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Soliton Solutions for (2+1)-Dimensional Breaking Soliton Equation: Three Wave Method

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Abstract

By means of the three-wave method one can solve some nonlinear partial differential equations (NLPDEs) in their bilinear forms. When an NLPDE has no bilinear closed form we can not use this method. We modify the idea of three-wave method to obtain some analytic solutions for the (2+1)-dimensional Breaking soliton equation by obtaining a bilinear closed form for it. By comparison of this method and other analytic methods, like HAM, HTA and EHTA, we can see that the new idea is very easy and straightforward.

Keywords: *Three-wave method; BS equation; Hirota's bilinear form.*

1 Introduction

Many important phenomena and dynamic processes in physics, mechanics, chemistry and biology can be represented by nonlinear partial differential equations. The study of exact solutions of nonlinear evolution equations plays an important role in soliton theory and explicit formulas of nonlinear partial differential equations play an essential role in the nonlinear science. Also, the explicit formulas may provide physical information and help us to understand the mechanism of related physical models. Recently, many kinds of powerful methods have been proposed to find exact solutions of nonlinear partial differential equations, e.g., the tanh-method [1], the homogeneous balance method [2], homotopy analysis method [3, 4, 5, 6, 7, 8], the $F-$expansion method [9], three-wave method [10, 11, 12], extended homoclinic test approach [13, 14, 15], the $(\frac{G'}{G})-$expansion method [16] and the exp-function method [17, 18, 19, 20, 21].

Dai et al. [22], suggested the three-wave method for nonlinear evolution equations. The basic idea of this method applies the Painlevé analysis to make a transformation as

$$u = T(f) \tag{1}$$

for some new and unknown function f.Then we use this transformation in a high dimensional nonlinear equation of the general form

$$F(u, u_t, u_x, u_y, u_z, u_{xx}, u_{yy}, u_{zz}, \cdots) = 0, \tag{2}$$

where $u = u(x, y, z, t)$ and F is a polynomial of u and its derivatives. By substituting (1) in (2), the first one converts into the Hirota's bilinear form, which it will solve by taking a special form for f and assuming that the obtained Hirota's bilinear form has three-wave solutions, we can specify the unknown function f. For more details see [22, 23].

2 Soliton Solutions to the (2+1)-dimensional Breaking Soliton equation

In this paper, we investigate explicit formula of solutions of the following (2+1)-Dimensional Breaking Soliton equation given in [24]

$$u_{xxxy} - 2\, u_y\, u_{xx} - 4\, u_x\, u_{xy} + u_{xt} = 0. \tag{3}$$

To solve eq. (3) authors in [24] used of N-soliton solution. In this paper, we use the idea of three-wave method [22, 23], to solve equation (3). By this idea we obtain some analytic solutions for the problem. The process of our method is very easy and more simple than the method of Ting et al. [24]. To solve eq. (3), we introduce a new dependent variable w by

$$w = -2(\ln f)_x \tag{4}$$

where $f(x, y, t)$ is an unknown real function which will be determined. Substituting eq. (4) into eq. (3), we have

$$2(\ln f)_{xxt} + 2(\ln f)xxxxy + 16(\ln f)_{xx}\,(\ln f)_{xxy} + 8(\ln f)_{xxx}\,(\ln f)_{xy} = 0, \tag{5}$$

which can be integrated once with respect to x to give

$$\begin{gathered} 2(\ln f)_{xt} + 2(\ln f)_{xxxy} + 12(\ln f)_{xx}\,(\ln f)_{xy} \\ +4\partial_x^{-1}((\ln f)_{xx}\,(\ln f)_{xxy} - (\ln f)_{xxx}\,(\ln f)_{xy}) = 0, \end{gathered} \tag{6}$$

Therefore, eq. (6) can be written as

$$(D_xD_t + D_yD_x^3)f \cdot f + 4\,f^2\,\partial_x^{-1}(D_x(\ln f)_{xx} \cdot (\ln f)_{xy}) = 0, \tag{7}$$

where the D-operator is defined by

$$D_x^m D_t^n f(x,t) \cdot g(x,t) =$$

$$(\frac{\partial}{\partial x_1} - \frac{\partial}{\partial x_2})^m(\frac{\partial}{\partial t_1} - \frac{\partial}{\partial t_2})^n[f(x_1,t_1)g(x_2,t_2)]\,|_{x_1=x_2=x,\ t_1=t_2=t}\ \cdot$$

We suppose that

$$\partial_x^{-1}(D_x(\ln f)_{xx} \cdot (\ln f)_{xy}) = 0,$$

then eq. (7) reduces to

$$(D_xD_t + D_yD_x^3)f \cdot f = 0, \tag{8}$$

Now we suppose the solution of eq. (8) as

$$f\,(x,y,t) = \mathrm{e}^{-\xi_1} + \delta_1\,\cos(\xi_2) + \delta_2\,\cosh(\xi_3) + \delta_3\,\mathrm{e}^{\xi_1} \tag{9}$$

where

$$\xi_i = a_ix + b_iy + c_it, \quad i = 1,2,3 \tag{10}$$

and a_i, c_i, δ_i are some constants to be determined later. Substituting eq. (9) into eq. (8), and equating all the coefficients of $\sin(a_2\,x + b_2\,y + c_2\,t)$, $\cos(a_2\,x + b_2\,y + c_2\,t)$, $\sinh(a_3\,x + b_3\,y + c_3\,t)$ and $\cosh(a_3\,x + b_3\,y + c_3\,t)$ to zero, we get the set of algebraic equation for a_i,b_i,c_i, δ_i, $(i = 1,2,3)$

$$-3\,{a_1}^2\,b_1\,a_3 - {a_1}^3\,b_3 - 3\,b_3{a_3}^2\,a_1 - {a_3}^3\,b_1 - c_3\,a_1 - c_1\,a_3 = 0,$$

$$3\,a_1\,b_1\,{a_3}^2 + c_1\,a_1 + a_3\,c_3 + {a_1}^3b_1 + b_3\,{a_3}^3 + 3\,b_3\,a_3\,{a_1}^2 = 0,$$

$$-a_2\,c_2 + b_2\,{a_2}^3 + {a_1}^3\,b_1 + c_1\,a_1 - 3\,b_2\,a_2\,{a_1}^2 - 3\,a_1\,b_1{a_2}^2 = 0,$$

$${a_1}^3\,b_2 + c_2\,a_1 + 3\,{a_1}^2\,b_1\,a_2 + c_1\,a_2 - {a_2}^3\,b_1 - 3\,b_2\,{a_2}^2\,a_1 = 0,$$

$$-{a_2}^3b_3 + c_2\,a_3 + c_3\,a_2 + {a_3}^3b_2 - 3\,b_2{a_2}^2a_3 + 3\,b_3{a_3}^2a_2 = 0,$$

$$a_3\,c_3 + b_3{a_3}^3 - a_2\,c_2 - 3\,b_3\,a_3{a_2}^2 + b_2\,{a_2}^3 - 3\,b_2\,a_2{a_3}^2 = 0,$$

$$16\,{a_1}^3b_1\delta_3 + 4\,c_1\,a_1\delta_3\ - {\delta_1}^2c_2\,a_2 + {\delta_2}^2c_3\,a_3 + 4\,{\delta_1}^2{a_2}^3b_2 + 4\,{\delta_2}^2{a_3}^3b_3 = 0 \tag{11}$$

Solving the system of equations (11) with the aid of Maple, we obtain the following cases:

2.1 CaseI:

$$a_1 = a_3, a_2 = 0, b_1 = -b_3, b_3 = -\frac{c_3}{{a_3}^2},$$
$$c_1 = -c_3, c_2 = -{a_3}^2\, b_2, \delta_1 = 0, \delta_3 = \frac{{\delta_2}^2}{4}, \tag{12}$$

for some arbitrary real constants a_3 ,c_3 ,b_2 and δ_2. Substitute eq. (12) into eq. (4) with eq. (9), we obtain the solution as

$$f\,(x, y, t) = \mathrm{e}^{-\xi_1} + \delta_2 \cosh\,(\xi_2) + \delta_3 \mathrm{e}^{\xi_1}$$

and

$$u\,(x, y, t) = \frac{-2(-a_3 \mathrm{e}^{-\xi_1} + \delta_2 \sinh\,(\xi_2)\, a_3 + \delta_3 a_3 \mathrm{e}^{\xi_1})}{\mathrm{e}^{-\xi_1} + \delta_2 \cosh\,(\xi_2) + \delta_3 \mathrm{e}^{\xi_1}} \tag{13}$$

for

$$\xi_1 = a_3 x - b_3 y - c_3 t \quad , \quad \xi_2 = a_3 x - \frac{c_3}{{a_3}^2} y + c_3 t \quad , \quad \delta_3 = \frac{1}{4}\,{\delta_2}^2$$

If $\delta_3 > 0$, then we obtain the exact breather cross-kink solution

$$u\,(x, y, t) = \frac{-2\, a_3 \left(2\,\sqrt{\delta_3} \sinh\,(\xi_1 - \theta) + \delta_2 \sinh\,(\xi_2)\right)}{2\,\sqrt{\delta_3} \cosh\,(\xi_1 - \theta) + \delta_2 \cosh\,(\xi_2)}$$

for

$$\theta = \frac{1}{2}\,\ln(\delta_3) \quad , \quad \delta_3 = \frac{1}{4}\,{\delta_2}^2$$

If $\delta_3 < 0$, then we obtain the exact breather cross-kink solution

$$u\,(x, y, t) = -2\,\frac{a_3 \left(2\,\sqrt{-\delta_3} \cosh\,(\xi_1 - \theta) + \delta_2 \sinh\,(\xi_2)\right)}{2\,\sqrt{-\delta_3} \sinh\,(\xi_1 - \theta) + \delta_2 \cosh\,(\xi_2)}$$

for

$$\theta = \frac{1}{2}\,\ln(-\delta_3) \quad , \quad \delta_3 = \frac{1}{4}\,{\delta_2}^2$$

2.2 CaseII:

$$a_1 = a_3, b_1 = b_3, c_1 = c_3 = -4\, b_3 {a_3}^2, \delta_1 = 0$$
$$c_2 = -\tfrac{1}{2}\,\frac{b_3\,({a_2}^4 + 6\,{a_3}^2{a_2}^2 - 3\,{a_3}^4)}{a_2 a_3}, b_2 = -\frac{1}{2}\,\frac{b_3\left({a_2}^2 + 3\,{a_3}^2\right)}{a_2 a_3} \tag{14}$$

for some arbitrary real constants $a_3, a_2, b_3, \delta_i,\ i = 1, 2$. Substitute eq. (14) into eq. (4) with eq. (9), we obtain the solution as follows

$$f(x, y, t) = \mathrm{e}^{-\xi_1} + \delta_2 \cosh(\xi_1) + \delta_3 \mathrm{e}^{\xi_1}$$

and

$$u(x, y, t) = \frac{-2(-a_3\mathrm{e}^{-\xi_1} + \delta_2 \sinh(\xi_1)\, a_3 + \delta_3 a_3 \mathrm{e}^{\xi_1})}{\mathrm{e}^{-\xi_1} + \delta_2 \cosh(\xi_1) + \delta_3 \mathrm{e}^{\xi_1}} \tag{15}$$

for

$$\xi_1 = a_3 x + b_3 y - 4\, b_3 {a_3}^2 t$$

If $\delta_3 > 0$ then we obtain the exact breather cross-kink solution

$$u(x, y, t) = \frac{-2\, a_3 \left(2\sqrt{\delta_3} \sinh(\xi_1 - \theta) + \delta_2 \sinh(\xi_1)\right)}{2\sqrt{\delta_3} \cosh(\xi_1 - \theta) + \delta_2 \cosh(\xi_1)}$$

for

$$\theta = \frac{1}{2} \ln(\delta_3)$$

If $\delta_3 < 0$ then we obtain the exact breather cross-kink solution

$$u(x, y, t) = \frac{-2\, a_3 \left(2\sqrt{-\delta_3} \cosh(\xi_1 - \theta) + \delta_2 \sinh(\xi_1)\right)}{-2\sqrt{-\delta_3} \sinh(\xi_1 - \theta) + \delta_2 \cosh(\xi_1)}$$

for

$$\theta = \frac{1}{2} \ln(-\delta_3)$$

2.3 CaseIII:

$$\begin{aligned} a_1 = a_3, a_2 = 0, b_1 = -b_3, b_3 = -\frac{c_3}{{a_3}^2}, \\ c_1 = -c_3, c_2 = -{a_3}^2\, b_2, \delta_1 = 0, \delta_3 = \frac{{\delta_2}^2}{4}, \end{aligned} \tag{16}$$

for some arbitrary real constants a_3 ,c_3 ,b_2 and δ_2. Substitute eq. (16) into eq. (4) with eq. (9), we obtain the solution as

$$f(x, y, t) = \mathrm{e}^{-\xi_1} + \delta_2 \cosh(\xi_2) + \delta_3 \mathrm{e}^{\xi_1}$$

and

$$u(x, y, t) = \frac{3}{2}\, \frac{-a_3\mathrm{e}^{-\xi_1} + \delta_2 \cosh(\xi_2)\, a_3 + \delta_3 a_3 \mathrm{e}^{\xi_1}}{\mathrm{e}^{-\xi_1} + \delta_2 \sinh(\xi_2) + \delta_3 \mathrm{e}^{\xi_1}} \tag{17}$$

for

$$\xi_1 = a_3x - b_3y - c_3t \quad , \quad \xi_2 = a_3x - \frac{c_3}{{a_3}^2}y + c_3t \quad , \quad \delta_3 = \frac{1}{4}\,{\delta_2}^2$$

If $\delta_3 > 0$, then we obtain the exact breather cross-kink solution

$$u\,(x,y,t) = \frac{3}{2}\,\frac{a_3\left(2\,\sqrt{\delta_3}\cosh\left(\xi_1-\theta\right)+\delta_2\cosh\left(\xi_2\right)\right)}{2\,\sqrt{\delta_3}\sinh\left(\xi_1-\theta\right)+\delta_2\sinh\left(\xi_2\right)}$$

for

$$\theta = \frac{1}{2}\,\ln(\delta_3) \quad , \quad \delta_3 = \frac{1}{4}\,{\delta_2}^2$$

If $\delta_3 < 0$, then we obtain the exact breather cross-kink solution

$$u\,(x,y,t) = \frac{3}{2}\,\frac{a_3\left(2\,\sqrt{-\delta_3}\sinh\left(\xi_1-\theta\right)+\delta_2\cosh\left(\xi_2\right)\right)}{2\,\sqrt{-\delta_3}\cosh\left(\xi_1-\theta\right)+\delta_2\sinh\left(\xi_2\right)}$$

for

$$\theta = \frac{1}{2}\,\ln(-\delta_3) \quad , \quad \delta_3 = \frac{1}{4}\,{\delta_2}^2$$

2.4 CaseIV:

$$\begin{gathered} a_1 = a_3, b_1 = b_3, c_1 = c_3 = -4\,b_3{a_3}^2, \delta_1 = 0 \\ c_2 = -\tfrac{1}{2}\,\frac{b_3\left({a_2}^4+6\,{a_3}^2{a_2}^2-3\,{a_3}^4\right)}{a_2a_3}, b_2 = -\frac{1}{2}\,\tfrac{b_3\left({a_2}^2+3\,{a_3}^2\right)}{a_2a_3} \end{gathered} \tag{18}$$

for some arbitrary real constants $a_3, a_2, b_3, \delta_i,\ i = 1, 2$. Substitute eq. (18) into eq. (4) with eq. (9), we obtain the solution as follows

$$f\,(x,y,t) = \mathrm{e}^{-\xi_1} + \delta_2\cosh\left(\xi_1\right) + \delta_3\mathrm{e}^{\xi_1}$$

and

$$u\,(x,y,t) = \frac{3}{2}\,\frac{-a_3\mathrm{e}^{-\xi_1}+\delta_2\cosh\left(\xi_1\right)a_3+\delta_3a_3\mathrm{e}^{\xi_1}}{\mathrm{e}^{-\xi_1}+\delta_2\sinh\left(\xi_1\right)+\delta_3\mathrm{e}^{\xi_1}} \tag{19}$$

for

$$\xi_1 = a_3x + b_3y - 4\,b_3{a_3}^2t$$

If $\delta_3 > 0$ then we obtain the exact breather cross-kink solution

$$u\,(x,y,t) = \frac{3}{2}\,\frac{a_3\left(2\,\sqrt{\delta_3}\cosh\left(\xi_1-\theta\right)+\delta_2\cosh\left(\xi_1\right)\right)}{2\,\sqrt{\delta_3}\sinh\left(\xi_1-\theta\right)+\delta_2\sinh\left(\xi_1\right)}$$

for

$$\theta = \frac{1}{2}\ \ln(\delta_3)$$

If $\delta_3 < 0$ then we obtain the exact breather cross-kink solution

$$u\left(x,y,t\right) = \frac{3}{2}\ \frac{a_3\left(2\sqrt{-\delta_3}\sinh\left(\xi_1-\theta\right)+\delta_2\cosh\left(\xi_1\right)\right)}{-2\sqrt{-\delta_3}\cosh\left(\xi_1-\theta\right)+\delta_2\sinh\left(\xi_1\right)}$$

for

$$\theta = \frac{1}{2}\ \ln(-\delta_3)$$

3 Conclusion

In this paper, using the three-wave solution method we obtained some explicit formulas of solutions for the (3+1)-dimensional Soliton equation. Three-wave solution method with the aid of a symbolic computation software like Maple or Mathematica is an easy and straightforward method which can be apply to other nonlinear partial differential equations. It must be noted that, all obtained solutions have checked in the (3+1)-dimensional Soliton equation. All solutions satisfy in the equations.

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