Performance Analysis of CO₂/NH₃ Cascade Refrigeration System Using ANNs

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Abstract

In this study, artificial neural networks (ANNs) have been used for performance analysis of a CO₂/NH₃ cascade refrigeration system using a data set, obtained from a thermodynamic model implemented in EES. Thermodynamic parameters of the system are estimated in terms of condensing temperature of ammonia, evaporating temperature of carbon dioxide, condensing temperature of carbon dioxide and temperature difference in cascade condenser. The computer program has been performed under MATLAB environment using neural network toolbox. New formulation obtained from ANN for this couple of refrigerants is presented for the calculation of target values. The total R value obtained when unknown data were used to the networks was 0.999992 which is very satisfactory. It can be used where a very accurate and fast estimation, simulation or optimization of the system performance is of interest to engineers.

Keywords: Ammonia, Carbon dioxide, Cascade refrigeration system, Neural Networks

1 Introduction

A great number of cases in industrial refrigeration systems, deal with low temperature applications such as storage of frozen food, liquefaction of petroleum vapor, manufacturing of dry ice, and rapid freezing systems, where an evaporation temperature between -30 °C and -55 °C is of interest to designers and engineers.
Since there is a high temperature difference between the heat source and heat sink, ordinary single stage vapor compression refrigeration systems are neither feasible nor economical to be utilized in these cases [1], due to the solidification temperature, low operating pressure of the refrigerant and difficulty of operating compressors which are expected to compress the refrigerants with extremely large specific volumes [2]. Cascade refrigeration systems divide the compression process into two separated steps, use two different refrigerants through two circuits linked together thermally via a heat exchanger.

Environmental problems and harmful effects of fluorocarbons on ozone depletion have led the manufacturers to replace them with natural refrigerants such as ammonia and carbon dioxide [1]. CO₂/NH₃ cascade refrigeration system uses ammonia in high temperature cycle and carbon dioxide in low temperature cycle. Ammonia is toxic with a pungent smell and flammable to some extent. It can not be used in low temperature cycle, because below -35 °C, it has a vapor pressure lower than atmosphere pressure which may cause air leakage into the system [3]. On the contrary, carbon dioxide is neither toxic nor flammable. Having a positive vapor pressure at temperatures below -35 °C, it is a suitable choice for low temperature cascade cycle [4].

Up to this time, CO₂/NH₃ cascade refrigeration system has been studied with various methods and viewpoints via experimental and numerical investigations. Bingming and Alberto Dopazo et al. [5, 6] carried out a series of experiments to analyze the performance of CO₂/NH₃ cascade refrigeration systems. They investigated the effect of some operation parameters on the system performance and also compared the performance of cascade system with CO₂/NH₃ with that of two-stage NH₃ system and single-stage NH₃ system with or without economizer. Rezayan et al. [1] optimized the system with respect to a set of system parameters, considering the total annual cost of the system, including costs of input exergy and annualized capital cost of the system. Lee et al. [4] thermodynamically analyzed the system, to determine the optimal condensing temperature of the cascade-condenser given various design parameters, to maximize the COP and minimize the exergy destruction. Getu and Bansal [7] presented a thermodynamic analysis of CO₂/NH₃ cascade refrigeration system to optimize a set of design and operating parameters of the system. In their work, a multilinear regression analysis was employed in terms of subcooling, superheating, evaporating, condensing, and cascade heat exchanger temperature difference in order to develop mathematical expressions for maximum COP, an optimum evaporating temperature of ammonia and an optimum mass flow ratio of ammonia to that of carbon dioxide in the cascade system.

Utilization of artificial neural networks with modeling and prediction aims particularly in energy systems is becoming increasingly popular in the last twenty years. Ertunc et al. [8] applied ANN approach to predict the performance of a refrigeration system with an evaporative condenser. Using a data set obtained from steady-state test runs of an experiment, they showed that refrigeration systems, even complex ones, can alternatively be modeled using ANNs within a
high degree of accuracy. Neural networks method has been applied to a variable speed vapor compression refrigeration system by Navarro-Esbría et al. [9]. They accurately predicted the performance of the system with low cost data requirement in terms of input variables and training data. Sencan [10] presented a new formulation for the analysis of ammonia-water absorption refrigeration system using artificial neural networks. The same author used artificial neural networks (ANNs) and adaptive neuro-fuzzy (ANFIS) for performance analysis of single-stage vapor compression refrigeration system with internal heat exchanger using refrigerants R134a, R404a, R407c. He inferred a new formulation obtained from ANN for the calculation of the COP [11].

From the previous studies, it can be seen that although artificial neural networks algorithm has been employed to model a limited number of single stage vapor compression refrigeration systems, it has not been applied to a CO$_2$/NH$_3$ cascade refrigeration system yet. In spite of the fact that CO$_2$/NH$_3$ cascade refrigeration system has been studied through various methods and viewpoints, a general explicit expression, capable for fast and accurately estimating the system parameters and easily be imported in different programming languages or spreadsheet programs has not been extracted so far.

In the present work, at first, a thermodynamic model is developed. By implementation and running the model in EES, the required data pattern in order to train and test a neural network is generated. Weight and bias matrixes will be presented to develop an explicit mathematical expression for a set of dependent variables such as COP, exergetic efficiency, exergy destruction, mass flow rate of both HTC and LTC circuits in terms of four independent variable temperatures: condensing temperature of ammonia, evaporating temperature of carbon dioxide, condensing temperature of carbon dioxide and temperature difference in cascade condenser.

## 2 Thermodynamic Model

A schematic diagram of cascade refrigeration system is shown in Fig. 1. The system consists of high temperature circuit (HTC) with ammonia, and low temperature circuit (LTC) with carbon dioxide as the refrigerant. These two circuits are thermally coupled via a heat exchanger called cascade condenser. This part of the system plays the role of condenser for LTC and evaporator for HTC. The LTC evaporator which has the temperature $T_E$ is exposed to the cold space which has the temperature $T_{CL}$ and absorbs the cooling load $Q_L$. HTC condenser, rejects the heat $Q_H$ at the condensing temperature $T_C$ to the ambient which has the temperature $T_0$. 
Evaporation of ammonia and condensation of carbon dioxide occur in cascade condenser at the temperatures $T_{ME}$ and $T_{MC}$ respectively. For convenience, cascade condenser temperature difference, defined by $\Delta T_{cas}=T_{MC}-T_{ME}$ is sometimes utilized instead of $T_{ME}$ or $T_{MC}$. T-s and P-h diagrams of the cycle are shown in Fig. 2 and Fig. 3 respectively. Energy and exergy balance equations for each components yield:

For evaporator:

$$\dot{Q}_L = \dot{m}_L (h_1 - h_4),$$  \hspace{1cm} (1)  

$$\dot{E}_{X,\text{evap}} = (1-\frac{T_0}{T_{CL}})\dot{Q}_L + \dot{m}_L (ex_4 - ex_1).$$  \hspace{1cm} (2)

For LTC compressor:

$$W_{\text{LTC, comp}} = \frac{\dot{m}_L (h_{3S} - h_1)}{\eta_s \eta_m \eta_e} = \frac{\dot{m}_L (h_2 - h_1)}{\eta_m \eta_e},$$  \hspace{1cm} (3)  

$$\dot{E}_{X,\text{LTC, comp}} = \dot{m}_L (ex_1 - ex_2).$$  \hspace{1cm} (4)

For LTC expansion valve:

$$h_3 = h_4,$$  \hspace{1cm} (5)  

$$\dot{E}_{X,\text{LTC, exp}} = \dot{m}_L (ex_3 - ex_4).$$  \hspace{1cm} (6)

For cascade condenser:
Fig. 2. T-s diagram of CO$_2$/NH$_3$ cascade refrigeration system.

Fig. 3. P-h diagram of CO$_2$/NH$_3$ cascade refrigeration system.
\[
\hat{Q}_m = \dot{m}_H (h_5 - h_8) = \dot{m}_L (h_2 - h_3) ,
\]
\[
\dot{E}_D_{\text{ex}, \text{cond}} = \dot{m}_L (ex_2 - ex_3) + \dot{m}_H (ex_8 - ex_5).
\]

For HTC compressor:
\[
W_{\text{HTC Comp}} = \frac{\dot{m}_H (h_{6S} - h_5)}{\eta_s \eta_m \eta_r}, \quad (9)
\]
\[
\dot{E}_D_{\text{HTC Comp}} = \dot{m}_H (ex_5 - ex_6).
\]

For HTC expansion valve:
\[
h_7 = h_8, \quad (11)
\]
\[
\dot{E}_D_{\text{HTC exp}} = \dot{m}_H (ex_7 - ex_8).
\]

For condenser:
\[
\hat{Q}_H = \dot{m}_H (h_7 - h_6),
\]
\[
\dot{E}_D_{\text{cond}} = (1 - \frac{T_o}{T_c}) \hat{Q}_L + \dot{m}_H (ex_6 - ex_7),
\]

where in equations (3) and (9) \(\eta_s \eta_r\) is the combined mechanical and compressor motor efficiency and is equal to 0.93 and \(\eta_r\), the isentropic efficiency, for screw compressors is determined using the following equations.

For the HTC compressor [12]:
\[
\eta_s = -0.00097r_p^2 - 0.01026r_p + 0.83955.
\]

For the LTC compressor [13]:
\[
\eta_s = 0.00476r_p^2 - 0.09238r_p + 0.89810.
\]

Power of condenser and evaporator fans can be neglected due to their small values in comparison to HTC and LTC compressors. The total exergy input to the system is equal to:
\[
\dot{E}_{\text{in}} = W_{\text{HTC Comp}} + W_{\text{LTC Comp}}.
\]

Exergy at each point of the cycle is defined by:
\[
ex_n = (h_n - h_0) - T_0 (s_n - s_0),
\]
where \(n = \{1, 2, 3, 4, 5, 6, 7, 8\}\)

And the total exergy output, or exergy product is:
\[
\dot{E}_x_{\text{out}} = \dot{Q}_L \left( \frac{T_{in}}{T_{CL}} - 1 \right).
\]  

The value of exergy destruction and exergetic efficiency of the overall system can be obtained by [14]:

\[
\dot{E}_x_{D,\text{tot}} = \dot{E}_x_{in} - \dot{E}_x_{\text{out}},
\]

\[
\varepsilon = 1 - \frac{\dot{E}_x_{D,\text{tot}}}{\dot{E}_x_{in}}.
\]

In order to simulate the cycle, the system of equations, consisting of the equations (1) to (21) was given to EES and solved simultaneously for a set of predefined values for condensing temperature of ammonia, evaporating temperature of carbon dioxide, condensing temperature of carbon dioxide and temperature difference in cascade condenser using a parametric table. The range of variable values used is presented in the Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Condensing temperature of ammonia (Tc), °C</td>
<td>40-64</td>
</tr>
<tr>
<td>Evaporating temperature of carbon dioxide (T_E), °C</td>
<td>-56 to -47</td>
</tr>
<tr>
<td>Condensing temperature of carbon dioxide (T_MC), °C</td>
<td>-11 to 1</td>
</tr>
<tr>
<td>Temperature difference in cascade condenser (ΔT_cas), °C</td>
<td>2-12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>COP</th>
<th>Exergetic efficiency (ε)</th>
<th>Exergy destruction (Ex_D), kW</th>
<th>Power consumption of HTC compressor (kW)</th>
<th>Power consumption of LTC compressor (kW)</th>
<th>Mass flow rate of HTC (m_H), kg/s</th>
<th>Mass flow rate of LTC (m_L), kg/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>COP</td>
<td>0.363 – 1.208</td>
<td>0.1114 – 0.3709</td>
<td>0.5207 – 2.448</td>
<td>0.3731 – 2.22</td>
<td>0.3426 – 0.8631</td>
<td>0.001248 – 0.001925</td>
<td>0.003856 – 0.004383</td>
</tr>
</tbody>
</table>

3 Artificial Neural Networks (ANNs)

Neural networks have been made up of simple elements that operate in parallel, inspired by biological nervous systems. There are several connections, so called “weights” between elements whose values tells the network how to perform our desired function. The process in which those values are adjusted is called “training”. Training a network causes a particular input to be led to a specific target output, it compares the output and target until the network output matches the target. Depending upon the complexity, sometimes it is necessary to have a large number of input/target pairs to train a network. ANNs differ from the traditional modeling approaches in that they are trained to learn solutions rather than being programmed to model a specific problem in the normal way. They are
usually used to address problems that are intractable or cumbersome to solve with traditional methods [15-17]. They can also be used as an alternative approach to obtain a simple mathematical correlation between input and output values with high accuracy.

4 ANN Development

In this work, a data set, composed of 5460 pairs of input and target values, obtained from thermodynamic analysis of the system was given to train and test the network. There were four inputs and seven outputs for the system. The input parameters are condensing temperature of ammonia, evaporating temperature of carbon dioxide, condensing temperature of carbon dioxide and temperature difference in cascade condenser. The output parameters were COP values, exergetic efficiency, exergy destruction, and mass flow rate of both HTC and LTC circuits. Henceforth, the input and target values are presented in matrix form as follows. The matrices X and Y are composed of the input and target values respectively.

\[
X = \begin{bmatrix} T_C & T_E & T_{MC} & \Delta T_{cas} \end{bmatrix}^T,
\]

\[
Y = \begin{bmatrix} \text{COP} & \varepsilon & \dot{E}_{X_{D, tot}} & \dot{W}_{HTC, Comp} & \dot{W}_{LTC, Comp} & \dot{m}_{HTC} & \dot{m}_{LTC} \end{bmatrix}^T.
\]

Neglecting the values of the fan power consumption, the main target values, COP and exergetic efficiency do not depend upon the cooling capacity. Hence, for convenience, the system has been analyzed in terms of a unit refrigeration capacity of 1 kW. It must be noted that exergy destruction and compressor power calculated values, are all a unit index and must be multiplied by the magnitude of actual cooling capacity in terms of kW in order to obtain the correct results. The range of input and output values are given in Table 1. In Fig. 4 the selected ANN structure was shown. It consists of an input layer, a hidden layer and an output layer. As shown in the figure, the number of input and output layers is four and seven respectively.

![Fig. 4. The selected ANN structure.](image-url)
Feed-forward backpropagation was used for learning algorithm with one hidden layer. The training function selected for the algorithm was Levenberg-Marquardt (LM). Adaptation learning function was selected to be gradient descent with momentum weight and bias learning function (LEARNGDM) which was standard for the network. A schematic diagram of the network is shown in Fig. 5.

![Network Diagram](image)

Fig. 5. Block diagram of the network in detail.

### 5 Main Results

In order to estimate the target values, the number of neurons in hidden layer varies in the range of 1-8 for determining the best approach. The best approach, which has minimum errors, is obtained from a network with 8 hidden neurons. Table 2 compares the accuracy of ANN model based on different number of hidden neurons in terms of the root absolute fraction of variance (R) and mean square error (MSE) values according to the equations given below:

<table>
<thead>
<tr>
<th>Number of the hidden neurons</th>
<th>MSE</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0038309900</td>
<td>0.924959000</td>
</tr>
<tr>
<td>2</td>
<td>0.0003182170</td>
<td>0.979573000</td>
</tr>
<tr>
<td>3</td>
<td>0.0000882414</td>
<td>0.989142000</td>
</tr>
<tr>
<td>4</td>
<td>0.0000049019</td>
<td>0.999497000</td>
</tr>
<tr>
<td>5</td>
<td>0.0000025572</td>
<td>0.999756000</td>
</tr>
<tr>
<td>6</td>
<td>0.0000007296</td>
<td>0.999960000</td>
</tr>
<tr>
<td>7</td>
<td>0.0000002542</td>
<td>0.999981000</td>
</tr>
<tr>
<td>8</td>
<td>0.0000000910</td>
<td>0.999992000</td>
</tr>
</tbody>
</table>
where $o$ is the output value, $t$ is the target value, and $N$ is the number of patterns [18]. Fig. 6 which was obtained from a network with 8 hidden neurons, shows the decrease of the MSE at each epoch during the training process.

Mathematical formulation in matrix form, obtained from ANN model is presented here. According to the structure of the ANN model, normalized target parameters including the COP values, exergetic efficiency, exergy destruction, power consumption and mass flow rate of HTC & LTC are all defined by the following equation:

$$
[Y_{norm}] = [W_I] \times (TANSIG([W_I] \times [X_{norm}] + [B_I])) + [B_{II}],
$$

(26)

where the $W_I$ and $W_{II}$ are the weight matrices of the layers. The $B_I$ and $B_{II}$ present the bias matrixes which must be added to the product of each layer corresponding to ANN methodology, illustrated in Fig. 5. Both weight and bias matrixes obtained from the trained network are presented in Appendix A. The matrix $X_{norm}$ contains the normalized form of the input parameters. Input data normalization should be done using a simple linear correlation according to the following equation in terms of the actual, minimum and maximum values listed in Table 1. It will map the actual value of each element onto a normal value between -1 and 1.
\[ X_{\text{norm}} = \frac{2(X - X_{\text{min}})}{X_{\text{max}} - X_{\text{min}}} - 1. \] (27)

where \( X \) is a general notation, refers to all the input matrix elements including \( T_C \), \( T_E \), \( T_{MC} \) and \( \Delta T_{\text{cas}} \). A hyperbolic tangent sigmoid transfer function (TANSIG) used in the first layer of ANN model. This function is defined as follows:

\[ TANSIG(a) = \tanh(a) = \frac{e^a - e^{-a}}{e^a + e^{-a}}. \] (28)

Elements of the matrix \( Y_{\text{norm}} \) are all in normal form and must be converted back to their actual values. This procedure requires another linear correlation in terms of the normal, maximum and minimum calculated values listed in Table 1.

\[ Y = \frac{(Y_{\text{norm}} + 1)(Y_{\text{max}} - Y_{\text{min}})}{2} + Y_{\text{min}}. \] (29)

In order to verify the results of this formulation and investigate the validity of the weight and bias matrixes, a set of input temperatures used for estimating the target values. Comparison of the actual and predicted values is shown in a set of diagrams shown in Fig. 7.
(b) Predicted exergetic efficiency vs. Actual exergetic efficiency

\[ R^2 = 0.999998947 \]

(c) Predicted exergy destruction (kW) vs. Actual exergy destruction (kW)

\[ R^2 = 0.999999463 \]
CO$_2$/NH$_3$ Cascade System ANN

\( R^2 = 0.99999902 \)

\( R^2 = 0.9999998829898840 \)

(d)

(e)
Fig. 7. COP vs. actual values of output parameters.

Fig. 8 compares the COP values obtained from the thermodynamic model, the COP values predicted by ANN method and the COP values measured in an experimental study done by Yabusita et. al. [19]. These results appear to be in good agreement with each other.
Target data matrix, whose elements are COP values, exergetic efficiency, exergy destruction, power consumption and mass flow rate of HTC & LTC cycles of a CO\textsubscript{2}/NH\textsubscript{3} cascade refrigeration system was estimated depending upon the input data matrix including condensing temperature of ammonia, evaporating temperature of carbon dioxide, condensing temperature of carbon dioxide and temperature difference in cascade condenser using ANNs. The R value for abovementioned parameters is 0.999992 which can be considered very satisfactory. From ANN model an explicit mathematical formulation was derived in matrix form. This correlates the matrix of target data to the matrix of input data using formulations of the ANN model via transfer functions, mapping procedures for normalizing the values, weights of neurons and bias matrixes. The values estimated by ANN formulations were found to be in good agreement with the values obtained form the thermodynamic model and experiment. Taking into account the assumptions applied to the analysis, this formulation, as an alternative method, can be easily implemented in all programming languages for the aims of simulation or optimization. It can also be used where a very accurate and fast estimation of system performance is of interest to engineers.

\textbf{References}

[1] O. Rezayan, A. Behbahaninia, Thermoeconomic optimization and exergy analysis of CO\textsubscript{2}/NH\textsubscript{3} cascade refrigeration systems, Energy, 36 (2011) 888-895.


Appendix A. Weight and Bias Matrixes Obtained from ANN Training Process Pertaining to the First and Second Layers.

\[ W_f = \begin{bmatrix} 0.065826 & 1.1038 & -0.084829 & -0.020896 \\ 0.61731 & -0.041128 & -0.48471 & 0.44689 \\ 0.11178 & -0.16489 & 0.1519 & 0.03041 \\ 0.10615 & -0.20187 & 0.15412 & 0.060076 \\ -0.00034338 & 0.12085 & -0.13562 & -0.00023162 \\ -0.12761 & 0.05723 & 0.012276 & -0.063339 \\ -0.045197 & -0.17771 & 0.28691 & -0.067987 \\ -1.3412 & 0.047576 & 1.2184 & -1.0564 \end{bmatrix} \]

\[ B_f = \begin{bmatrix} -0.86516 \\ -0.38385 \\ 1.1846 \\ 0.33917 \end{bmatrix} \]

\[ W_I = \begin{bmatrix} -0.01317 & -0.28922 & -5.2772 & -0.65291 & 0.80925 & 0.8425 & 3.8971 & 0.31418 \\ -0.013206 & -0.28907 & -5.2723 & -0.65353 & 0.80897 & 0.84346 & 3.8952 & 0.31605 \\ 0.0080187 & 0.85372 & 0.45193 & -0.18945 & -2.393 & -4.4452 & -0.35637 & -2.7688 \\ 0.0031803 & 0.88795 & 0.48513 & -0.20724 & 0.20763 & -4.6598 & -0.38417 & -2.8949 \\ 0.018452 & 0.011741 & -0.051671 & 0.036221 & -9.5929 & 0.078149 & 0.044944 & 0.024326 \\ 0.11883 & 0.053839 & 0.35037 & 2.0963 & -0.36133 & 0.34697 & 0.080193 & -0.53813 \\ 0.60771 & -1.157 & 0.45422 & 1.9521 & -2.2032 & -0.12935 & 1.1707 & -0.61424 \end{bmatrix} \]

\[ B_I = \begin{bmatrix} -0.63029 \\ -0.63462 \\ 8.4648 \\ 6.6957 \\ 7.1615 \\ -0.61433 \\ -0.42102 \end{bmatrix} \]