



On complementary quadratic fractional programming problem

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Abstract

In this paper, a class of optimization problems has been considered where quadratic fractional programming problem has an additional characteristic, i.e Complementary quadratic fractional programming problem (CQFPP) and consequently a convergent algorithm has been developed in the following discussion. Numerical examples have been provided in support of the theory. By using Matlab 2011 version 7.12.0.635 (R2011a).

Keywords: CQFPP, Solving CQFPP by new technique modified simplex method.

1. Introduction

In (1969-1971) Ibaraki ([5], [6]) studied complementary programming, and defined a new type of optimization problems, known as complementary programming problems. In (1982) Gupta and Sharma had developed an algorithm for solving a quadratic complementary programming problem with indefinite [4]. In (2009) Arora and Narang are studies a bilevel fractional programming problem with independent followers [2]. In (2011) Judice studies algorithms for linear programming with linear complementarity constraints [8]. In (2010) Jahan and Islam are studied a complementary slackness theorem for linear fractional programming problem [7]. In (1989) Sulaiman [11] studied to quadratic complementary programming algorithms and their computer applications. In (2009) Fang, Gao, Sheu and Xing are studies to global optimization for class of fractional programming problems and quadratic fractional programming problem [3]. In (2011) Arora and Arora are studies to solving linear quadratic bilevel programming problem using Kuhn-Tucker conditions [1].

To extend this work, we have been defined CQFPP and investigated technique to generate the best compromising optimal solution.

2. Complementary QFPP

Ibaraki [6] defined complementary programming problem and Gupta and Sharma [4] defined complementary quadratic programming problem (CQPP), complementary quadratic fractional programming problem (CQFPP) can be defined as follows:

$$\left. \begin{array}{l} \text{Max. } Z = \frac{(c^t x + s^t u + s^t v + \gamma)(e^t x + o^t u + o^t v + \delta)}{(d^t x + w^t u + w^t v + \beta)(f^t x + G^t u + g^t v + \epsilon)} \\ \text{Subject to:} \\ \quad Ax + Bu + Cv = b \\ \quad uv = 0 \\ \quad u, v, x \geq 0 \end{array} \right\} \quad (1)$$

Where x, u, v are n, m -dimensional and m -dimensional vectors of variables respectively; c, e, f and d are n -dimensional and S, O, W, G, s, o, w, g are m -dimensional vectors of constants respectively; b is p -dimensional vector of constants; A, B, C is $p \times n, p \times m$ and $p \times m$ matrix of constraints respectively, $\gamma, \delta, \beta, \varepsilon$ are constants

3. Modified simplex method development

Simplex method is developed by Dantzig in (1947). The simplex method provides a systematic algorithm which consists of moving from one basic feasible solution (one vertex) to another in prescribed manner such that the value of the objective function is improved. This procedure of jumping from vertex to vertex is repeated. If the objective function is improved at each jump, then no basis can ever be repeated and there is no need to go back to vertex already covered. Since the number of vertices is finite, the process must lead to the optimal vertex in a finite number of steps. The simplex algorithm is an iterative (step by step) procedure for solving linear programming problems. It consists of:

- i) Having a trial basic feasible solution to constraint equations.
- ii) Testing whether is an optimal solution.
- iii) Improving the first trial solution by a set of rules, and repeating the process till an optimal solution is obtained.

For more details [9]. Modified simplex method to solve linear fractional programming problem and to solve produced to linear programming problem [10]. Using new technique to modified simplex method to solve the numerical example to apply simplex process [10]. First we find $\Delta_{j1}, \Delta_{j2}, \Delta_{j3}$ and Δ_{j4} from the coefficients of numerator and denominator of objective function respectively, by using the following formula:

$$\begin{aligned} \Delta_{ji} &= C_{ji} - C_{Bi}x_{ji}, i = 1, 2, 3, 4, j = 1, 2, \dots, m + n, \\ z_1 &= C_{B1}V_B + \gamma, z_2 = C_{B2}V_B + \delta, z_3 = C_{B3}V_B + \beta, z_4 = C_{B4}V_B + \varepsilon, \\ \gamma, \delta, \beta, \varepsilon &\text{ are constants, } Z_1 = z_1z_2, Z_2 = z_3z_4, Z = \frac{z_1}{z_2} \\ \mu_{j1} &= \mu_{j2} = \min \left[\frac{V_B}{x_j}, x_j > 0 \right] \text{ for non - basic variables} \\ \Delta_{ja} &= z_1\Delta_{j2} + z_2\Delta_{j1} + \mu_{j1}\Delta_{j1}\Delta_{j2}, \Delta_{jb} = z_3\Delta_{j4} + z_4\Delta_{j3} + \mu_{j2}\Delta_{j3}\Delta_{j4} \end{aligned}$$

In this approach we define the formula to find Δ_j from Z_1, Z_2, Δ_{ja} and Δ_{jb} as follows: $\Delta_j = Z_2\Delta_{ja} - Z_1\Delta_{jb}$
Here C_{ji} are the coefficients of the basic and non-basic variables in the objective function and C_{Bi} are the coefficients of the basic variables in the objective function, $j = 1, 2, \dots, m + n, i = 1, 2, 3, 4$

For testing optimality solution must be all $\Delta_j \leq 0$ but here all Δ_j not lesser than zero, and then the solution is not optimal. Repeat the same approach to find next feasible solution.

4. Algorithm for CQFPP

The following algorithm is to obtain the optimal solution for the CQFPP by new technique to modified simplex method which can be summarized as follows:

- Step1:** Write the standard form of the problem without complementary condition $uv = 0$, by introducing slack and artificial variables to constraints, and write starting new technique to modified simplex table.
- Step2:** Calculate the Δ_j by the following formula $\Delta_j = Z_2\Delta_{ja} - Z_1\Delta_{jb}$, then write it in the starting new technique to modified simplex table.
- Step3:** Find the solution by using new technique to modified simplex process without complementary condition $uv = 0$, examine whether the complementary condition holds or not.
- Step4:** If the complementary condition $uv = 0$, then go to step7 otherwise go to step5.
- Step5:** If the complementary condition $uv = 0$, is not satisfied use branch and bounded algorithm proposed by Ibaraki [6] modified by Gupta and Sharma [4] for quadratic objective function, and ensure that $uv = 0$, go to step6.
- Step6:** Check the solution for feasibility in step3, if it is feasible then go to step7, otherwise use dual new technique to modified simplex method to remove infeasibility.
- Step7:** Check solution for optimality if all $\Delta_j \leq 0$, then the solution is optimal, otherwise go to step3.

5. Numerical examples and results

In this section we have presented only two examples among several examples that we have solved, to show the accuracy of the new technique and which new technique to modified simplex method is better and more convenience.

Example 5.1: We consider the following CQFP as

$$\text{Max. } Z = \frac{(4x_1+2x_2)(2x_1+x_2)}{(x_1+x_2+2)(2x_1+x_2+4)}$$

Subject to:

$$\begin{aligned} x_1 + x_2 &\leq 6 \\ 8x_1 + 4x_2 &\leq 32 \\ x_1 &\leq 3 \\ x_2 &\leq 5 \\ x_1x_2 &= 0 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Solution 5.1: Solving the example 5.1 by new technique to modified simplex method and applied an algorithm in the section 4, after 2 steps we obtained the initial table as follows in table 5.1.1, after two iterations, we obtained the result in the following table 5.1.2

Table 5.1.1: Initial table for example 5.1 by new technique to modified simplex method

						C_{j1}	4	2	0	0	0	0	
						C_{j2}	2	1	0	0	0	0	
						C_{j3}	1	1	0	0	0	0	
						C_{j4}	2	1	0	0	0	0	
B.V.	C_{B1}	C_{B2}	C_{B3}	C_{B4}	V_B	x_1	x_2	x_3	x_4	x_5	x_6	Min ratio	
x_3	0	0	0	0	6	1	1	1	0	0	0	$6/1 = 6$	
x_4	0	0	0	0	32	8	4	0	1	0	0	$32/8 = 4$	
x_5	0	0	0	0	3	1	0	0	0	1	0	$3/1 = 3$	
x_6	0	0	0	0	5	0	1	0	0	0	1	—	
$z_1 = 0$						Δ_{j1}	4	2	0	0	0	0	
$z_2 = 0$						Δ_{j2}	2	1	0	0	0	0	
$Z_1 = z_1z_2 = 0$						μ_{j1}	3	5	0	0	0	0	
$z_3 = 2$						Δ_{j3}	1	1	0	0	0	0	
$z_4 = 4$						Δ_{j4}	2	1	0	0	0	0	
$Z_2 = z_3z_4 = 8$						μ_{j2}	3	5	0	0	0	0	
$Z_1 = 0$						Δ_{ja}	24	10	0	0	0	0	
$Z_2 = 8$						Δ_{jb}	14	11	0	0	0	0	
$Z = \frac{z_1}{z_2} = 0$						Δ_j	192	80	0	0	0	0	

Table 5.1.2: Final table for example 5.1 by new technique to modified simplex method

					C_{j1}	4	2	0	0	0	0	
					C_{j2}	2	1	0	0	0	0	
					C_{j3}	1	1	0	0	0	0	
					C_{j4}	2	1	0	0	0	0	
B.V.	C_{B1}	C_{B2}	C_{B3}	C_{B4}	V_B	x_1	x_2	x_3	x_4	x_5	x_6	Min ratio
x_3	0	0	0	0	1	0	0	1	$-1/4$	1	0	
x_2	2	1	1	1	2	0	1	0	$1/4$	-2	0	
x_1	4	2	1	2	3	1	0	0	0	1	0	
x_6	0	0	0	0	3	0	0	0	$-1/4$	2	1	
$z_1 = 16$					Δ_{j1}	0	0	0	$-2/4$	0	0	
$z_2 = 8$					Δ_{j2}	0	0	0	$-1/4$	0	0	
$Z_1 = z_1 z_2 = 128$					μ_{j1}	0	0	0	8	1	0	
$z_3 = 7$					Δ_{j3}	0	0	0	$-1/4$	1	0	
$z_4 = 12$					Δ_{j4}	0	0	0	$-1/4$	0	0	
$Z_2 = z_3 z_4 = 84$					μ_{j2}	0	0	0	8	1	0	
$Z_1 = 128$					Δ_{ja}	0	0	0	-7	0	0	
$Z_2 = 84$					Δ_{jb}	0	0	0	$-17/4$	12	0	
$Z = \frac{z_1}{z_2} = \frac{128}{84}$					Δ_j	0	0	0	-44	-1536	0	

After solving it by new technique to modified simplex method, without complementary condition ($x_1 x_2 = 0$) we get $Max. Z = \frac{128}{84}$ and $x_1 = 3, x_2 = 2$

Now, we going to satisfied the complementary condition ($x_1 x_2 = 0$), because $x_1 x_2 \neq 0$. The optimal solution is $Max. Z = \frac{72}{50}$ and $x_1 = 3, x_2 = 0$

Example 5.2: We consider the following CQFPP as

$$Max. Z = \frac{(6x_1 + 10x_2 + 9x_3)(x_1 + x_2 + x_3)}{(2x_1 + 2x_2 + 2x_3 + 4)(x_1 + x_2 + x_3 + 5)}$$

Subject to:

$$\begin{aligned} x_1 + 3x_2 + 2x_3 &\leq 9 \\ 3x_1 + 2x_2 + x_3 &\leq 8 \\ 2x_1 + x_2 + 3x_3 &\leq 7 \\ x_2 x_3 &= 0 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

Solution 5.2: Solving the example 5.2 by new technique to modified simplex method and applied an algorithm in the section 4, after 2 steps we obtained the initial table as follows in table 5.2.1, after three iterations, we obtained the result in the following table 5.2.2

Table 5.2.1: Initial table for example 5.2 by new technique to modified simplex method

					C_{j1}	6	10	9	0	0	0	
					C_{j2}	1	1	1	0	0	0	
					C_{j3}	2	2	2	0	0	0	
					C_{j4}	1	1	1	0	0	0	
B.V.	C_{B1}	C_{B2}	C_{B3}	C_{B4}	V_B	x_1	x_2	x_3	x_4	x_5	x_6	Min ratio
x_4	0	0	0	0	9	1	3	2	1	0	0	$9/3 = 3$
x_5	0	0	0	0	8	3	2	1	0	1	0	$8/2 = 4$
x_6	0	0	0	0	7	2	1	3	0	0	1	$7/1 = 7$
$z_1 = 0$					Δ_{j1}	6	10	9	0	0	0	
$z_2 = 0$					Δ_{j2}	1	1	1	0	0	0	
$Z_1 = z_1 z_2 = 0$					μ_{j1}	$8/3$	3	$7/3$	0	0	0	
$z_3 = 4$					Δ_{j3}	2	2	2	0	0	0	
$z_4 = 5$					Δ_{j4}	1	1	1	0	0	0	
$Z_2 = z_3 z_4 = 20$					μ_{j2}	$8/3$	3	$7/3$	0	0	0	
$Z_1 = 0$					Δ_{ja}	16	30	21	0	0	0	
$Z_2 = 20$					Δ_{jb}	$58/3$	20	$56/3$	0	0	0	
$Z = \frac{z_1}{z_2} = 0$					Δ_j	320	600	420	0	0	0	

Table 5.2.2: Final table for example 5.2 by new technique to modified simplex method

					C_{j1}	6	10	9	0	0	0	
					C_{j2}	1	1	1	0	0	0	
					C_{j3}	2	2	2	0	0	0	
					C_{j4}	1	1	1	0	0	0	
B.V.	C_{B1}	C_{B2}	C_{B3}	C_{B4}	V_B	x_1	x_2	x_3	x_4	x_5	x_6	Min ratio
x_2	10	1	2	1	2	0	1	0	$7/18$	$1/18$	$-5/18$	
x_1	6	1	2	1	1	1	0	0	$-5/18$	$7/18$	$1/18$	
x_3	9	1	2	1	1	0	0	1	$1/18$	$-5/18$	$7/18$	
$z_1 = 35$					Δ_{j1}	0	0	0	$-49/18$	$-7/18$	$-19/18$	
$z_2 = 4$					Δ_{j2}	0	0	0	$-3/18$	$-3/18$	$-3/18$	
$Z_1 = z_1 z_2 = 140$					μ_{j1}	0	0	0	$36/7$	$18/7$	$18/7$	
$z_3 = 12$					Δ_{j3}	0	0	0	$-6/18$	$-6/18$	$-6/18$	
$z_4 = 9$					Δ_{j4}	0	0	0	$-3/18$	$-3/18$	$-3/18$	
$Z_2 = z_3 z_4 = 108$					μ_{j2}	0	0	0	$36/7$	$18/7$	$18/7$	
$Z_1 = 140$					Δ_{ja}	0	0	0	$-259/18$	$-130/18$	$-1210/126$	
$Z_2 = 108$					Δ_{jb}	0	0	0	$-594/126$	$-612/126$	$-612/126$	
$Z = \frac{z_1}{z_2} = \frac{140}{108}$					Δ_j	0	0	0	$-16092/18$	$-1800/18$	$-45000/126$	

After solving it by new technique to modified simplex method, without complementary condition ($x_2 x_3 = 0$) we get $Max. Z = \frac{140}{108}$ and $x_1 = 1$, $x_2 = 2$, $x_3 = 1$

Now, we going to satisfied the complementary condition ($x_2 x_3 = 0$), because $x_2 x_3 \neq 0$. The optimal solution is $Max. Z = \frac{78}{80}$ and $x_1 = 1$, $x_2 = 2$, $x_3 = 0$

6. Discussion

In this paper find CQFPP by the new technique to modified simplex method. The optimal solution must be at one of the complementary points of the polygon of the feasible region, sometimes it may be need to use technique for finding best complementary point solution for the problem.

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