Exact solutions of the ZK-MEW equation and the Davey-Stewartson equation

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Abstract

In this paper we introduce a new version of the trial equation method for solving non-integrable partial differential equations in mathematical physics. Some exact solutions including soliton solutions, rational and elliptic function solutions to the generalized (2+1)-dimensional ZK-MEW equation and the generalized Davey-Stewartson equation with the complex coefficients are obtained by this method.

Keywords: Extended trial equation method, generalized (2+1)-dimensional ZK-MEW equation, Davey-Stewartson equation, soliton solution, elliptic solutions

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1 Introduction

The investigation of exact solutions of nonlinear evolution equations (NLEEs) plays a crucial role in the analysis of some physical phenomena. It is difficult to obtain the exact solution for these problems. In recent decades, there has been great development in exact solution for nonlinear partial differential equations (PDEs). Many powerful methods, such as the Backlund transformation, the inverse scattering method [1], bilinear transformation, the tanh-sech method [2], the extended tanh method, the pseudo-spectral method [3], the trial function and the sine-cosine method [4], the Hirota method [5], the tanh-coth method [6-7], the exponential function method [8], the (G'/G)-expansion method [9-13], the homogeneous balance method [14], the F-expansion method [15-20], the trial equation method [21-31] have been used to investigate nonlinear partial differential equations problems. The types of solutions of NLEEs, that are integrated using various mathematical techniques, are very important and appear in various areas of physics, applied mathematics and engineering.

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The spatially one-dimensional KdV equation

$$u_t + \alpha u u_x + u_{xxx} = 0,$$

governs the one-dimensional propagation of small-amplitude weakly dispersive waves, and plays a major role in the soliton concept. The term soliton was coined by Zabusky and Kruskal [32] who found particle-like waves which retained their shapes and velocities after collisions. The balance between the non-linear convection term uu_x and the dispersion effect term u_{xxx} in the KdV equation gives rise to solitons. Solitons are defined as non-linear waves characterized as follows [33]:

- localized waves that propagate without change of its shape, velocity, etc.;
- localized waves that are stable against mutual collisions and retain their identities to indicate that soliton has the property of a particle.

In this paper, extended trial equation methods is used to obtain a generalized soliton solution with some free parameters of the generalized (2+1)-dimensional Zakharov-Kuznetsov-Modified Equal-Width (ZK-MEW) equation [34,35]

$$u_t + \alpha(u^n)_x + (\beta u_{xt} + \gamma u_{yy})_x = 0. \tag{1}$$

and generalized Davey-Stewartson equation (DSE) that arises in the study of fluid dynamics [36,37]

$$iq_t + a(q_{xx} + q_{yy}) + b|q|^{2n} q = \alpha q r \tag{2}$$

$$r_{xx} + r_{yy} + \beta(|q|^{2n})_{xx} = 0. \tag{3}$$

Exact solutions of the ZK-MEW equation were obtained both by using the tanh and sine-cosine methods by Wazwaz [34] and the modified simple equation method by Zayed and Arnous [35]. The Cauchy problem of the generalized Davey-Stewartson systems and the global solvability and existence of self-similar solutions to a generalized Davey-Stewartson system were studied in some sense by Zhao [38]. Ebadi and Biswas studied by applying the (G'/G)-method carry out the integration of Davey-Stewartson equation [36] while Bekir and Cevikel have solved them using the sine-cosine and the exp-function methods [37]. Subsequently, using the ansatz method this equation is integrated in $(1 + 2)$-dimensions with power law nonlinearity. Here, we use the extended trial equation method to solve the soliton solutions of generalized $(1+2)$-dimensional ZK-MEW equation and generalized Davey-Stewartson equation with the complex coefficients. The extended trial equation method will be employed to back up our analysis in obtaining exact solutions with distinct physical structures.

2 The extended trial equation method

Step 1. For a given nonlinear partial differential equation with rank inhomogeneous

$$P(u, u_t, u_x, u_{xx}, \dots) = 0, \tag{4}$$

take the wave transformation

$$u(x_1, \dots, x_N, t) = u(\eta), \quad \eta = \lambda\left(\sum_{j=1}^{N} x_j - ct\right), \tag{5}$$

where $\lambda \neq 0$ and $c \neq 0$. Substituting Eq. (5) into Eq. (4) yields a nonlinear ordinary differential equation,

$$N(u, u', u'', ...) = 0. \tag{6}$$

Step 2. Take transformation and trial equation as follows:

$$u = \sum_{i=0}^{\delta} \tau_i \Gamma^i, \tag{7}$$

in which

$$(\Gamma')^2 = \Lambda(\Gamma) = \frac{\Phi(\Gamma)}{\Psi(\Gamma)} = \frac{\xi_\theta \Gamma^\theta + ... + \xi_1 \Gamma + \xi_0}{\zeta_\epsilon \Gamma^\epsilon + ... + \zeta_1 \Gamma + \zeta_0}, \tag{8}$$

where τ_i $(i = 0, ..., \delta)$, ξ_i $(i = 0, ..., \theta)$ and ζ_i $(i = 0, ..., \epsilon)$ are constants. Using the relations (7) and (8), we can find

$$(u')^2 = \frac{\Phi(\Gamma)}{\Psi(\Gamma)} \left(\sum_{i=0}^{\delta} i\tau_i \Gamma^{i-1} \right)^2, \tag{9}$$

$$u'' = \frac{\Phi'(\Gamma)\Psi(\Gamma) - \Phi(\Gamma)\Psi'(\Gamma)}{2\Psi^2(\Gamma)} \left(\sum_{i=0}^{\delta} i\tau_i \Gamma^{i-1} \right) + \frac{\Phi(\Gamma)}{\Psi(\Gamma)} \left(\sum_{i=0}^{\delta} i(i-1)\tau_i \Gamma^{i-2} \right), \tag{10}$$

where $\Phi(\Gamma)$ and $\Psi(\Gamma)$ are polynomials. Substituting these terms into Eq. (6) yields an equation of polynomial $\Omega(\Gamma)$ of Γ :

$$\Omega(\Gamma) = \varrho_s \Gamma^s + ... + \varrho_1 \Gamma + \varrho_0 = 0. \tag{11}$$

According to the balance principle we can determine a relation of θ, ϵ, and δ. We can take some values of θ, ϵ, and δ.

Step 3. Let the coefficients of $\Omega(\Gamma)$ all be zero will yield an algebraic equations system:

$$\varrho_i = 0, \quad i = 0, ..., s. \tag{12}$$

Solving this equations system (12), we will determine the values of $\xi_0, ..., \xi_\theta; \zeta_0, ..., \zeta_\epsilon$ and $\tau_0, ..., \tau_\delta$.

Step 4. Reduce Eq. (8) to the elementary integral form,

$$\pm(\eta - \eta_0) = \int \frac{d\Gamma}{\sqrt{\Lambda(\Gamma)}} = \int \sqrt{\frac{\Psi(\Gamma)}{\Phi(\Gamma)}} d\Gamma. \tag{13}$$

Using a complete discrimination system for polynomial to classify the roots of $\Phi(\Gamma)$, we solve the infinite integral (13) and obtain the exact solutions to Eq. (6). Furthermore, we can write the exact traveling wave solutions to Eq. (4) respectively.

3 Applications

To illustrate the necessity of our new view concerning the trial equation method, we introduce two case studies.

Example 1. *Application to the generalized (2+1)-dimensional ZK-MEW equation*

The generalized (2+1)-dimensional ZK-MEW equation [34,35] is in the form of

$$u_t + \alpha(u^n)_x + (\beta u_{xt} + \gamma u_{yy})_x = 0, \quad (n > 1)$$

where α, β and γ are arbitrary constants.

In order to look for travelling wave solutions of Eq. (1), we make the transformation

$$u(x, y, t) = u(\eta), \quad \eta = \kappa_1 x + \kappa_2 y - ct,$$

where κ_1, κ_2 and c are real constants. Then, integrating the resulting equation with respect to η and setting the integration constant to zero yield the ordinary differential equation

$$-cu + \alpha\kappa_1 u^n + (\gamma\kappa_1\kappa_2^2 - c\beta\kappa_1^2)u'' = 0, \tag{14}$$

Eq. (14), with the transformation

$$u = \omega^{\frac{1}{n-1}}, \tag{15}$$

reduces to

$$(\gamma\kappa_2^2 - c\beta\kappa_1)Q\omega\omega'' + (\gamma\kappa_2^2 - c\beta\kappa_1)W(\omega')^2 - c\omega^2 + \alpha\kappa_1\omega^3 = 0, \tag{16}$$

where

$$Q = \kappa_1/(n-1), \quad W = \kappa_1(2-n)/(n-1)^2.$$

Substituting Eqs. (9) and (10) into Eq. (16) and using balance principle yields $\theta = \epsilon + \delta + 2$. If we take $\theta = 3$, $\epsilon = 0$ and $\delta = 1$, then

$$(\omega')^2 = \frac{\tau_1^2(\xi_3\Gamma^3 + \xi_2\Gamma^2 + \xi_1\Gamma + \xi_0)}{\zeta_0},$$

where $\xi_3 \neq 0$, $\zeta_0 \neq 0$. Respectively, solving the algebraic equation system (12) yields

$$\xi_0 = \frac{\tau_0(2\alpha\kappa_1\tau_0(\beta\kappa_1\xi_1\tau_1(Q+W) - \zeta_0\tau_0) - \gamma\kappa_2^2\xi_1\tau_1(3Q+2W))}{2\tau_1^2(3\alpha\beta\kappa_1^2\tau_0(Q+W) - \gamma\kappa_2^2(3Q+2W))}, \quad \xi_3 = \frac{\alpha\kappa_1\tau_1(2\zeta_0\tau_0 + \beta\kappa_1\xi_1\tau_1(Q+W))}{\tau_0(3\alpha\beta\kappa_1^2\tau_0(Q+W) - \gamma\kappa_2^2(3Q+2W))},$$

$$\xi_2 = \frac{6\alpha\kappa_1\tau_0(\zeta_0\tau_0 + \beta\kappa_1\xi_1\tau_1(Q+W)) - \gamma\kappa_2^2\xi_1\tau_1(3Q+2W)}{6\alpha\beta\kappa_1^2\tau_0^2(Q+W) - 2\gamma\kappa_2^2\tau_0(3Q+2W)}, \quad c = \frac{(Q+W)(6\alpha\kappa_1\zeta_0\tau_0^2 + \gamma\kappa_2^2\xi_1\tau_1(3Q+2W))}{(3Q+2W)(2\zeta_0\tau_0 + \beta\kappa_1\xi_1\tau_1(Q+W))},$$

$$\xi_1 = \xi_1, \quad \zeta_0 = \zeta_0, \qquad \tau_0 = \tau_0, \quad \tau_1 = \tau_1.$$

Substituting these results into Eq. (8) and Eq. (13), we can write

$$\pm(\eta - \eta_0) = \sqrt{\frac{\zeta_0\tau_0(3\alpha\beta\kappa_1^2\tau_0(Q+W) - \gamma\kappa_2^2(3Q+2W))}{\alpha\kappa_1\tau_1(2\zeta_0\tau_0 + \beta\kappa_1\xi_1\tau_1(Q+W))}} \times \int \frac{d\Gamma}{\sqrt{\Gamma^3 + \ell_2\Gamma^2 + \ell_1\Gamma + \ell_0}}, \tag{17}$$

where

$$\ell_2 = \frac{6\alpha\kappa_1\tau_0(\zeta_0\tau_0 + \beta\kappa_1\xi_1\tau_1(Q+W)) - \gamma\kappa_2^2\xi_1\tau_1(3Q+2W)}{2\alpha\kappa_1\tau_1(2\zeta_0\tau_0 + \beta\kappa_1\xi_1\tau_1(Q+W))},$$

and

$$\ell_1 = \frac{\xi_1\tau_0(3\alpha\beta\kappa_1^2\tau_0(Q+W) - \gamma\kappa_2^2(3Q+2W))}{\alpha\kappa_1\tau_1(2\zeta_0\tau_0 + \beta\kappa_1\xi_1\tau_1(Q+W))}, \quad \ell_0 = \frac{\tau_0^2(2\alpha\kappa_1\tau_0(\beta\kappa_1\xi_1\tau_1(Q+W) - \zeta_0\tau_0) - \gamma\kappa_2^2\xi_1\tau_1(3Q+2W))}{2\alpha\kappa_1\tau_1^3(2\zeta_0\tau_0 + \beta\kappa_1\xi_1\tau_1(Q+W))}.$$

Integrating Eq. (17), we obtain the solutions to the Eq. (1) as follows:

$$\pm(\eta - \eta_0) = -2\sqrt{A}\frac{1}{\sqrt{\Gamma - \alpha_1}}, \tag{18}$$

$$\pm(\eta - \eta_0) = 2\sqrt{\frac{A}{\alpha_2 - \alpha_1}} \arctan\sqrt{\frac{\Gamma - \alpha_2}{\alpha_2 - \alpha_1}}, \quad \alpha_2 > \alpha_1, \tag{19}$$

$$\pm(\eta - \eta_0) = \sqrt{\frac{A}{\alpha_1 - \alpha_2}} \ln\left|\frac{\sqrt{\Gamma - \alpha_2} - \sqrt{\alpha_1 - \alpha_2}}{\sqrt{\Gamma - \alpha_2} + \sqrt{\alpha_1 - \alpha_2}}\right|, \quad \alpha_1 > \alpha_2, \tag{20}$$

$$\pm(\eta - \eta_0) = 2\sqrt{\frac{A}{\alpha_1 - \alpha_3}} F(\varphi, l), \quad \alpha_1 > \alpha_2 > \alpha_3,$$

where

$$A = \frac{\zeta_0\tau_0(3\alpha\beta\kappa_1^2\tau_0(Q+W) - \gamma\kappa_2^2(3Q+2W))}{\alpha\kappa_1\tau_1(2\zeta_0\tau_0 + \beta\kappa_1\xi_1\tau_1(Q+W))}, \quad F(\varphi, l) = \int_0^{\varphi} \frac{d\psi}{\sqrt{1 - l^2\sin^2\psi}},$$

and

$$\varphi = \arcsin\sqrt{\frac{\Gamma - \alpha_3}{\alpha_2 - \alpha_3}}, \quad l^2 = \frac{\alpha_2 - \alpha_3}{\alpha_1 - \alpha_3}.$$

Also α_1, α_2 and α_3 are the roots of the polynomial equation

$$\Gamma^3 + \frac{\xi_2}{\xi_3}\Gamma^2 + \frac{\xi_1}{\xi_3}\Gamma + \frac{\xi_0}{\xi_3} = 0.$$

Substituting the solutions (18)-(20) into (7) and (15), denoting $\overline{\tau} = \tau_0 + \tau_1\alpha_1$, and setting

$$v = \frac{(Q+W)(6\alpha\kappa_1\zeta_0\tau_0^2 + \gamma\kappa_2^2\xi_1\tau_1(3Q+2W))}{(3Q+2W)(2\zeta_0\tau_0 + \beta\kappa_1\xi_1\tau_1(Q+W))},$$

we get, respectively,

$$u(x,y,t) = \left[\overline{\tau} + \frac{4\tau_1 A}{(\kappa_1 x + \kappa_2 y - vt - \eta_0)^2}\right]^{\frac{1}{n-1}}, \tag{21}$$

$$u(x,y,t) = \left\{\overline{\tau} + \tau_1(\alpha_2 - \alpha_1)\left[1 - \tanh^2\left(\mp\frac{1}{2}\sqrt{\frac{\alpha_1 - \alpha_2}{A}}(\kappa_1 x + \kappa_2 y - vt - \eta_0)\right)\right]\right\}^{\frac{1}{n-1}}, \tag{22}$$

$$u(x,y,t) = \left\{\overline{\tau} + \tau_1(\alpha_1 - \alpha_2)\mathrm{cosech}^2\left(\frac{1}{2}\sqrt{\frac{\alpha_1 - \alpha_2}{A}}(\kappa_1 x + \kappa_2 y - vt)\right)\right\}^{\frac{1}{n-1}}. \tag{23}$$

If we take $\tau_0 = -\tau_1\alpha_1$, that is $\overline{\tau} = 0$, and $\eta_0 = 0$, then the solutions (21)-(23) can reduce to rational function solution

$$u(x,y,t) = \left[\frac{2\sqrt{\tau_1 A}}{\kappa_1 x + \kappa_2 y - vt}\right]^{\frac{2}{n-1}}, \tag{24}$$

1-soliton solution

$$u(x,y,t) = \frac{A_1}{\cosh^{\frac{2}{n-1}}[\mp B(\kappa_1 x + \kappa_2 y - vt)]}, \tag{25}$$

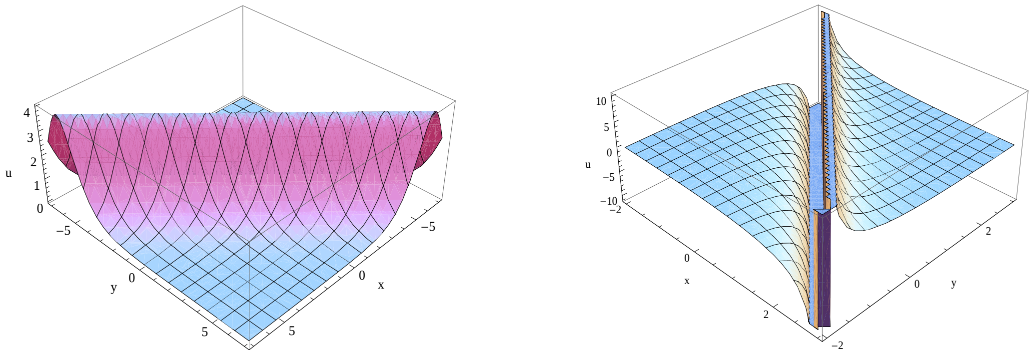
and singular soliton solution

$$u(x,y,t) = \frac{A_2}{\sinh^{\frac{2}{n-1}}[B(\kappa_1 x + \kappa_2 y - vt)]}, \tag{26}$$

where

$$A_1 = [\tau_1(\alpha_2 - \alpha_1)]^{\frac{1}{n-1}}, \quad A_2 = [\tau_1(\alpha_1 - \alpha_2)]^{\frac{1}{n-1}}, \quad B = \frac{1}{2}\sqrt{\frac{\alpha_1 - \alpha_2}{A}}.$$

Here, A_1 and A_2 are the amplitudes of the solitons, κ_1 is the inverse width of solitons in the $x-$direction and κ_2 is the inverse width of solitons in the $y-$direction and v is the velocity of the solitons. Thus, we can say that the solitons exist for $\tau_1 > 0$.



(a) Profile of 1-soliton solution (b) Profile of singular soliton solution

Figure 1: Figure 1 respectively is shown numerical solutions of 1-soliton solution and singular soliton solution at $n = 3,\ \kappa_1 = \kappa_2 = 1,\ A_1 = A_2 = 4,\ B = 1$ while $vt = 1$.

Example 2. *Application to the DSE in (1+2) dimensions*

In (2) and (3), q and r are the dependent variables while x, y and t are the independent variables. The first two of the independent variables are the spatial variables while t represents time. The exponent n is the power law parameter. It is necessary to have $n > 0$. In (2) and (3), q is a complex valued function while r is a real valued function. Also, a, b, α and β are all constant coefficients. For solving the Eqs. (2) and (3) with the trial equation method, using the wave variables

$$q(x, y, t) = u(\eta)e^{i\phi}, \quad r(x, y, t) = v(\eta) \tag{27}$$

$$\phi = \phi_1 x + \phi_2 y + \phi_3 t, \quad \eta = \eta_1 x + \eta_2 y + \eta_3 t \tag{28}$$

where $\phi_1, \phi_2, \phi_3, \eta_1, \eta_2$ and η_3 are real constants, converts (2) and (3) to the system of ODEs

$$(\eta_3 + 2a\phi_1\eta_1 + 2a\phi_2\eta_2)u(\eta) = 0, \tag{29}$$

$$-(\phi_3 + a{\phi_1}^2 + a{\phi_2}^2)u(\eta) + a({\eta_1}^2 + {\eta_2}^2)u''(\eta) + bu^{2n+1}(\eta) - \alpha u(\eta)v(\eta) = 0, \tag{30}$$

$$({\eta_1}^2 + {\eta_2}^2)v''(\eta) + \beta{\eta_1}^2(u^{2n})''(\eta) = 0 \tag{31}$$

where primes denote the derivatives with respect to η . Eq. (31) is then integrated term by term two times where integration constants are considered zero. This converts it into

$$v(\eta) = \frac{-\beta{\eta_1}^2}{{\eta_1}^2 + {\eta_2}^2}u^{2n}(\eta). \tag{32}$$

Substituting (32) into (30) gives

$$-(\phi_3 + a{\phi_1}^2 + a{\phi_2}^2)u(\eta) + a({\eta_1}^2 + {\eta_2}^2)u''(\eta) + \left(b + \alpha\beta\frac{{\eta_1}^2}{{\eta_1}^2 + {\eta_2}^2}\right)u^{2n+1}(\eta) = 0. \tag{33}$$

Eq. (33), with the transformation

$$u(\eta) = V^{\frac{1}{n}}(\eta) \tag{34}$$

reduces to

$$QVV'' + P(V')^2 - \left[\phi_3 + a{\phi_1}^2 + a{\phi_2}^2\right]RV^2 + WV^4 = 0, \tag{35}$$

where

$$Q = an\left({\eta_1}^2 + {\eta_2}^2\right)^2, \quad P = a(1-n)\left({\eta_1}^2 + {\eta_2}^2\right)^2, \quad R = n^2\left({\eta_1}^2 + {\eta_2}^2\right), \quad W = n^2\left[b\left({\eta_1}^2 + {\eta_2}^2\right) + \alpha\beta{\eta_1}^2\right].$$

Substituting Eqs. (9) and (10) into Eq. (35) and using balance principle yields $\theta = \epsilon + 2\delta + 2$. If we take $\theta = 4$, $\epsilon = 0$ and $\delta = 1$, then

$$(V')^2 = \frac{\tau_1^2(\xi_4\Gamma^4 + \xi_3\Gamma^3 + \xi_2\Gamma^2 + \xi_1\Gamma + \xi_0)}{\zeta_0},$$

where $\xi_4 \neq 0$, $\zeta_0 \neq 0$. Respectively, solving the algebraic equation system (12) yields

$$\xi_0 = \left(\frac{\tau_0}{\tau_1}\right)^2\left(\xi_2 + \frac{5\zeta_0\tau_0^2W}{P+2Q}\right), \quad \xi_1 = \frac{2\tau_0}{\tau_1}\left(\xi_2 + \frac{4\zeta_0\tau_0^2W}{P+2Q}\right), \quad \xi_2 = \xi_2, \quad \xi_3 = -\frac{4\zeta_0\tau_0\tau_1W}{P+2Q}, \quad \xi_4 = -\frac{\zeta_0\tau_1^2W}{P+2Q},$$

$$\phi_1 = \phi_1, \quad \phi_2 = \phi_2, \quad \phi_3 = \frac{\xi_2(P+Q)(P+2Q) + \zeta_0(6\tau_0^2W(P+Q) - aR(P+2Q)(\phi_1^2 + \phi_2^2))}{\zeta_0R(P+2Q)}$$

$$\zeta_0 = \zeta_0, \quad \tau_0 = \tau_0, \quad \tau_1 = \tau_1.$$

Also from Eq. (29), it can be seen that $\eta_3 = -2a(\phi_1\eta_1 + \phi_2\eta_2)$. Substituting these results into Eq. (8) and Eq. (13), we can write

$$\pm(\eta - \eta_0) = \sqrt{-\frac{P+2Q}{\tau_1^2W}} \times \int \frac{d\Gamma}{\sqrt{\Gamma^4 + \ell_3\Gamma^3 + \ell_2\Gamma^2 + \ell_1\Gamma + \ell_0}}, \tag{36}$$

where

$$\ell_3 = \frac{4\tau_0}{\tau_1}, \quad \ell_2 = -\frac{\xi_2(P+2Q)}{\zeta_0\tau_1^2W}, \quad \ell_1 = -\frac{2\tau_0(\xi_2(P+2Q) + 4\zeta_0\tau_0^2W)}{\zeta_0\tau_1^3W}, \quad \ell_0 = -\frac{\tau_0^2(\xi_2(P+2Q) + 5\zeta_0\tau_0^2W)}{\zeta_0\tau_1^4W}.$$

Integrating Eq. (36), we obtain the solutions to the Eqs. (2) and (3) as follows:

$$\pm(\eta - \eta_0) = -\frac{B}{\Gamma - \alpha_1}, \tag{37}$$

$$\pm(\eta - \eta_0) = \frac{2B}{\alpha_1 - \alpha_2}\sqrt{\frac{\Gamma - \alpha_2}{\Gamma - \alpha_1}}, \quad \alpha_2 > \alpha_1, \tag{38}$$

$$\pm(\eta - \eta_0) = \frac{B}{\alpha_1 - \alpha_2}\ln\left|\frac{\Gamma - \alpha_1}{\Gamma - \alpha_2}\right|, \tag{39}$$

$$\pm(\eta - \eta_0) = \frac{B}{\sqrt{(\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3)}}\ln\left|\frac{\sqrt{(\Gamma - \alpha_2)(\alpha_1 - \alpha_3)} - \sqrt{(\Gamma - \alpha_3)(\alpha_1 - \alpha_2)}}{\sqrt{(\Gamma - \alpha_2)(\alpha_1 - \alpha_3)} + \sqrt{(\Gamma - \alpha_3)(\alpha_1 - \alpha_2)}}\right|, \quad \alpha_1 > \alpha_2 > \alpha_3, \tag{40}$$

$$\pm(\eta - \eta_0) = 2\sqrt{\frac{B}{(\alpha_1 - \alpha_3)(\alpha_2 - \alpha_4)}}F(\varphi, l), \quad \alpha_1 > \alpha_2 > \alpha_3 > \alpha_4,$$

where

$$B = \sqrt{-\frac{P+2Q}{\tau_1^2W}}, \quad F(\varphi, l) = \int_0^{\varphi}\frac{d\psi}{\sqrt{1 - l^2\sin^2\psi}},$$

and

$$\varphi = \arcsin\sqrt{\frac{(\Gamma - \alpha_1)(\alpha_2 - \alpha_4)}{(\Gamma - \alpha_2)(\alpha_1 - \alpha_4)}}, \quad l^2 = \frac{(\alpha_2 - \alpha_3)(\alpha_1 - \alpha_4)}{(\alpha_1 - \alpha_3)(\alpha_2 - \alpha_4)}.$$

Also α_1, α_2, α_3 and α_4 are the roots of the polynomial equation

$$\Gamma^4 + \frac{\xi_3}{\xi_4}\Gamma^3 + \frac{\xi_2}{\xi_4}\Gamma^2 + \frac{\xi_1}{\xi_4}\Gamma + \frac{\xi_0}{\xi_4} = 0.$$

Substituting the solutions (37)-(40) into (7) and (34), we obtain

$$q(x,y,t) = \left\{\tau_0 + \tau_1\alpha_1 \pm \frac{\tau_1 B}{\eta_1 x + \eta_2 y + \eta_3 t - \eta_0}\right\}^{\frac{1}{n}} e^{i\phi}, \tag{41}$$

$$r(x,y,t) = -\frac{\beta\eta_1^2}{\eta_1^2+\eta_2^2}\left\{\tau_0 + \tau_1\alpha_1 \pm \frac{\tau_1 B}{\eta_1 x + \eta_2 y + \eta_3 t - \eta_0}\right\}^2, \tag{42}$$

$$q(x,y,t) = \left\{\tau_0 + \tau_1\alpha_1 + \frac{4B^2(\alpha_2-\alpha_1)\tau_1}{4B^2 - \left[(\alpha_1-\alpha_2)(\eta_1 x + \eta_2 y + \eta_3 t - \eta_0)\right]^2}\right\}^{\frac{1}{n}} e^{i\phi}, \tag{43}$$

$$r(x,y,t) = -\frac{\beta\eta_1^2}{\eta_1^2+\eta_2^2}\left\{\tau_0 + \tau_1\alpha_1 + \frac{4B^2(\alpha_2-\alpha_1)\tau_1}{4B^2 - \left[(\alpha_1-\alpha_2)(\eta_1 x + \eta_2 y + \eta_3 t - \eta_0)\right]^2}\right\}^2, \tag{44}$$

$$q(x,y,t) = \left\{\tau_0 + \tau_1\alpha_2 + \frac{(\alpha_2-\alpha_1)\tau_1}{\exp\left(\frac{\alpha_1-\alpha_2}{B}(\eta_1 x + \eta_2 y + \eta_3 t - \eta_0)\right) - 1}\right\}^{\frac{1}{n}} e^{i\phi}, \tag{45}$$

$$r(x,y,t) = -\frac{\beta\eta_1^2}{\eta_1^2+\eta_2^2}\left\{\tau_0 + \tau_1\alpha_2 + \frac{(\alpha_2-\alpha_1)\tau_1}{\exp\left(\frac{\alpha_1-\alpha_2}{B}(\eta_1 x + \eta_2 y + \eta_3 t - \eta_0)\right) - 1}\right\}^2, \tag{46}$$

$$q(x,y,t) = \left\{\tau_0 + \tau_1\alpha_1 + \frac{(\alpha_1-\alpha_2)\tau_1}{\exp\left(\frac{\alpha_1-\alpha_2}{B}(\eta_1 x + \eta_2 y + \eta_3 t - \eta_0)\right) - 1}\right\}^{\frac{1}{n}} e^{i\phi}, \tag{47}$$

$$r(x,y,t) = -\frac{\beta\eta_1^2}{\eta_1^2+\eta_2^2}\left\{\tau_0 + \tau_1\alpha_1 + \frac{(\alpha_1-\alpha_2)\tau_1}{\exp\left(\frac{\alpha_1-\alpha_2}{B}(\eta_1 x + \eta_2 y + \eta_3 t - \eta_0)\right) - 1}\right\}^2, \tag{48}$$

$$q(x,y,t) = \left\{\tau_0 + \tau_1\alpha_1 - \frac{2(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)\tau_1}{2\alpha_1 - \alpha_2 - \alpha_3 + (\alpha_3-\alpha_2)\cosh\left(\frac{\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}{B}(\eta_1 x + \eta_2 y + \eta_3 t)\right)}\right\}^{\frac{1}{n}} e^{i\phi}, \tag{49}$$

$$r(x,y,t) = -\frac{\beta\eta_1^2}{\eta_1^2+\eta_2^2}\left\{\tau_0 + \tau_1\alpha_1 - \frac{2(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)\tau_1}{2\alpha_1 - \alpha_2 - \alpha_3 + (\alpha_3-\alpha_2)\cosh\left(\frac{\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}{B}(\eta_1 x + \eta_2 y + \eta_3 t)\right)}\right\}^2. \tag{50}$$

If we take $\tau_0 = -\tau_1\alpha_1$ and $\eta_0 = 0$, then the solutions (41)-(50) can reduce to rational function solutions

$$q(x,y,t) = \left(\pm\frac{\tau_1 B}{\eta_1 x + \eta_2 y + \eta_3 t}\right)^{\frac{1}{n}} e^{i(\phi_1 x + \phi_2 y + \phi_3 t)}, \tag{51}$$

$$r(x,y,t) = \Upsilon\left(\frac{\tau_1 B}{\eta_1 x + \eta_2 y + \eta_3 t}\right)^2, \tag{52}$$

$$q(x,y,t) = \left\{\frac{4B^2(\alpha_2-\alpha_1)\tau_1}{4B^2 - \left[(\alpha_1-\alpha_2)(\eta_1 x + \eta_2 y + \eta_3 t)\right]^2}\right\}^{\frac{1}{n}} e^{i(\phi_1 x + \phi_2 y + \phi_3 t)}, \tag{53}$$

$$r(x,y,t) = \Upsilon \left\{ \frac{4B^2(\alpha_2 - \alpha_1)\tau_1}{4B^2 - \left[(\alpha_1 - \alpha_2)(\eta_1 x + \eta_2 y + \eta_3 t)\right]^2} \right\}^2, \tag{54}$$

traveling wave solutions

$$q(x,y,t) = \left\{ \frac{(\alpha_2 - \alpha_1)\tau_1}{2} \left\{ 1 \mp \coth\left[\frac{\alpha_1 - \alpha_2}{2B} (\eta_1 x + \eta_2 y + \eta_3 t) \right] \right\} \right\}^{\frac{1}{n}} e^{i(\phi_1 x + \phi_2 y + \phi_3 t)}, \tag{55}$$

$$r(x,y,t) = \Upsilon \left\{ \frac{(\alpha_2 - \alpha_1)\tau_1}{2} \left\{ 1 \mp \coth\left[\frac{\alpha_1 - \alpha_2}{2B} (\eta_1 x + \eta_2 y + \eta_3 t) \right] \right\} \right\}^2, \tag{56}$$

and soliton solutions

$$q(x,y,t) = \frac{A_3}{\left(D + \cosh\left[B_1(\eta_1 x + \eta_2 y + \eta_3 t)\right]\right)^{\frac{1}{n}}} e^{i(\phi_1 x + \phi_2 y + \phi_3 t)}, \tag{57}$$

$$r(x,y,t) = \Upsilon \frac{A_4}{\left(D + \cosh\left[B_1(\eta_1 x + \eta_2 y + \eta_3 t)\right]\right)^2}, \tag{58}$$

where

$$\eta_3 = -2a(\phi_1\eta_1 + \phi_2\eta_2), \quad \phi_3 = \frac{\xi_2(P+Q)(P+2Q) + \zeta_0(6\tau_0^2 W(P+Q) - aR(P+2Q)(\phi_1^2 + \phi_2^2))}{\zeta_0 R(P+2Q)},$$

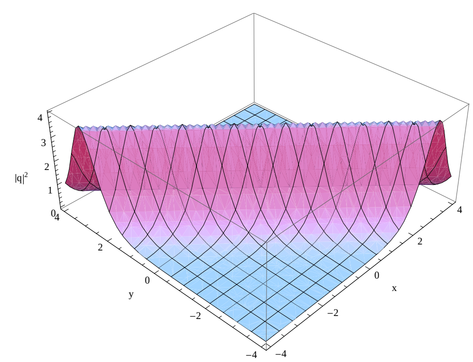
and

$$\Upsilon = -\frac{\beta{\eta_1}^2}{{\eta_1}^2 + {\eta_2}^2}, \quad A_3 = \left(\frac{2(\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3)\tau_1}{\alpha_3 - \alpha_2} \right)^{\frac{1}{n}}, \quad A_4 = A_3^{2n},$$

and

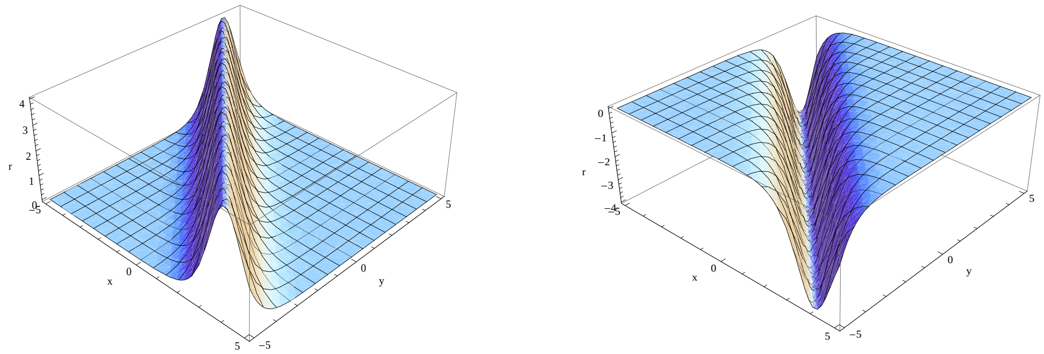
$$B_1 = \frac{\sqrt{(\alpha_1 - \alpha_2)(\alpha_1 - \alpha_3)}}{B}, \quad D = \frac{2\alpha_1 - \alpha_2 - \alpha_3}{\alpha_3 - \alpha_2}.$$

From (28), η_1 and η_2 are the widths of the solitons in the $x-$ and $y-$directions respectively while η_3 is the velocity of the solitons. From the phase component given by ϕ, ϕ_1 and ϕ_2 are the phase frequencies in the x- and y-directions respectively while ϕ_3 is the wave numbers of the solitons. Also, A_3 and A_4 are the amplitudes of the solitons. Thus, we can say that the solitons exist for $\tau_1 < 0$.



(a) Profile of (57) solution

Figure 2: Numerical solution of (57) at $n = 1,\ \eta_1 = \eta_2 = 1,\ \eta_3 t = 1,\ A_3 = 2,\ B_1 = 2$ and $D < 0$.



(a) Profile of (58) solution for $\Upsilon > 0$ (b) Profile of (58) solution for $\Upsilon < 0$

Figure 3: Numerical solutions of (58) at $\eta_1 = \eta_2 = 1,\ \eta_3 t = 1,\ A_4 = 4,\ B_1 = 1$ and $D < 0$.

4 Conclusion

In this paper we have used the extended trial equation method to derive exact solutions with distinct physical structures. This method with symbolic computation on the computer is used for constructing broad classes of periodic and soliton solutions of two nonlinear equations arising in nonlinear physics. Basic features of the 1-soliton solution and singular soliton solution were analytically and numerically discussed. We proposed more general trial equation method as an alternative approach to obtain the analytic solutions of nonlinear partial differential equations with generalized evolution in mathematical physics. We use the extended trial equation method aided with symbolic computation to construct the soliton solutions, the elliptic function and rational function solutions for generalized $(2+1)$-dimensional ZK-MEW equation and generalized Davey-Stewartson system.

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