

Edge cordial and total edge cordial labeling for eight sprocket graph

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Abstract

This paper deals with the Edge cordial labeling of newly introduced eight sprocket graph. This graph is already proven as cordial and graceful in graph labelling. In our study we have further proved that Eight Sprocket graph related families of connected edge-cordial graphs. Also the path union of Eight Sprocket graph, cycle of Eight Sprocket graph and star of Eight Sprocket graph are Edge-cordial.

Keywords: Edge Cordial; Eight- Sprocket Graph; Path Union of Graphs; Cycle of Graphs; Star of a Graph.

1. Introduction

The concept of cordial labeling of graph was introduced by Cahit [3] in 1987 and for numbering in graph was defined by S. W. Golomb [2], [7]. It is found from Gallian [4] that many researchers have studied cordialness of several graphs. The new graph named Eight Sprocket graph was introduced by J. C. Kanani and V. J. Kaneria [5]. The graceful labeling and cordial labeling for the said graph is already proven by the same authors. In this paper, the notions and definitions are followed from Harary [1]. Let us recall some basic definitions, which are used in this paper.

Definition 1.1: A binary edge labeling of a graph G is called an edge cordial labeling if $|\partial f(0) - \partial f(1)| \leq 1$ and $|ef(0) - ef(1)| \leq 1$. A graph G is edge cordial if it admits cordial labeling.

Definition 1.2: Cahit [3] introduced edge-cordial labeling as a binary edge labeling $: E(G) \rightarrow \{0,1\}$, with the induced vertex labeling given by $f(v) = \sum_{uv \in E} f(uv) \pmod{2}$ for each $\partial \in V$ such that $|\partial f(0) - \partial f(1)| \leq 1$ and $|ef(0) - ef(1)| \leq 1$ where $ef(i)$ and $vf(i)$ ($i = 0,1$) denote the number of edges and vertices labeled with 0 and 1 respectively.

Definition 1.3: As an extension of the above, we define a total edge-cordial labeling of a graph G with vertex set V and edge set E as an edge-cordial labeling such that number of vertices and edges labeled with 0 and the number of vertices and edges labeled with 1 differ by at most 1 (i.e.) $|(vf(0) + ef(0)) - (vf(1) + ef(1))| \leq 1$. A graph with a total edge-cordial labeling is called a total edge-cordial graph.

Definition 1.4: Let G be a graph and $G_1, G_2, \dots, G_n$ $n \geq 2$ be n copies of graph G . Let $\partial \in V(G)$ Then the graph obtained by joining vertex v of $G^{(i)}$ with the same vertex of $G^{(i+1)}$ by an edge, $\forall i = 1, 2, 3, \dots, n-1$ is called a path union of n copies of a graph G . Also if the same vertex v of $G^{(n)}$ join by an edge with v of $G^{(1)}$ then such graph is known as cycle graph of n copies of G . These are denoted by $P(n; G)$ and $C(n; G)$ respectively. Obviously $P(n; K_1) = P_n$ and $C(n; K_n) = C_n$.

Definition 1.5: Let G be a graph on n vertices. The graph obtained by replacing each vertex of the star $K_{1,n}$ by a copy of G is called a star of G and is denoted by G^* .

Definition 1.6: Eight Sprocket graph is an union of eight copies of C_{4n} . If $V_{i,j}$ ($\forall i = 1, 2, \dots, 8; \forall j = 1, 2, \dots, 4n$) be vertices of i^{th} copy of C_{4n} then we shall combine $V_{1,4n}$ and $V_{2,1}$, $V_{2,4n}$ and $V_{3,1}$, $V_{3,4n}$ and $V_{4,1}$, $V_{4,4n}$ and $V_{5,1}$, $V_{5,4n}$ and $V_{6,1}$, $V_{6,4n}$ and $V_{7,1}$, $V_{7,4n}$ and $V_{8,1}$ and $V_{8,4n}$ and $V_{1,1}$ by a single vertex. Where $n \in N - 1$. So, Graph becomes sprocket shaped, and here the number of sprockets are eight. Hence, named as Eight Sprocket. It is denoted by S_{Cn} of n size, Where $n \in N - 1$. $|V(S_{Cn})| = 16n - 8$, $|E(S_{Cn})| = 16n$. The coordinates of Eight Sprocket graph, path union of Eight Sprocket graph, cycle of Eight Sprocket graph and star of Eight Sprocket graph are already well define by J. C. Kanani and V. J. Kaneria [6].

2. Main results

Theorem 2.1: An Eight Sprocket graph is edge cordial Graph , where $n \in N - \{1\}$.

Proof: let $G = S_{C_n}$ be any Eight Sprocket graph of size n , where $n \in N - \{1\}$. We mention each edges of S_{C_n} like $E_{i,j}$ ($\forall i = 1, 2, 3, \dots, 8; \forall j = 1, 2, 3, \dots, 4n$). We see the number of vertices in G is $|V(S_{C_n})| = p = 16n - 8$ and $|E(S_{C_n})| = q = 16n$. We define labeling function $f: E(G) \rightarrow \{0, 1\}$ as follows.

$$f(e_{1,j}) = \begin{cases} 0 & ; \text{if } j = 3, 4, 7, 8, 11, 12, \dots, 4n - 1, 4n. \\ 1 & ; \text{if } j = 1, 2, 5, 6, 9, 10, \dots, 4n - 3, 4n - 2. \end{cases}$$

$$f(e_{2,j}) = \begin{cases} 0 & : \text{if } j = 1, 2, 5, 6, 9, 10, \dots, 4n - 3, 4n - 2 \\ 1 & ; \text{if } j = 3, 4, 7, 8, 11, 12, \dots, 4n - 1, 4n. \end{cases}$$

$$f(e_{3,j}) = \begin{cases} 0 & ; \text{if } j = 3, 4, 7, 8, 11, 12, \dots, 4n - 1, 4n. \\ 1 & ; \text{if } j = 1, 2, 5, 6, 9, 10, \dots, 4n - 3, 4n - 2. \end{cases}$$

$$f(e_{4,j}) = \begin{cases} 0 & ; \text{if } j = 1, 2, 5, 6, 9, 10, \dots, 4n - 3, 4n - 2 \\ 1 & ; \text{if } j = 3, 4, 7, 8, 11, 12, \dots, 4n - 1, 4n. \end{cases}$$

$$f(e_{5,j}) = \begin{cases} 0 & ; \text{if } j = 1, 2, 5, 6, 9, 10, \dots, 4n - 3, 4n - 2 \\ 1 & ; \text{if } j = 3, 4, 7, 8, 11, 12, \dots, 4n - 1, 4n. \end{cases}$$

$$f(e_{6,j}) = \begin{cases} 0 & ; \text{if } j = 1, 2, 5, 6, 9, 10, \dots, 4n - 3, 4n - 2 \\ 1 & ; \text{if } j = 3, 4, 7, 8, 11, 12, \dots, 4n - 1, 4n. \end{cases}$$

$$f(e_{7,j}) = \begin{cases} 0 & ; \text{if } j = 3, 4, 7, 8, 11, 12, \dots, 4n - 1, 4n. \\ 1 & ; \text{if } j = 1, 2, 5, 6, 9, 10, \dots, 4n - 3, 4n - 2. \end{cases}$$

$$f(e_{8,j}) = \begin{cases} 0 & ; \text{if } j = 3, 4, 7, 8, 11, 12, \dots, 4n - 1, 4n. \\ 1 & ; \text{if } j = 1, 2, 5, 6, 9, 10, \dots, 4n - 3, 4n - 2. \end{cases}$$

The above pattern gives rise of an edge cordial labeling of the graph G , Hence, G is Edge cordial graph.

Corollary: 2.2 Total graph G holds the definition of Total edge cordial labeling by satisfying the condition that $|(vf(0) + ef(0)) - (vf(1) + ef(1))| \leq 1$. Hence G is total Edge cordial Graph.

Illustration 2.3: Eight Sprocket graph S_{C_n} is shown consisting $n = 8$ sprockets with Edge cordial Labeling with $p = 120$ and $q = 128$ where $vf(0) = vf(1) = 60$ and $ef(0) = ef(1) = 64$

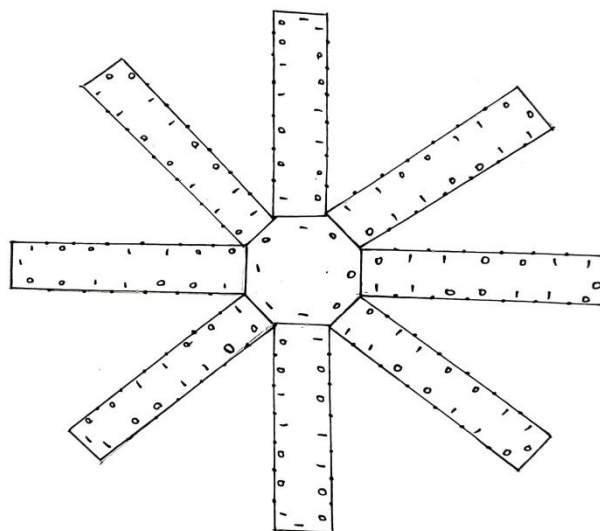


Fig. 1: Edge Cordial Labeling of Eight Sprocket Graph with $P = 120$ and $Q = 128$.

Theorem 2.4: Path union of finite copies of the Eight Sprocket graph S_{C_n} is an edge cordial graph, where $n \in N - \{1\}$.

Proof: Let $G = P(r, S_{C_n})$ be a path union of r copies of the eight sprocket graph S_{C_n} , where $n \in N - \{1\}$.

Let f be the edge cordial labeling of S_{C_n} , as mentioned in theorem 2.1. In graph G , we observe that the number of vertices

$|v(G)| = P = r(16(n) - 8)$ and the number of edges $|e(G)| = q = (r - 1) + r 16(n)$.

Let $v_{k,i,j}$ ($\forall i = 1, 2, \dots, 8; \forall j = 1, 2, 3, \dots, 4n$) be the vertices of k^{th} copy of S_{C_n} ($\forall k = 1, 2, 3, \dots, r$) where the number of vertices $p = 16(n) - 8$ and number of edges $q = 16n$. Join vertices $v_{k,1,2n+1}$ with $v_{k+1,1,2n+1}$ for $k = 1, 2, 3, \dots, r - 1$ by an edge to form the path union of r copies of eight Sprocket graph.

The labeling of the function $g: E(G) \rightarrow \{0, 1\}$ as follows:

$$g(e_{1,i,j}) = f(e_{1,i,j})$$

$$g(e_{2,i,j}) = \begin{cases} g(e_{1,i,j}) + 1; & \text{if } j = 1, 2, 5, 6, 9, 10, \dots, 4n - 3, 4n - 2 \\ g(e_{1,i,j}) - 1; & \text{if } j = 3, 4, 7, 8, \dots, 4n - 1, 4n \end{cases}$$

$$g(e_{3,i,j}) = f(e_{2,i,j})$$

$$g(e_{k,i,j}) = g(e_{k-3,i,j}) \text{ if } k = 4, 5, 6, 7, \dots, 3n + 1, 3n + 2, 3n + 3.$$

Above labeling pattern gives rise of Edge cordial labeling to graph G. Hence, the path union of finite copies of the Eight Sprocket graph is edge cordial graph.

Corollary: 2.5 The path union of finite copies of the graph G holds the definition of total edge cordial labeling by satisfying the condition that $|(vf(0) + ef(0)) - (vf(1) + ef(1))| \leq 1$. Hence the path union of k^{th} copy of graph S_{C_n} is total Edge cordial for $k = 1, 2, 3, \dots, r - 1$

Illustration 2.6: Path union of 3 copies of S_{C_3} and its edge cordial labeling is shown in figure 2. With $p = 3(88)$ and $q = 3(96) + 2$ where $vf(0) = vf(1) = 3(44)$ and $ef(0) = ef(1) = 3(48) + 1$

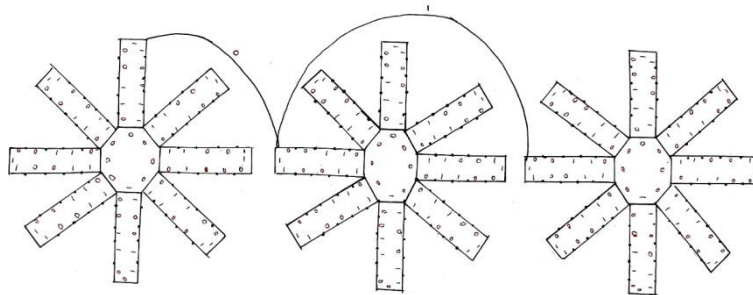


Fig. 2: A Path Union of 3 Copies of S_{C_3} and Its Edge Cordial Labeling.

Theorem 2.7: Cycle of r copies of Eight Sprocket graph $C(r, S_{C_n})$ is an edge cordial graph, where $n \in \mathbb{N} - \{1\}$ and $r \equiv 0, 3 \pmod{4}$.

Proof: Let $G = C(r, S_{C_n})$ be a cycle of Eight Sprocket graph S_{C_n} where $n \in \mathbb{N} - \{1\}$. Let f be the edge cordial labeling of S_{C_n} as proved in theorem 2.1. In graph G , we observe that the number of vertices $|V(G)| = p = r(16(n) - 8)$ and the number of edges $|E(G)| = q = r(16(n) + 1)$. Let $v_{k,i,j}$ ($\forall i = 1, 2, \dots, 8; \forall j = 1, 2, 3, \dots, 4n$) be the vertices of k^{th} copy of S_{C_n} ($\forall k = 1, 2, 3, \dots, r$), where the number of vertices $p = 16(n) - 8$ and number of edges $q = 16n$. Join the vertices $v_{k,1,2n+1}$ with $v_{k+1,1,2n+1}$ for $k = 1, 2, 3, \dots, r - 1$ and $v_{r,1,2n+1}$ with $v_{1,1,2n+1}$ by an edge to form $C(r, S_{C_n})$

We define edge labeling function $g: E(G) \rightarrow \{0, 1\}$ as follows.

$$g(e_{1,i,j}) = f(e_{1,i,j})$$

$$g(e_{2,i,j}) = \begin{cases} g(e_{1,i,j}) + 1; & \text{if } j = 1, 2, 5, 6, 9, 10, \dots, 4n - 3, 4n - 2 \\ g(e_{1,i,j}) - 1; & \text{if } j = 3, 4, 7, 8, \dots, 4n - 1, 4n \end{cases}$$

$$g(e_{3,i,j}) = f(e_{2,i,j})$$

$$g(e_{4,i,j}) = f(e_{1,i,j})$$

$$g(e_{k,i,j}) = g(v_{k-3,i,j}); \text{ if } k = 5, 6, 7, \dots, 4n + 1, 4n + 2, 4n + 3$$

$$(e_{k,i,j}) = g(v_{k-4,i,j}); \text{ if } k = 8, 12, 16, \dots, 4n + 4$$

Above labeling Pattern proves that the cycle of r copies of eight Sprocket graph is edge cordial.

Corollary: 2.8 The cycle of r copies of the graph G holds good with the definition of Total edge cordial labeling by satisfying the condition that $|(vf(0) + ef(0)) - (vf(1) + ef(1))| \leq 1$. Hence Cycle of r copies of Eight Sprocket graph $C(r, S_{C_n})$ is total edge cordial graph, where $n \in \mathbb{N} - \{1\}$ and $r \equiv 0, 3 \pmod{4}$. for $k = 1, 2, 3, \dots, r - 1$

Illustration 2.9: Cycle of 4 copies for eight Sprocket graph S_{C_4} and its edge cordial labeling is shown in figure 2. With $p = 4(56)$ and $q = 4(64) + 3$ where $vf(0) = vf(1) = 4(28)$ and $ef(0) = ef(1) = 4(32) + 2 \mp r$

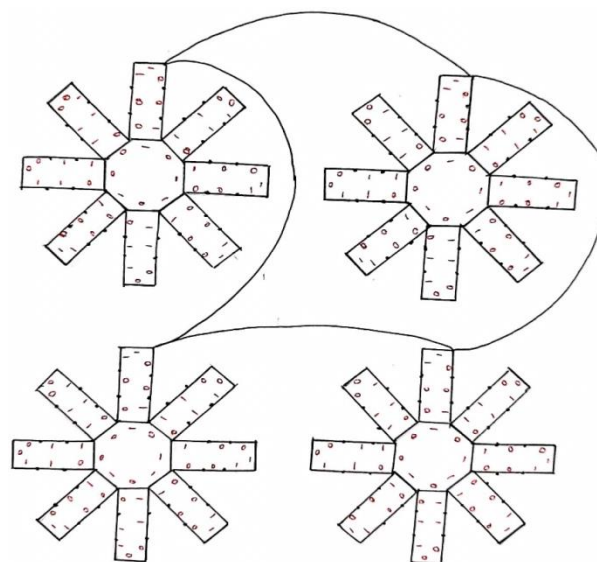


Fig. 3: Cycle of 4 Copies of Eight Sprocket Graph S_{C4} Is Edge Cordial.

Theorem 2.10: Star of Eight Sprocket graph $(S_{Cn})^*$ is an edge cordial graph, where $n \in N - \{1\}$.

Proof: Let $G = (S_{Cn})^*$ be a star graph of eight Sprocket graph S_{Cn} , where $n \in N - \{1\}$. Let f be the edge cordial labeling of S_{Cn} as proved in theorem 2.1

In graph G , we observe that the number of vertices $|V(G)| = P = p(p+1)$ and the number of edges $|E(G)| = Q = q(p+1) + p$ where $p = 16(n) - 8$ and $q = 16(n)$.

Let $v_{k,i,j}$ ($\forall i = 1, 2, \dots, 8; \forall j = 1, 2, 3, \dots, 4n$) be the vertices of k^{th} copy of S_{Cn} ($\forall k = 1, 2, 3, \dots, p$). Where the number of vertices $p = 16(n) - 8$ and number of edges $q = 16(n)$. The central copy of $(S_{Cn})^*$ is $(S_{Cn})^0$ and other copies of $(S_{Cn})^*$ is $(S_{Cn})^{(k)}$ ($\forall k = 1, 2, 3, \dots, p$)

We define edge labeling function $g: E(G) \rightarrow \{0, 1\}$ as follows.

$$g(e_{1,i,j}) = f(e_{1,i,j})$$

$$g(e_{1,i,j}) = \begin{cases} g(e_{0,i,j}) + 1; & \text{if } j = 1, 2, 5, 6, 9, 10, \dots, 4n - 3, 4n - 2 \\ g(e_{0,i,j}) - 1; & \text{if } j = 3, 4, 7, 8, \dots, 4n - 1, 4n \end{cases}$$

$$g(e_{2,i,j}) = f(e_{0,i,j})$$

$$g(e_{3,i,j}) = f(e_{1,i,j})$$

$$g(e_{k,i,j}) = g(v_{k-2,i,j}); \text{ if } k = 2, 3, 4, 5, \dots, 4n - 2, 4n - 1, 4n$$

We observe that difference of vertices and edges for the center copy $(S_{Cn})^0$ of G and its other copies $(S_{Cn})^{(k)}$, ($1 \leq k \leq p$) is G . Using this sequence we can construct required edge label by joining corresponding vertices of $(S_{Cn})^0$ with its other copies $(S_{Cn})^{(k)}$, ($1 \leq k \leq p$) in G . Thus it holds well with edge cordial labeling and total edge cordial labeling.

3. Concluding

The present work contributes with some new results. We discussed Edge cordialness and Total Edge cordial labeling of Eight Sprocket graphs, path union of Eight Sprocket graph, and cycle of Eight Sprocket graph and star of Eight Sprocket graph. The labeling pattern is demonstrated by means of illustrations which provide better understanding to derived results.

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