the ODE parameters were obtained from Ghana Veterinary Medical Association Report (GVMAR), [9]. We simulate and perform analysis based from these standard results shown in table 5.1.

Table 5.1: Parameter Description for the ODE's

Parameter	Description	Standard Value
β	Transmission Coefficient	$3.0417 \times 10^{-3}(Dogsmonth)^{-1}$
ν	Latency Rate	$2.1429 \times 10^{-3}(month)^{-1}$
$\gamma = \mu$	Rate Of Death	$2.293 \times 10^{-3}(month)^{-1}$
δ	Rate Of Birth	$0.1975 kits/female\ month^{-1}$
ϵ	Rate Of Disease Induced Mortality	$4.9167 \times 10^{-3}(month)^{-1}$
κ	Rate Of Weakening Immunity	$1.9177 \times 10^{-3}(month)^{-1}$
α	Rate Of Vaccination	$2.975 \times 10^{-3}(month)^{-1}$

5.1. Estimating the basic reproductive ratio (R_0) of rabies transmission with vaccination

Our model uses Basic Reproductive Ratio of

$R_0 = \frac{\beta\nu}{(\alpha+\mu+\nu)(\mu+\epsilon)}$

$= \frac{3.0417\times10^{-3}\times2.1429\times10^{-3}}{(2.975\times10^{-3}+2.293\times10^{-3}+2.1429\times10^{-3})(2.293\times10^{-3}+4.9167\times10^{-3})}$

$= 0.3755 < 1$

Which means the infected population does not die out.

5.2. Stability analysis of disease-free equilibrium points of rabies transmission with vaccination

The $SEIR$ value is given as $S, E, I, R = \left(\frac{(\mu+\kappa)\delta}{\mu^2+(\alpha+\kappa)\mu}, 0,0, \frac{\alpha\delta}{\mu^2+(\alpha+\kappa)\mu}\right)$

$= \left(\frac{4.5325\times10^{-4}}{1.2084\times10^{-5}}, 0, 0, \frac{5.8756\times10^{-4}}{1.2084\times10^{-5}}\right)$

$(37.5085, 0, 0, 48.6233)$

But we know that $J(S,E,I,R) = \begin{bmatrix} -\alpha-\mu & 0 & -\beta S & \kappa \\ 0 & -\alpha-\mu-\nu & \beta S & 0 \\ 0 & \nu & -\mu-\epsilon & 0 \\ \alpha & \alpha & 0 & -\mu-\kappa \end{bmatrix}$

Inserting the values of $\alpha, \beta, \gamma, \epsilon, \mu$ and κ into the characteristic equation we obtain

$$\begin{vmatrix} -5.268\times10^{-3}-\lambda & 0 & -3.0417\times10^{-3} & 1.9177\times10^{-3} \\ 0 & -7.4109\times10^{-3}-\lambda & 3.0417\times10^{-3} & 0 \\ 0 & 2.149\times10^{-3} & -2.3421\times10^{-3}-\lambda & 0 \\ 2.975\times10^{-3} & 2.975\times10^{-3} & 0 & -2.2949\times10^{-3}-\lambda \end{vmatrix} = 0$$

$\lambda_1 = -2.835\times10^{-3} + 0.9945i, \lambda_2 = -4.91\times10^{-3} + 0.1044i, \lambda_3 = -4.91\times10^{-3} - 0.1044i$ and $\lambda_4 = -2.8353 - 0.9945i$. Since all the roots are complex with negative real part, we therefore say that the equilibrium point of the disease-free equilibrium for dog rabies with vaccination in the Bolgatanga district is asymptotically stable.

5.3. Stability analysis of the endemic equilibrium point of rabies with vaccination

From the Jacobian matrix we know that from equation (3.17).

$I = \frac{\delta\beta\nu^2 - [\alpha(k-1)+\mu)](\alpha+\mu+\nu)(\mu+\epsilon)}{(\mu+\epsilon)[\beta\nu(\alpha+\mu+\nu)(\mu+\epsilon)-\alpha\beta\nu]}$

$= \frac{(2.7586\times10^{-9}+1.4614\times10^{-6})(1.7357\times10^{-5})}{(2.3421\times10^{-3})(1.1314\times10^{-10})-(1.9391\times10^{-8})}$
$= -1.3104 \times 10^{-3}$
$\beta I = (3.0417)(-1.3104 \times 10^{-3})$
$= -3.9859 \times 10^{-6}$
Also from equation (3.18)
$S = \frac{(\alpha+\mu+\nu)(\mu+\epsilon)}{\beta\nu}$
$= \frac{(7.4109\times10^{-3})(2.342\times10^{-3})}{6.5181\times10^{-6}}$
$= 2.6630$
$\beta S = (3.0417 \times 10^{-3})(2.6630)$
$= 8.1 \times 10^{-3}$
Putting βI and βS into the Jacobian matrix the characteristic equation is

$$\begin{bmatrix} -5.264\times10^{-3}-\lambda & 0 & -8.1\times10^{-3} \\ -3.9859\times10^{-6} & -7.4109\times10^{-3}-\lambda & -8.1\times10^{-3} \\ 0 & 2.1429\times10^{-3} & -2.3422\times10^{-3}-\lambda \end{bmatrix} = 0$$

$\lambda_1 = -1.0253 \times 10^{-9}, \lambda_2 = -7.8 \times 10^{-3}, \lambda_3 = -7.8 \times 10^{-3}$
Since $\lambda_1, \lambda_2, \lambda_3 \in \mathbb{R} < 0$ the equilibrium point is said to be asymptotically stable.

5.4. Point estimation of herd immunity threshold (H_1)

Rabies which is contagious is likely to be disrupted when large numbers of a population are immune or less susceptible to the disease. We use the estimation
$H_1 = 1 - \frac{1}{R_0}$
$= 1 - \frac{1}{1.3267}$
$= 0.2463$
Therefore to control the epidemic in Bolgatanga district, about 24.63% of the population need to be vaccinated.

5.5. Sensitivity analysis of R_0 for rabies transmission with vaccination

If β is increased to (say) $\beta = 8.802 \times 10^{-3}$ then
$R_0 = \frac{\beta\nu}{(\alpha+\mu+\nu)(\mu+\epsilon)}$
$= \frac{8.802\times10^{-3}\times2.1429\times10^{-3}}{(2.975\times10^{-3}+2.293\times10^{-3}+2.1429\times10^{-3})(2.293\times10^{-3}+4.9167\times10^{-3})}$
$= 1.0897 > 1$
Thus keeping $\alpha, \mu, \nu, \epsilon$ unchanged and choosing any number for $\beta \geq 8.102 \times 10^{-3}$ will cause $R_0 \geq 1$. That is a transmission coefficient of 8.102×10^{-3} or more will produce a higher basic reproductive ratio which will wipe out the population.

5.6. Sensitivity analysis of disease-free equilibrium point of rabies transmission with vaccination

We know decouple the Jacobian matrix J and obtain the determinant

$$\begin{bmatrix} -\alpha-\mu-\lambda & 0 & -\beta \\ 0 & -\alpha-\mu-\nu-\lambda & \beta \\ 0 & \nu & -\mu-\epsilon-\lambda \end{bmatrix} = 0$$

Inserting the values of $\alpha, \mu, \beta, \epsilon$ and ν into the above, we obtain

$$\begin{bmatrix} -5.268\times10^{-3}-\lambda & 0 & -3.0417\times10^{-3} \\ 0 & -7.4109\times10^{-3}-\lambda & 3.0417\times10^{-3} \\ 0 & 2.1429\times10^{-3} & -2.3421\times10^{3}-\lambda \end{bmatrix} = 0$$

$\lambda_1 = -1.4741 \times 10^{-3}, \lambda_2 = -6.7734 \times 10^{-3}$ and $\lambda_3 = -6.7734 \times 10^{-3}$ And since $\lambda_1, \lambda_2, \lambda_3 < 0$ we conclude that the disease-free equilibrium point for Rabies transmission with vaccination is asymptotically stable.

5.7. Sensitivity analysis of disease-free equilibrium point of rabies transmission with vaccination by simulation

We perform simulation to our model using the data in table 4.1. The values of S, E, I and R are altered and the change that occur in the model are observed. We consider a period of six months and give a matlab plot of each compartment of the rabies model with vaccination.

Case I: For $S = 500$, $E = 0$, $I = 0$ and $R = 0$

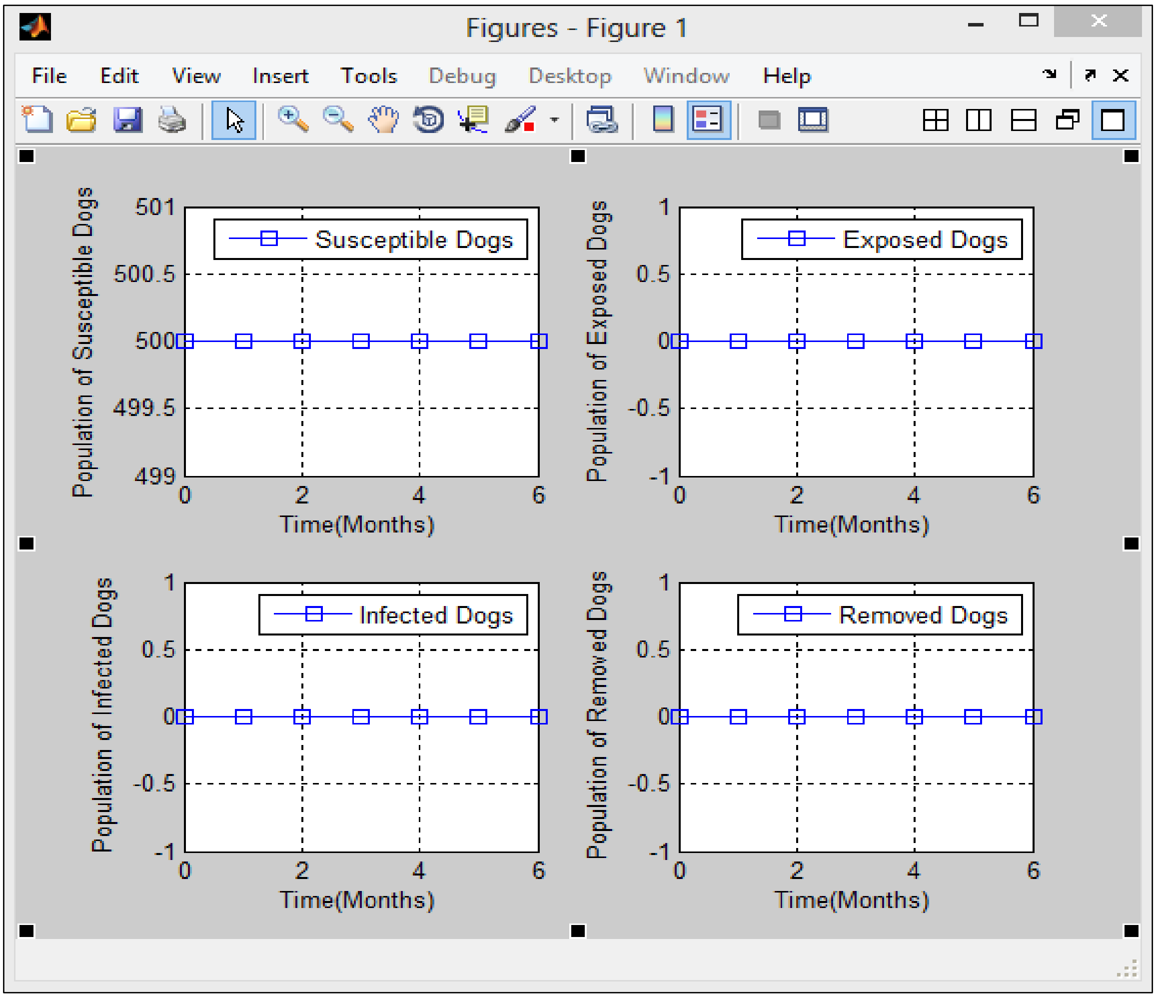

Fig. 5.1: Graphs For $S = 500, E = 0, I = 0$ and $R = 0$

From figure 5.1 the number of susceptible dogs over a period of 6 months remain at 500 where as at this same time frame the number of exposed, infected and removed cases remain at zero.

Case II: For $S \rightarrow 8 \rightarrow 0$, $E \rightarrow 120$, $I \rightarrow 140$ and $R \rightarrow 232$

Assuming 100 dogs have been vaccinated, introducing one(1) infective dog into the 500 susceptible population, the number of exposed increase to about 350 dogs by the first month and reduces to about 120 by the beginning of the sixth month. At this same time frame, the number of infective rise continuously to about 140 by the beginning of the 6^{th} month then afterwards diminishing returns set in. The recovered dogs then increase to about 232 dogs by the beginning of this sixth month. This is shown in figure 5.2.

Case III: For $S \rightarrow 0$, $E \rightarrow 97$, $I \rightarrow 105$ and $R \rightarrow 298$.

Increasing the number of dogs which have been vaccinated to 200, it is observed in figure 5.3 that by 6 weeks, the number of exposed dogs decrease from 350 in our previous simulation to 97 dogs within the six weeks and starts reducing. By the beginning of the sixth month, the number of exposed dogs will be about 100 dogs. The number of infected dog's decreases from 140 dogs as shown in figure 5.2 to about 105 dogs and the number of recovered dogs increase from 232 dogs in figure 5.2 to about 298 dogs.

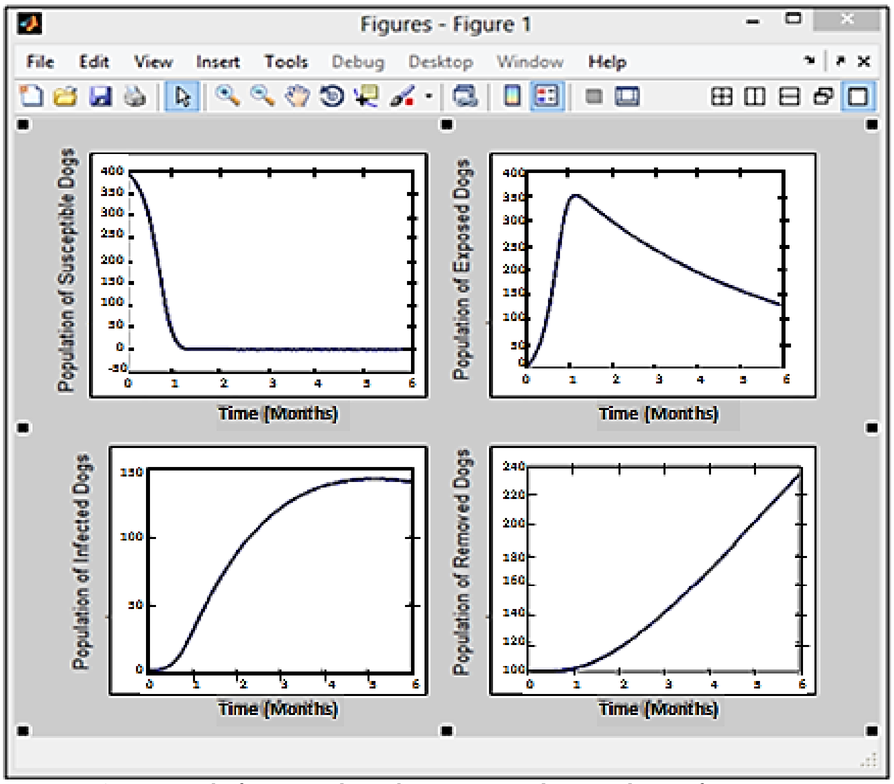

Fig. 5.2: Graphs for S,E,I and R with 100 Vaccinated Dogs and One Infective Dogs

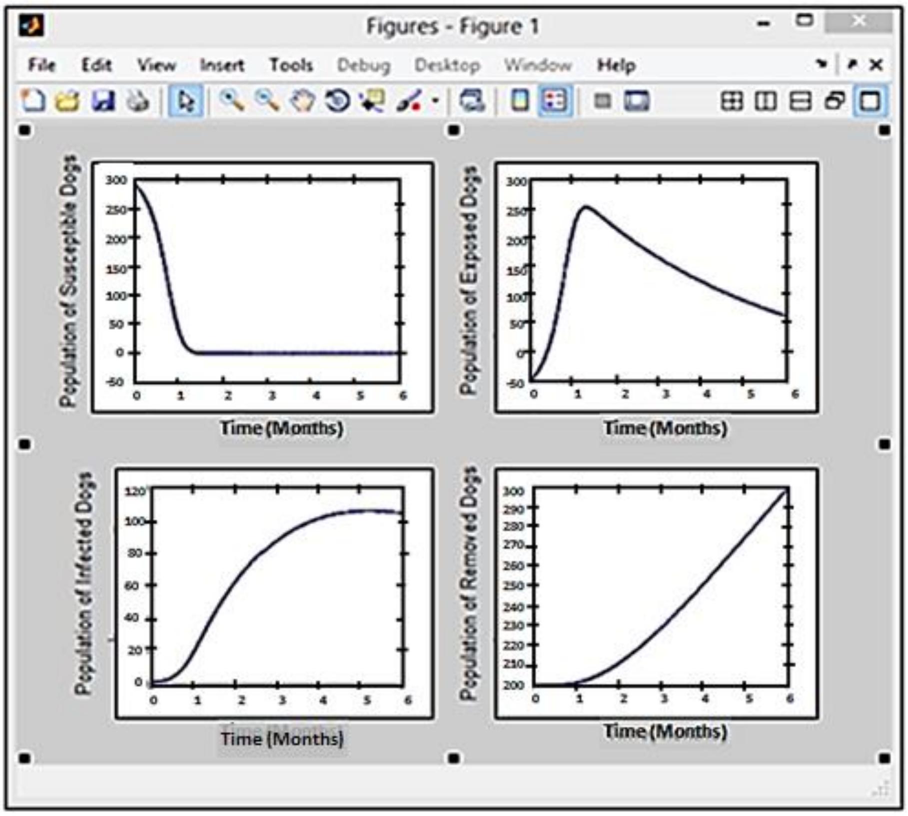

Fig 5.3: Graphs for S, E, I and R with 200 Vaccinated Dogs and One (1) Infected Dog

Case IV: For $S \to 0$, $E = 35$, $I = 35$ and $R = 430$

Increasing the number of vaccinated dogs to 400, with still an infective introduced in the system, it is observed in figure 5.4 that it takes about two (2) months for the number of susceptible to decreased to zero. The number of exposed dogs decreases from 250 dogs in figure 6.4 to 80 dogs within the first two months. By the beginning of the sixth month, the number of exposed dogs will have decreased to about 35 dogs. The recovered dogs increase from 298 dogs in Figure 5.4 to about 430 dogs. This is shown in figure 5.5.

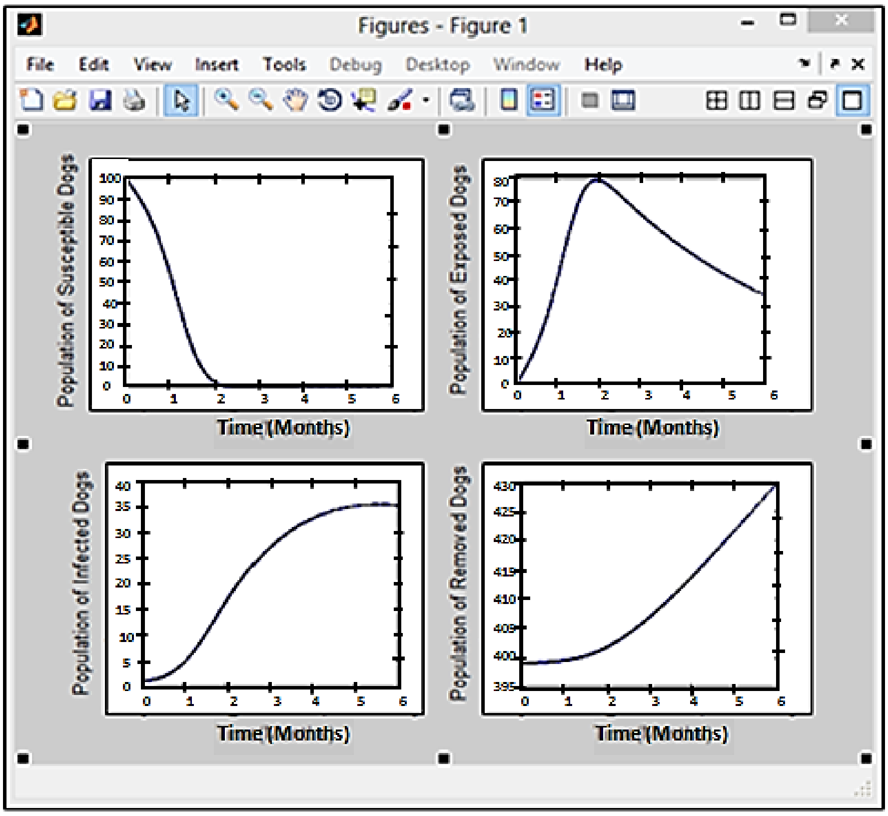

Fig. 5.5: Graphs for S, E, I and R with 400 Vaccinated Dogs and One (1) Infective Dog.

6. Discussion of results

In this paper we use standard SEIR model to predict the spread of rabies in Bolgatanga District of Ghana. We discussed the existence and stability of the disease free and disease endemic equilibria of the model and performed sensitivity analysis by varying the number of infectious which is introduced into the model. Based on the data provided in table 5.1. We also considered Herd immunity as the sole immunization strategy in our study. We estimated the basic reproductive number of the rabies with vaccination to be $R_o = 0.3755$ which not an epidemic is. This decrease (less than one) is as a result of the introduction of vaccination in the model. Herd immunity of 24.63% of the population need to be vaccinated in order to control the spread of the disease. Unvaccinated individuals are indirectly protected by vaccinated individuals as the later will not contract and transmit the disease between infected and susceptible individuals. If the proportion of immune individuals exceeds the Herd Immunity Threshold level by mass vaccination, the disease will die out. Thus 24.63% represents the minimum proportion of the population that must be immunized regularly for the infection to die out in the population.

From our simulation, it was found that when the number of vaccinated dogs is increased, the number of dogs that will attained a level of immunity also increase. If vaccination is done on regular basis then we are sure to have a lot of rabies immuned dogs in our system thereby decreasing the spread of dog rabies amongst dogs and human population.

7. Conclusion and recommendations

In our study, when sensitivity analysis was performed on the rabies transmission with vaccination, we found that increasing the use of rabies vaccine has a significant impact on the rate of spread of rabies transmission by increasing the number of recovered in the model and reducing the use of the rabies vaccines increased the number of recovered in the model.

Also sensitivity anlaysis for rabies vaccination on disease free-equilibria resulted in an asymptotically stable condition of the diseease in Bolgatanga district.

Increasing rabies vaccination coverage in Bolgatanga District and at large in Ghana will decrease the prevalence of rabies even if there is an increase in the number of dog rabies in the locality.

To eradicate rabies in the Bolgatanga district we recommend that government should reintroduce the free anti-rabies vaccination program to undertake a mass vaccination which is to be followed by the consistent re-vaccination of dogs in the Bolgatanga district. Also, governement should commit funds to procure anti-rabies vaccines which is cheaper instead of importing millions of doses of post-exposure rabies vaccines in anticipation of an exposure. Finally, there should be enforcement of laws on dog owners to ensure regular vaccination of their dogs.

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