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Viscous Dissipation and Buoyancy Effects on Laminar Convection in a Vertical Channel with Transpiration.

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Abstract

The viscous dissipation and buoyancy effects on laminar convection with transpiration are investigated. Uniform and asymmetric temperatures are prescribed at the channel walls. The velocity field is considered as parallel. A perturbation method is employed to solve the momentum balance equation and the energy balance equation. A comparison with the velocity and temperature profiles in the case of laminar forced convection with various dissipation is performed in order to point out the effect of buoyancy. The case of convective boundary condition is also discussed.

Keywords: *Buoyancy, Convection, Laminar, Transpiration, Viscous dissipation.*

1 Introduction

Viscous dissipation is often neglected in the studies of laminar convection in channels. On the other hand, this effect can be relevant if highly viscous fluids with low thermal conductivity are considered.

A comprehensive review of the literature on this subject matter can be found in Aung [1]. Analytical and numerical solutions for the temperature and the velocity field have been obtained for both prescribed wall temperatures and for prescribed wall heat fluxes.

The paper presented by Tao [13] was the earliest analyses of laminar and fully developed mixed convection in a parallel – plate vertical channel with prescribed uniform temperatures at the boundaries.

Berardi et al [8] studied viscous dissipation in duct flow of molten polymers. Flow reversal and heat transfer of fully developed mixed convection in vertical channels was studied by Cheng et al [9]. Hamadah and Wirtz [10] made an analysis of laminar fully developed mixed convection in a vertical channel with opposing buoyancy. These authors pointed out that the temperature distribution in the fluid is uniform when both boundaries are at the same temperature and is a linear function of the transverse coordinate when the boundaries are kept at different temperatures. In asymmetric heating, heat transfer between the two boundaries of the channel occurs by pure conduction. Moreover, the buoyancy force influences the velocity profile and can give rise to flow reversal both for upward flow and downward flow.

In literature, both for prescribed wall temperatures and for prescribed wall heat fluxes, the analyses of fully developed mixed convection are performed by analytical methods [11]. While the developing flow is analyzed by numerical techniques, such are presented by Aung and Worku [2,4], they studied the developing flow and flow reversal in a vertical channel with asymmetric wall temperatures.

Aung and Worku [3] also used a numerical technique to analyzed mixed convection in ducts with asymmetric wall heat fluxes.

In work of Barletta [6], a solution of the problem of mixed convection with viscous dissipation in a parallel – plate vertical channel with uniform and equal temperatures is obtained. The velocity and temperature profiles are expressed as a perturbation series with respect to a dimensionless parameter which accounts for buoyancy.

Recently, Barletta and Nield [7] analyzed the buoyant laminar flow in a square lid – driven enclosure. The governing equations are solved numerically by employing a Galerkin finite element method.

Moreover, some papers deal with the internal mixed convection with viscous dissipation. For instance, Iqbal et al [12] consider the fully-developed combined force and free convection in a vertical circular tube with a uniform boundary heat flux. These authors employed three different mathematical techniques for the solution of the coupled momentum and energy balance equations: an extended Frobenius method, the Galerkin method and the Runge-Kutta method.

This paper is to extend the studies available in the literature on fully developed laminar convection in a vertical channel. Here, viscous dissipation and buoyancy effects on laminar convection in a vertical channel are considered and investigated. The prescribed wall temperatures are asymmetric. These effects are expected to be relevant for fluids with high – velocity flows. In the case of asymmetric heating, a consequence of this coupling is that no conduction regime is present when viscous dissipation is taken into account. Here, the velocity profile and the temperature profile are obtained by a perturbation method based on the perturbation parameter ε defined by $\varepsilon = \frac{Gr}{Re}$ where Gr is the Grashof number and Re is the Reynolds number.

2 Formulation of the Problem and Governing Equations

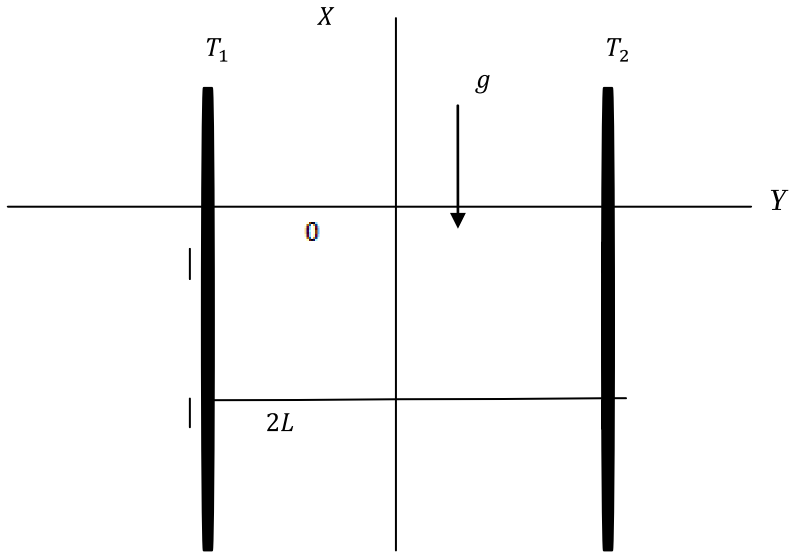

Fig. 1: Configuration of the system

Let us consider a Newtonian fluid which steadily flows in a plane-parallel channel whose walls are separated by a distance$2L$. The flow is assumed to be laminar and parallel, i.e., the only nonvanishing component of the velocity $\boldsymbol{U}$ is the axial component$U$. The temperature field on the boundary walls are uniform and asymmetric with values $T_1 and\ T_2$. as shown in Fig.1, the axial coordinate X is directed upward, i.e., it has a direction opposite to the gravitational acceleration vector. Since only the axial U component of is nonzero, the mass balance equation is

$$\frac{\partial U}{\partial X} = 0 \tag{2.1}$$

On account of equation (2.1), U depends on the transverse coordinate Y

Let us assume the equation of state

$$\rho = \rho_b[1 - \beta(T - T_b)], \tag{2.2}$$

As well as the Boussinesq approximation. In equation (2.2), β is the thermal expansion coefficient and the temperature is the bulk temperature T_b defined as

$$T_b = \frac{1}{LU_m}\int_0^L UTdY \tag{2.3}$$

Where the mean velocity is given by

$$U_m = \frac{1}{L}\int_0^L UdY \tag{2.4}$$

Then, the momentum balance equation is given by

$$w_0 U''(Y) + \frac{g\beta}{\nu}(T - T_b) - \frac{1}{\nu\rho_b}\frac{\partial P}{\partial X} = 0 \tag{2.5}$$

$$\frac{\partial P}{\partial Y} = 0\ , \tag{2.6}$$

Where w_0 is the transpiration parameter g is the gravitational acceleration, ν is the kinematic viscosity defined by $\nu = \frac{\mu}{\rho_b}$and $P = p + \rho_b gX$ is the difference between the pressure p and the hydrostatic pressure P.

As a consequence of equations (2.5), (2.6) and of the boundary condition on T, it can be easily proved that: P dependson X; T depends only on Y; T_b is a constant; $\frac{\partial P}{\partial X}$ is a constant.

The energy balance equation is given by

$$kT''(Y) + \mu[U'(Y)]^2 = 0 , \quad (2.7)$$

wHere k is thermal conductivity and $\mu = \rho_b\nu$ is the dynamic viscosity.
The boundary conditions on the velocity U and temperature T are

$$U(0) = 0 , \qquad T(0) = T_1,$$
$$U(L) = 0 , \qquad T(L) = T_2 \quad (2.8)$$

Note that, on account of equations (2.2) to (2.4), the mass flow rate M per unit width can be expressed as

$$M = \int_0^L \rho U dY$$
$$M = \int_0^L \{\rho_b[1 - \beta(T - T_b)]\} U dY$$
$$M = \int_0^L [\rho_b U - \beta U(T - T_b)] dY$$
$$M = \int_0^L \left[\rho_b U - \beta U\left(T - \frac{UT}{U_m}\right)\right] dY$$
$$M = \int_0^L \left[\rho_b U_m - \beta U_m\left(T - \frac{U_m T}{U_m}\right)\right] dY$$
$$= \int_0^L \rho_b U_m dY$$
$$= L\rho_b U_m \quad (2.9)$$

Introducing the dimensionless parameters

$$u = \frac{U}{U_m}, \quad \theta = \frac{T - T_b}{\Delta T}, \quad y = \frac{Y}{L}, \ \gamma = \frac{-L^2}{\mu U_m}\frac{dP}{dX}, \ Gr = \frac{L^3 g\beta\Delta T}{\nu^2}$$
$$Re = \frac{LU_m}{\nu}, \quad \varepsilon = \frac{Gr}{Re}, \quad Br = \frac{\mu U_m^2}{k\Delta T}, \ w_0 = \frac{Rev}{L}, \ \Delta T = \frac{\mu U_m^2}{k} \quad (2.10)$$

While the Grashof number Gr is always positive, the Reynolds number Re and the parameter E can be either positive or negative. In particular, in the case of upward flow $(U_m > 0)$, both Re and E are positive, while, for downward flow $(U_m < 0)$, these dimensionless parameters are negative.
Equation (2.5) and (2.7) can be written in non – dimensionless form as

$$Reu''(y) + \varepsilon\theta(y) + \gamma = 0 \quad (2.11)$$

And

$$\theta''(y) + Br[u'(y)]^2 = 0 \quad (2.12)$$

The corresponding boundary conditions are

$$u(0) = 0, u(1) = 0, \theta(0) = R, \theta(1) = 1 \quad (2.13)$$

Moreover, equations (2.3), (2.4) and (2.11) yield

$$\int_0^1 u\theta dy = 0 \ , \ \int_0^1 u dy = 1 \quad (2.14)$$

For any prescribed value of the dimensionless parameterε, equations (2.13) to (2.15) allow one to determine the functions $u(y)\ and\ \theta(y)$ as well as the constants γ and . In particular, it is easily verified that the choice $\varepsilon = 0$ correspond to the absence of buoyancy forces, i.e., to forced convection.

3 Perturbation Series Solution

In this section, the perturbation method which leads to the solutions of equations (2.11) to (2.14) is described .Let us expand the functions $u(y), \theta(y)$ and the constants γ and R as a power series in the parameter ε , namely

$$u(y) = u_0(y) + u_1(y)\varepsilon + u_2(y)\varepsilon^2 + \cdots = \sum_{n=0}^{\infty} u_n(y)\varepsilon^n \quad (3.1)$$

$$\theta(y) = \theta_0(y) + \theta_1(y)\varepsilon + \theta_2(y)\varepsilon^2 + \cdots = \sum_{n=0}^{\infty} \theta_n(y)\varepsilon^n \quad (3.2)$$

$$\gamma = \gamma_0 + \gamma_1\varepsilon + \gamma_2\varepsilon^2 + \cdots = \sum_{n=0}^{\infty} \gamma_n\varepsilon^n \quad (3.3)$$

$$\varphi = R_0 + R_1\varepsilon + R_2\varepsilon^2 + \cdots = \sum_{n=0}^{\infty} R_n\varepsilon^n \quad (3.4)$$

These equations (3.1) to (3.4) are substituted in equations (2.11) to (2.14), the coefficients of the increasing power of ε were equated to zero, thus:
For order zero

$$Reu_0''(y) + \gamma_0 = 0 , u_0(0) = 0 , u_0(1) = 0 , \ \int_0^1 u_0 dy = 1 \quad (3.5)$$

$$\theta_0''(y) + 2Br[u_0'(y)]^2 = 0, \theta_0(0) = 0 , \theta_0(1) = R_0 , \int_0^1 u_0\theta_0 dy = 0 \quad (3.6)$$

The boundary value problem which correspond to $n = 1$ is the following

$$Reu_1''(y) + \theta_0(y) + \gamma_1 = 0\ , u_1\ (0) = 0\ , u_1(1) = 0\ , \int_0^1 u_1 dy = 0 \quad (3.7)$$

$$\theta_1''(y) - w_0\theta_1'(y) = -[2u_1'u_0'], \theta_1(0) = 0\ , \theta_1(1) = R_1\ , \int_0^1 (u_0\theta_1 + u_1\theta_0) = 0 \quad (3.8)$$

Equations (3.5) and (3.6) are easily solved because the function $\theta_0(y)$ does not affect the function$u_0(y)$. The latter, together with the constantγ_0, is determined by solving equation (3.5) namely;

$$u_0(y) = 6(y - y^2), \quad (3.9)$$

$$\gamma_0 = 12Re \quad (3.10)$$

Similarly, $\theta_0(y)$ is also to be determined together with the constant R_0 by substituting equation (3.9) into equation (3.6), namely

$$\theta_0(y) = -Br(18y^2 - 24y^3 + 12y^4) + (1 + 6Br)y + \frac{1}{70}(5118Br - 140) \quad (3.11)$$

$$R_0 = \frac{1}{140}(5118Br - 140) \quad (3.12)$$

Equations (3.1) and (3.2) reveal that $u_0(y)\ and\ \theta_0(y)$ are the dimensionless velocity and temperature profiles in the case $\varepsilon\ of\ order\ 0$, i.e., in the case of forced convection. As expected, $u_0(y)$ is the usual Hagen – Poiseuille velocity of laminar forced convection. Obviously, $\gamma_0 and\ R_0$ are the values of $\gamma\ and\ R$ in the case $\varepsilon = 0.$

For the case $\varepsilon = 1$, by substituting equation (3.11) into equation (3.7), one obtains the function $u_1(y)$ and the constant γ_1 as follows

$$u_1(y) = \frac{1}{Re}\left(\frac{133}{336} + \frac{21856}{280}Br\right)y - \frac{1}{Re}\left(\frac{20}{32} + \frac{9243}{56}Br\right)y^2 + \frac{1}{24Re}(1 + 6Br)y^3 - \frac{Br}{4Re}\left(\frac{3}{2}y^4 - \frac{6}{5}y^5 + \frac{2}{5}y^6\right) \quad (3.13)$$

$$\gamma_1 = \frac{12}{32} + \frac{24387}{140}Br \quad (3.14)$$

Also, by substituting equations (3.9), (3.10) and (3.12) into equation (3.8), one obtains the function $\theta_1(y) and\ the\ constant\ \varphi_1$ as follows

$$\theta_1(y) = \frac{3}{20}\boldsymbol{\lambda}Br(1 + 6Br)y^5 - \boldsymbol{\lambda}Br\left(\frac{21}{8} + \frac{18507}{28}Br\right)y^4 + 4\boldsymbol{\lambda}Br\left(\frac{343}{336} + \frac{68071}{280}Br\right)y^3 - 6\boldsymbol{\lambda}Br\left(\frac{133}{336} + \frac{21856}{280}Br\right)y^2 + 3\boldsymbol{\lambda}Br^2\left(\frac{2}{30}y^5 - \frac{1}{5}y^6 + \frac{2}{105}y^7\right) - 6\boldsymbol{\lambda}Br^2\left(\frac{1}{10}y^6 - \frac{1}{7}y^7 + \frac{1}{70}y^8\right) + \left(\frac{293691}{840}\boldsymbol{\lambda}Br + \frac{21709}{140}\boldsymbol{\lambda}Br^2 + \frac{196704}{175}Br^2 + \frac{1538}{336}\boldsymbol{\lambda}\right)y - \left(\frac{36053905}{840}\boldsymbol{\lambda}Br^2 + \boldsymbol{\lambda}Br + \frac{196704}{175}Br^2 + \frac{4788}{840}Br + \frac{1538}{336}\boldsymbol{\lambda}\right) \quad (3.15)$$

$$R_1 = -\frac{36053905}{840}\boldsymbol{\lambda}Br^2 - \frac{293051}{840}\boldsymbol{\lambda}Br - \frac{196704}{175}Br^2 - \frac{4788}{840}Br - \frac{1538}{336}\lambda \quad (3.16)$$

By employing equations (3.9), (3.10), (3.12), (3.13), (3.14), (3.15) and (3.16) one evaluate the functions $u_n(y), \theta_n(y),$ $and\ the\ constants\ \gamma_n\ and\ R_n\ for\ n = 0{,}1.$ although the expansions of $u(y), \theta(y),\ \gamma\ and\ R$ given by equations (3.1) to (3.4) have infinite number of terms, in practice one can deal with truncated perturbation series. In particular, if the first two terms of each series are considered, one obtains

$$u(y) \cong 6(y - y^2) + \varepsilon\begin{bmatrix}\frac{1}{Re}\left(\frac{133}{336} + \frac{21856}{280}Br\right)y - \frac{1}{Re}\left(\frac{20}{32} + \frac{9243}{56}Br\right)y^2 + \frac{1}{24Re}(1 + 6Br)y^3 \\ -\frac{Br}{4Re}\left(\frac{3}{2}y^4 - \frac{6}{5}y^5 + \frac{2}{5}y^6\right)\end{bmatrix} \quad (3.17)$$

$$\theta(y) \cong -Br(18y^2 - 24y^3 + 12y^4) + (1 + 6Br)y + \frac{1}{70}(5118Br - 140) + \varepsilon\begin{bmatrix}\frac{3}{20}\boldsymbol{\lambda}Br(1 + 6Br)y^5 - \boldsymbol{\lambda}Br\left(\frac{21}{8} + \frac{18507}{28}Br\right)y^4 + 4\boldsymbol{\lambda}Br\left(\frac{343}{336} + \frac{68071}{280}Br\right)y^3 - \\ 6\boldsymbol{\lambda}Br\left(\frac{133}{336} + \frac{21856}{280}Br\right)y^2 + 3\boldsymbol{\lambda}Br^2\left(\frac{2}{30}y^5 - \frac{1}{5}y^6 + \frac{2}{105}y^7\right) \\ -6\boldsymbol{\lambda}Br^2\left(\frac{1}{10}y^6 - \frac{1}{7}y^7 + \frac{1}{70}y^8\right) + \left(\frac{293691}{840}\boldsymbol{\lambda}Br + \frac{21709}{140}\boldsymbol{\lambda}Br^2 + \frac{196704}{175}Br^2 + \frac{1538}{336}\boldsymbol{\lambda}\right)y - \\ \left(\frac{36053905}{840}\boldsymbol{\lambda}Br^2 + \boldsymbol{\lambda}Br + \frac{196704}{175}Br^2 + \frac{4788}{840}Br + \frac{1538}{336}\boldsymbol{\lambda}\right)\end{bmatrix} \quad (3.18)$$

$$\gamma \cong 12Re + \varepsilon\left[\frac{12}{32} + \frac{24387}{140}Br\right] \quad (3.19)$$

$$R \cong \frac{1}{140}(5118Br - 140) + \varepsilon\left[-\left(\frac{36053905}{840}\boldsymbol{\lambda}Br^2 + \frac{293051}{840}\boldsymbol{\lambda}Br + \frac{196704}{175}Br^2 + \frac{4788}{840}Br + \frac{1538}{336}\boldsymbol{\lambda}\right)\right] \quad (3.20)$$

The Nusselt number Nu which is the coefficient of heat transfer or the ratio of convective and conductive heat is given by

$$Nu_0 = \frac{d\theta}{dy}\Big|_{y=0} \quad (3.21)$$

And

$$Nu_1 = \frac{d\theta}{dy}\Big|_{y=1} \quad (3.22)$$

On differentiating equation (3.18) and substituting into equations (3.21) and (3.22), we obtain

$$Nu_0 = \frac{d\theta}{dy}\Big|_{y=0} = (1 + 6Br) + \varepsilon\left[\left(\frac{1538}{336}\boldsymbol{\lambda} + \frac{196704}{175}Br^2 + \frac{293691}{840}\boldsymbol{\lambda}Br + \frac{21709}{140}\boldsymbol{\lambda}Br^2\right)\right] \quad (3.23)$$

And

$$Nu_1 = \frac{d\theta}{dy}\Big|_{y=1} = (1-6Br) + \varepsilon\left[\frac{1538}{336}\boldsymbol{\lambda} + \frac{196704}{175}Br^2 + \frac{291801}{840}\boldsymbol{\lambda}Br - \frac{140094}{280}\boldsymbol{\lambda}Br^2\right] \tag{3.24}$$

4 Analysais and Discussion of the Results

In this section, 2-terms perturbation series are employed to evaluate the dimensionless velocity profiles and the dimensionless temperature profiles. The radius of convergence of the perturbation series can be easily obtained by estimating the value of D'Alembert's ratio limit [5].Since the temperatures T_1 and T_2 are not the same (asymmetrical), both the dimensionless velocity u and dimensionless temperature θ depend on the dimensionless parameterε. When the flow is upward ε is positive. On the other hand, when the flow is downward ε is negative.

In Table 1, the values of γ and R are obtained by means of 2-terms perturbation series, for values of ε which lie in the interval $-400 \le \varepsilon \le 400$. The pressure drop parameter γ increases for both for downward flow $(\varepsilon < 0)$ and for upward flow$(\varepsilon > 0)$. The temperature difference ratio R decreases for both $\varepsilon < 0$ and $\varepsilon > 0$.

Tables 2 and 3, show the numerical values of Nusselt numbers$Nu_0 and$ Nu_1. These numerical values are obtained by 2 -terms perturbation series for values of ε which lie in the interval $-400 \le \varepsilon \le 400$. it can be observed from table 2 that the values for both $Nu_0 and$ Nu_1 are the same. While in table 3, both $Nu_0 and$ Nu_1 increase for both < 0 and $\varepsilon > 0$.

The buoyancy effect for downward flow, enhances the pressure drop and the bulk temperature and the temperature on the axis of the tube, while it lowers the convection coefficient. On the other hand, for upward flow the convection coefficient and the pressure drop are enhanced by the buoyancy effect, while the bulk temperature and the temperature at the axis are lowered.

In figures 2 – 10, the dimensionless velocity and temperature profiles are plotted for different values of ε. The plots have been obtained by employing 2 – term perturbation series. In these figures comparison between the behavior of u and θ in the case of mixed convection and in the case of forced convection (i.e. $\varepsilon = 0$) is performed.

Figures 2 – 4 clearly reveal that the plot of $u(y)$ with different values of ε is asymmetrical. The graphs consist of two parts. The first part which is to the left shows that if both $\varepsilon < 0$ and $\varepsilon > 0$ (downward and upward flow), the velocity u increases and vice – versa in the second part of the graph.

Figure 5 reveals the effect of Reynolds number on the velocity u in the case of asymmetric heating. It can be observed from the graph that the velocity increases asymmetrically with increases in Re.

Figures 6 and 7 respectively show the effect of Brinkman number Br on velocity u in the case of asymmetric heating for $\varepsilon > 0$ and the effect of Brinkman number Br on velocity u in the case of asymmetric heating for $\varepsilon < 0$. It can also be observed that the u increases asymmetrically.

Figure 8 depicts that, for upward flow $(\varepsilon > 0)$, the buoyancy effect decreases the temperature θ, while this effect increases the temperature profile for downward flow $(\varepsilon < 0)$. The lines of the graph converge to the points 0 and 1, this is due to forced convection $(\varepsilon = 0)$.

Figures 9 and 10 demonstrate the effect of Re on temperature profile for $\varepsilon < 0$ and the effect of Re on temperature profile for $\varepsilon > 0$ respectively. It is shown in figure 9 that, the temperature θ increases as Re decreases. While the temperature θ decreases as Re increases. On account of Tables 1 – 3 and figures 2 - 10, it can be pointed out that the changes induced by buoyancy on the velocity profile and on the temperature profile are more apparent in the case of downward than in the case of upward flow.

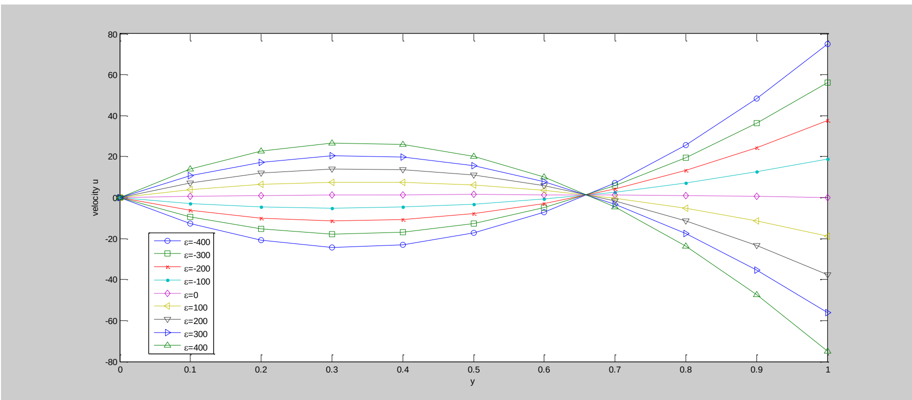

Fig 2: Plots of $u(y)$ in the case of asymmetric heating, for different values of ε and for $Br = 0$ and $Re = 1$

Table1: Values of γ and R for various values of ε

ε	γ	R
-400	-6.9185×10^4	1.7762×10^7
-350	-6.1087×10^4	1.5542×10^7
-300	-5.2358×10^4	1.3321×10^7
-250	-4.3630×10^4	1.1101×10^7
-200	-3.4902×10^4	8.8809×10^6
-150	-2.6173×10^4	6.6607×10^6
-100	-1.7445×10^4	4.4405×10^6
-50	-8.7164×10^3	2.2203×10^6
0	12	35.5571
50	8.7404×10^3	-2.2202×10^6
100	1.7469×10^4	-4.4404×10^6
150	2.6197×10^4	-6.6606×10^6
200	3.4926×10^4	-8.8809×10^6
250	4.3654×10^4	-1.1101×10^7
300	5.2382×10^4	-1.3321×10^7
350	6.1111×10^4	-1.5542×10^7
400	6.9839×10^4	-1.7762×10^7

Table2: Values of Nu_0 and Nu_1 for $Br = 0$ and $Re = 1$

ε	Nu_0	Nu_1
-400	-1.8300×10^3	-1.8300×10^3
-350	-1.6011×10^3	-1.6011×10^3
-300	-1.3722×10^3	-1.3722×10^3
-250	-1.1433×10^3	-1.1433×10^3
-200	-914.4762	-914.476
-150	-685.6071	-685.6071
-100	-456.7381	-456.7381
-50	-227.8690	-227.8690
0	1	1
50	229.8690	229.8690
100	458.738	458.7381
150	687.6071	687.6071
200	916.4762	916.4762
250	1.1453×10^3	1.1453×10^3
300	1.3742×10^3	1.3742×10^3
350	1.6031×10^3	1.6031×10^3
400	1.8320×10^3	1.8320×10^3

Table 3: Values of Nu_0 and Nu_1 for $Br = 1$ and $Re = 1$

ε	Nu_0	Nu_1
-400	-6.5331×10^5	-3.9026×10^5
-350	-5.7165×10^5	-3.4148×10^5
-300	-4.8998×10^5	-2.9269×10^5
-250	-4.0332×10^5	-2.4392×10^5
-200	-3.2665×10^5	-1.9513×10^5
-150	-2.4499×10^5	-1.4635×10^5
-100	-1.6332×10^5	-9.7570×10^4
-50	-8.1658×10^4	-4.8787×10^4
0	7	-5
50	8.1672×10^4	4.8770×10^4
100	1.6334×10^5	9.7560×10^4
150	2.4500×10^5	1.4634×10^5
200	3.2667×10^5	1.9512×10^5
250	4.0833×10^5	2.4391×10^5
300	4.9000×10^5	2.9269×10^5
350	5.7166×10^5	3.4147×10^5
400	6.5333×10^5	3.9025×10^5

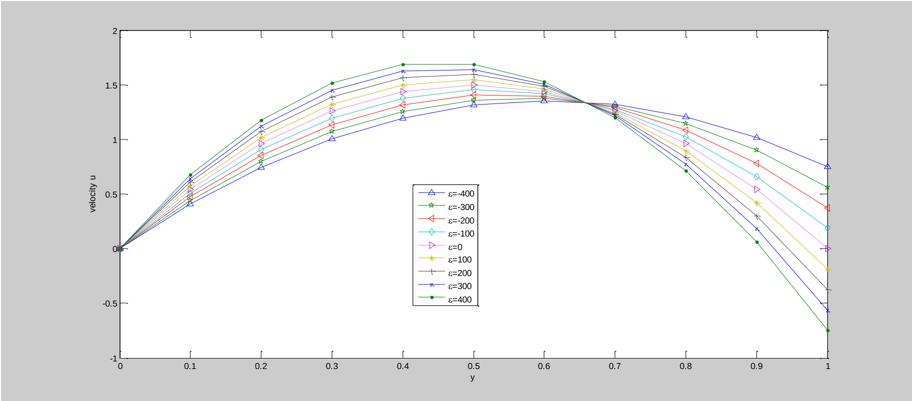

Fig 3: Plots of $u(y)$ in the case of asymmetric heating, for different values of ε and for $Br = 0 \; and \; Re = 100$

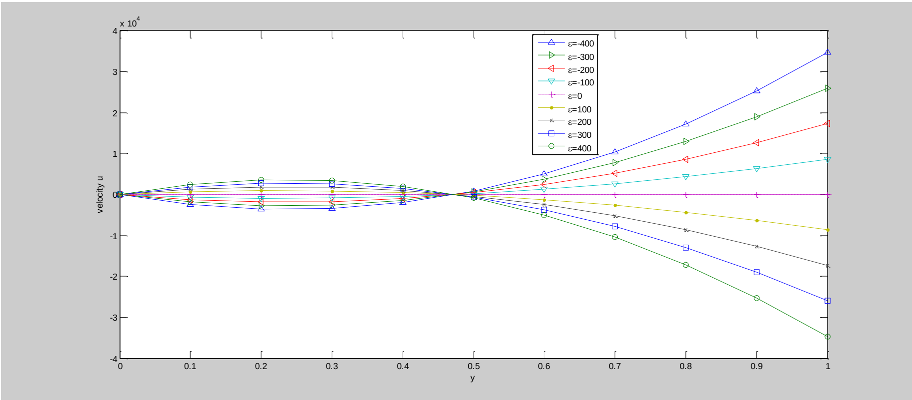

Fig 4: Plots of $u(y)$ in the case of asymmetric heating, for different values of ε and for $Br = 1 \; and \; Re = 1$

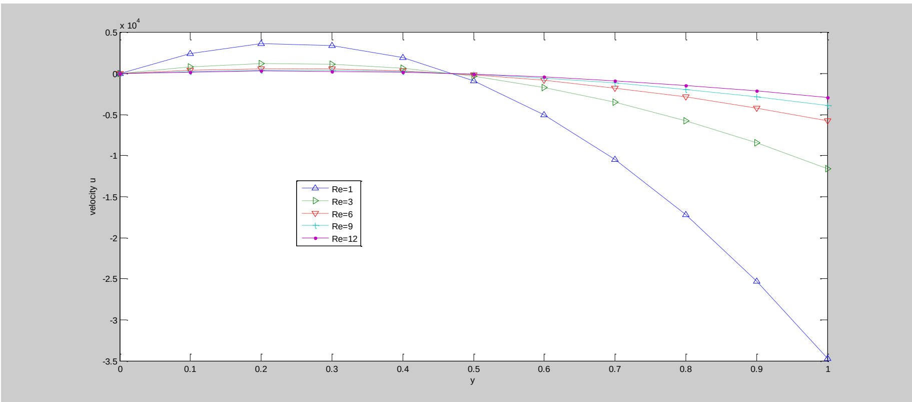

Fig 5: Effect of Reynolds number on the velocity u in the case of asymmetric heating

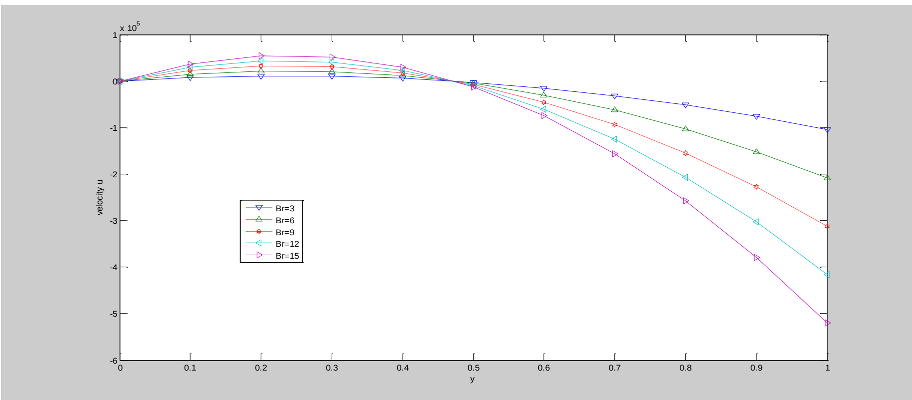

Fig 6: Effect of Brinkman number Br on velocity u in the case of asymmetric heating for $\varepsilon > 0$

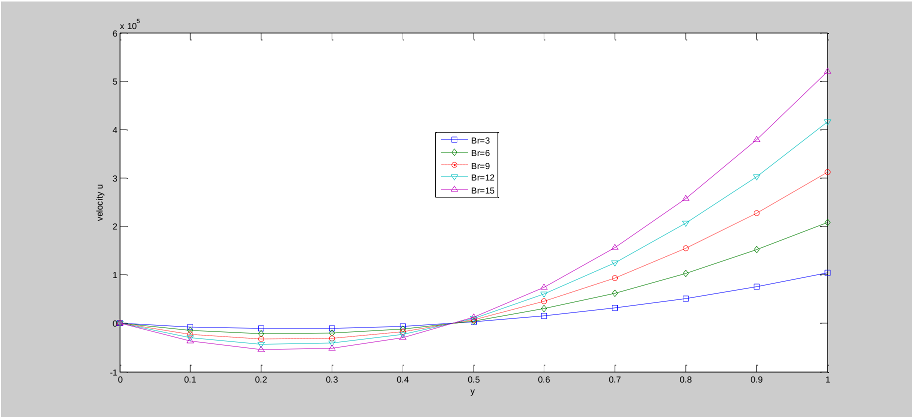

Fig 7: Effect of Brinkman number Br on velocity u in the case of asymmetric heating for $\varepsilon < 0$

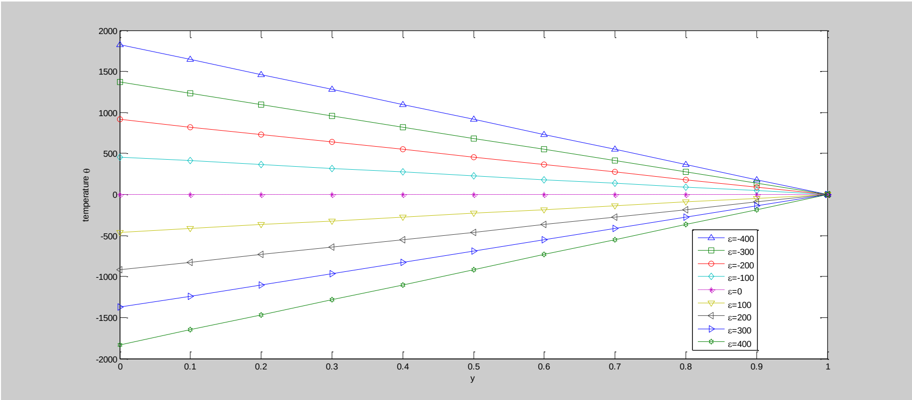

Fig 8: Plots of $\theta(y)$ in the case of asymmetric heating, for different values of ε and for $Br = 0$ and $Re = 1$

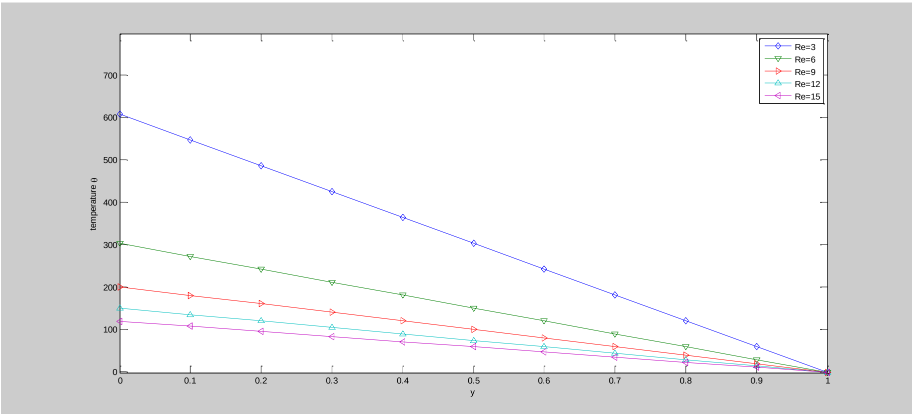

Fig 9: Effect of Re on temperature profile for $\varepsilon < 0$

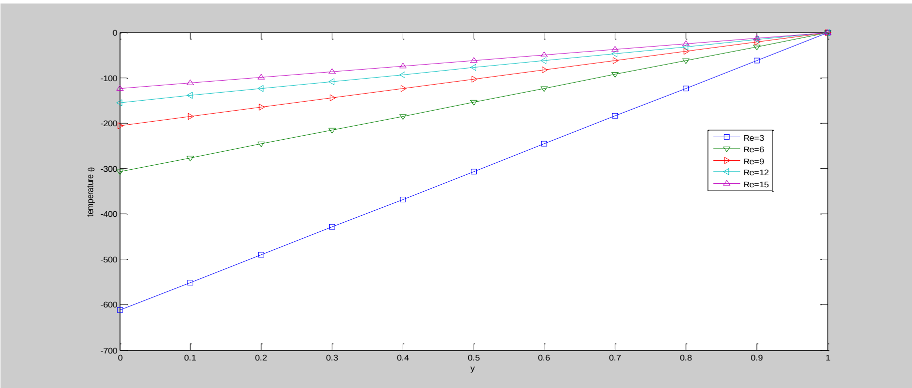

Fig 10: Effect of Re on temperature profile for $\varepsilon > 0$

5 Summary and conclusions

Viscous dissipation and Buoyancy effects on laminar convection in a vertical plate channel with transpiration have been studied. The wall temperatures are uniform and asymmetric. The mass flow rate has been considered as prescribed and the bulk temperature has been chosen as reference fluid temperature in the Boussinesq approximation.
The governing equations (the momentum balance equation and the temperature balance equation) have been written in a dimensionless form.
A perturbation method has been employed to evaluate the dimensionless velocity$u$, the dimensionless temperatureθ, the Nusselt numbersNu_0 and Nu_1, the pressure drop parameter γ and the temperature difference ratio R. The solution of u and θ has been determined by the perturbation parameter $= \frac{Gr}{Re}$. Moreover, 2 – terms perturbation series has been used to obtain the values ofγ,R Nu_0 and Nu_1 as well as to plot the functions $u(y)and$ $\theta(y)$ for some values ofε.
It has been pointed out that the viscous dissipation and the buoyancy effects enhance the flow in the case of downward flow, while it lowers this effect in the case of upward flow.

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