

Graphical representation of a real valued function in the interval where it gives imaginary value

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Abstract

We are introduced to the graph of a real valued function $y = f(x)$. But when values of $f(x)$ are imaginary numbers at some points of real axis, we have no graphical representation of the function at those points. This article demonstrates a model where we construct a 3D graph of the function at those points where values of the function are imaginary numbers.

Keywords: Complex Plane; Graph of a Function; Imaginary Number; Real Valued Function; Three-Dimensional Rectangular Coordinate System.

1. Introduction

Graph of function is very important to understand the behaviour of a function. Graphs can provide valuable visual information about a function. Everywhere graph of function is used. In mathematics graph of function is widely used to describe others mathematical phenomenon. If f is a real-valued function of a real variable, then the graph of f in the xy -plane is defined to be the graph of the equation $y = f(x)$.^[1] If value of a function at some points is imaginary number, then the graph of the equation $y = f(x)$ doesn't show this imaginary part. In this article we are trying to have a visual presentation of the part of a function where it gives imaginary number. As example, $\ln x$, $\sqrt{x}$ functions have no graph on the negative x axis, since $\ln x$, $\sqrt{x}$ functions give imaginary number when x is a negative number. Here, we represent graphs of $\ln x$, $\sqrt{x}$ and $\sec^{-1} x$ functions on the interval where these functions give imaginary value along with usual graph.

2. Materials and methods

At first, we take the three-dimensional rectangular coordinate system to describe this model. At the point $x = x_0$ where value of the function is a complex number, at that point we consider a complex plane (we may denote this complex plane by $CP_{x=x_0}$) that is perpendicular to the x -axis. Real axis of the complex plane $CP_{x=x_0}$ is the straight line $x = x_0$ in the xy plane that is parallel to the y axis and the imaginary axis of the complex plane $CP_{x=x_0}$ is the perpendicular line to the xy plane at the point $(x_0, 0, 0)$ that is parallel to the z -axis. Part of the real axis of the complex plane $CP_{x=x_0}$ in the side of the positive y -axis is considered as positive real axis and part of the real axis of the complex plane $CP_{x=x_0}$ in the side of the negative y -axis is considered as negative real axis. Positive and negative imaginary axes of the complex plane $CP_{x=x_0}$ are determined same as z -axis. Origin of the complex plane $CP_{x=x_0}$ is on the x -axis at the point $(x_0, 0, 0)$. If $f(x_0) = u + iv$, then the point (x_0, u, v) is a point on the graph of the function. The graph created by combining all these points may be called imaginary part of the graph of the function $f(x)$ and the as usual graph may be called real part of the graph. Real part of the graph of a function always stays in the xy plane. But imaginary part always stays outside of the xy plane.

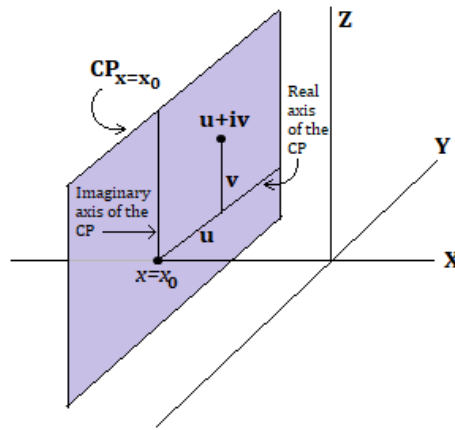


Fig. 1: Complex Plane $CP_{x=x_0}$ at $X = X_0$.

3. Results and discussions

3.1. Graph of $\ln x$ function

We know that, ^[2]

$$\ln x = \ln |x| + i\pi = u + iv \quad x \in (-\infty, 0)$$

Therefore, $u = \ln |x|$ and $v = \pi$

Parametric equations of the graph of $\ln x$ function:

Parametric equations of the real part:

$$x(t) = t; \quad y(t) = \ln t; \quad z(t) = 0; \quad t \in (0, \infty)$$

Real part is created by combining all the points of the set: $\{(t, \ln t, 0) \mid t \in (0, \infty)\}$.

Parametric equations of imaginary part:

$$x(t) = t; \quad y(t) = \ln |t|; \quad z(t) = \pi; \quad t \in (-\infty, 0)$$

Imaginary part is created by combining all the points of the set: $\{(t, \ln |t|, \pi) \mid t \in (-\infty, 0)\}$.

The following graph of $\ln x$ function is generated by Mathematica. The code of Mathematica is given below-

```
a = ParametricPlot3D[{t, Log[t], 0}, {t, 0, 5}, PlotStyle -> Blue];
```

```
b = ParametricPlot3D[{t, Log[Abs[-t]], π}, {t, -5, 0}, PlotStyle -> Red];
```

```
Show[{a, b}, PlotRange -> {{-5, 5}, {-5, 3}, {0, 5}}, Axes -> True, AxesOrigin -> {0, 0, 0}, Ticks -> {{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5}, {-5, -4, -3, -2, -1, 1, 2, 3}, {1, 2, 3, 4, 5}}, TicksStyle -> Directive[Black, 20]]
```

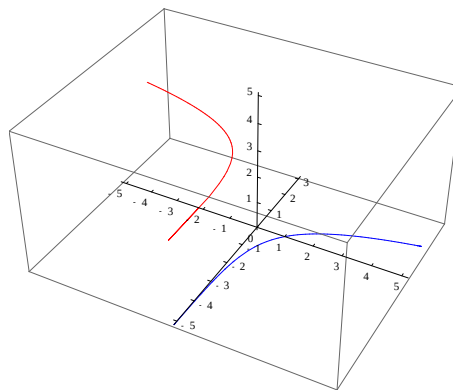


Fig. 2: Graph of $\ln x$ Function.

Blue curve is the real part of the graph of the $\ln x$ function where outputs of $\ln x$ are real numbers. The red one is the imaginary part of the graph of $\ln x$ function where outputs of $\ln x$ are imaginary numbers. Axis with ticks from -5 to 5 is the x -axis, axis with ticks from -5 to 3 is the y -axis and axis with ticks from 0 to 5 is the z -axis. Imaginary part lies in the plane $z = \pi$.

Other views of the graph of $\ln x$ function:

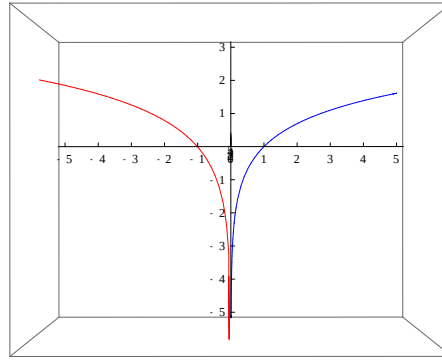


Fig. 3: Top View of the Graph of $\ln x$ Function.

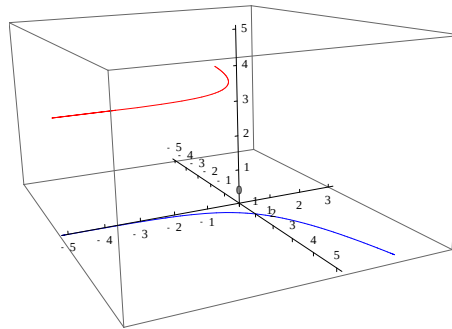


Fig. 4: Side View of the Graph of $\ln x$ Function.

3.2. Graph of $\sqrt{x}$ function

$$\sqrt{x} = i\sqrt{|x|} = u + iv \quad x \in (-\infty, 0)$$

Therefore, $u = 0$ and $v = \sqrt{|x|}$.

Parametric equations of the graph of $\sqrt{x}$ function:
Parametric equations of the real part:

$$x(t) = t; \quad y(t) = \sqrt{t}; \quad z(t) = 0; \quad t \in [0, \infty)$$

Real part is created by combining all the points of the set: $\{(t, \sqrt{t}, 0) \mid t \in [0, \infty)\}$.

Parametric equations of imaginary part:

$$x(t) = t; \quad y(t) = 0; \quad z(t) = \sqrt{|t|}; \quad t \in (-\infty, 0)$$

Imaginary part is created by combining all the points of the set: $\{(t, 0, \sqrt{|t|}) \mid t \in (-\infty, 0)\}$.

The following graph of $\sqrt{x}$ function is generated by Mathematica. The code of Mathematica is given below-

```
a = ParametricPlot3D[{t, Sqrt[t], 0}, {t, 0, 5}, PlotStyle -> Blue];
```

```
b = ParametricPlot3D[{t, 0, Sqrt[Abs[t]]}, {t, 0, -5}, PlotStyle -> Red];
```

```
Show[{a, b}, PlotRange -> {{-5, 5}, {-5, 3}, {0, 5}}, Axes -> True, AxesOrigin -> {0, 0, 0}, Ticks -> {{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5}, {-5, -4, -3, -2, -1, 1, 2, 3}, {1, 2, 3, 4, 5}}, TicksStyle -> Directive[Black, 20]]
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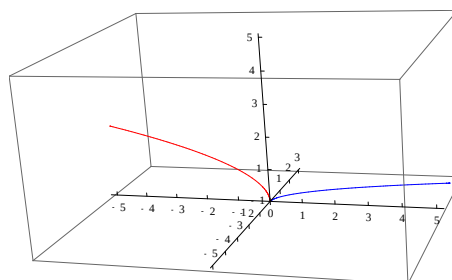


Fig. 5: Graph of $\sqrt{x}$ Function.

Blue curve is the real part of the graph of the $\sqrt{x}$ function where outputs of $\sqrt{x}$ are real numbers. The red one is the imaginary part of the graph of $\sqrt{x}$ function where outputs of $\sqrt{x}$ are imaginary numbers. Axis with ticks from -5 to 5 is the x-axis, axis with ticks from -5 to 3 is the y-axis and axis with ticks from 0 to 5 is the z-axis. Imaginary part lies in the plane $y = 0$.

Other views of the graph of $\sqrt{x}$ function:

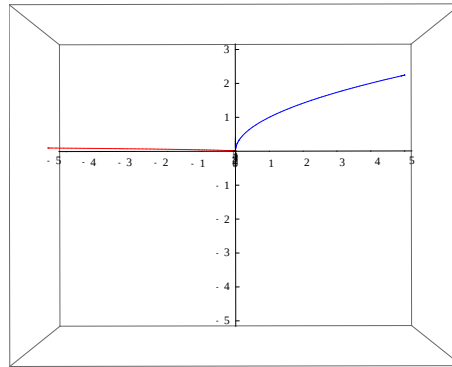


Fig. 6: Top View of the Graph of $\sqrt{x}$ Function.

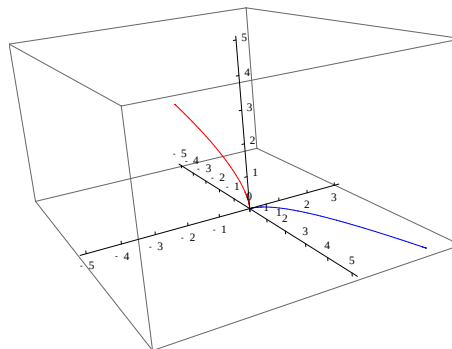


Fig. 7: Side View of the Graph of $\sqrt{x}$ Function.

3.3. Graph of $\sec^{-1} x$ function

Actually, this function is the source of my interest. Since, it has no graph in the interval $(-1,1)$. We know that, for $x \in (0,1)$

if $\sec^{-1} x = ai$ then $\sec^{-1}(-x) = \pi - ai$.

Now, using the proposed model graph of $\sec^{-1} x$ function is generated by Mathematica which is given below along with code-

```
a = ParametricPlot3D[{t, ArcSec[t], 0}, {t, -5, 5}, PlotStyle -> Blue];
```

```
b = ParametricPlot3D[{t, 0, Im[ArcSec[t]]}, {t, 0, 1}, PlotStyle -> Red];
```

```
c = ParametricPlot3D[{t, pi, Im[ArcSec[t]]}, {t, -1, 0}, PlotStyle -> Red];
```

```
Show[{a, b, c}, PlotRange -> {{-5, 5}, {-5, 5}, {-5, 5}}, Axes -> True, AxesOrigin -> {0, 0, 0}, Ticks -> {{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5}, {-5, -4, -3, -2, -1, 1, 2, 3}, {1, 2, 3, 4, 5}}, TicksStyle -> Directive[Black, 20]]
```

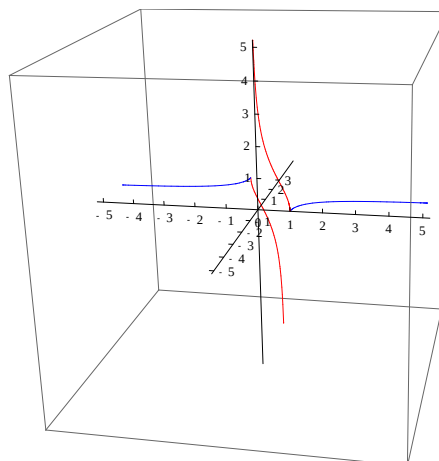


Fig. 8: Graph of $\sec^{-1} x$ Function.

Blue curve is the real part of the graph of the $\sec^{-1} x$ function where outputs of $\sec^{-1} x$ are real numbers. The red one is the imaginary part of the graph of $\sec^{-1} x$ function where outputs of $\sec^{-1} x$ are imaginary numbers. Axis with ticks from -5 to 5 is the x-axis, axis with ticks from -5 to 3 is the y-axis and axis with ticks from 0 to 5 is the z-axis. Imaginary part in the interval $(0,1)$ lies in the plane $y = 0$ and imaginary part in the interval $(-1,0)$ lies in the plane $y = \pi$. From the graph we can see that when x tends to 0 from right hand side, then value of the function tends to $+\infty$ along with positive z axis in the plane $y = 0$ and when x tends to 0 from left hand side, then value of the function tends to $-\infty$ along with negative z axis in the plane $y = \pi$.

Other views of the graph of $\sec^{-1} x$ function:

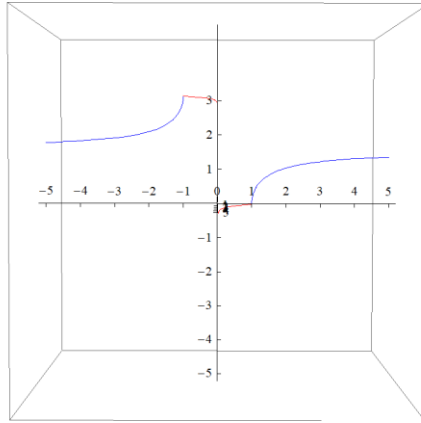


Fig. 9: Top View of the Graph of $\sec^{-1} x$ Function.

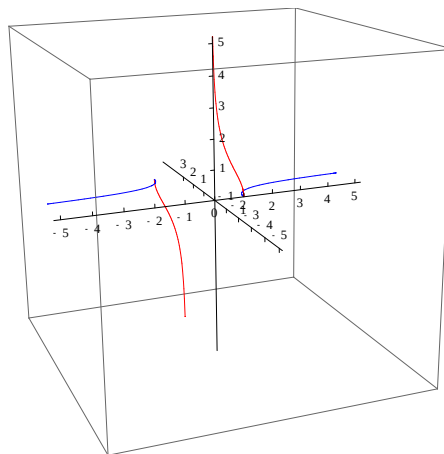


Fig. 10: Side View of the Graph of $\sec^{-1} x$ Function.

4. Conclusions

By this model we can create graph of any function with imaginary output at some points of x axis and can analysis behavior of the function in this portion. For the function $y = f(x)$, simply we can use parametric plot of the parametric equations-

$$x = t; \quad y = \operatorname{Re}(f(t)); \quad z = \operatorname{Im}(f(t)).$$

Acknowledgements

My thinking is started from seeing the graph of $\sec^{-1} x$ function. I see that there is no graph of this function from -1 to 1 . Why? Because, in this interval values of this function are imaginary numbers. Then a question is arisen in my mind that is there no way to represent this portion of this function graphically? At last, I find this technique by the grace of Almighty.

References

- [1] Howard Anton, Irl Bivens, Stephen Davis, *Calculus Early Transcendentals*, 10th Edition, John Wiley & Sons, Inc., New York, USA, 2012, page# 04.
- [2] James Ward Brown and Ruel V. Churchill, *Complex Variables and Applications*, 8th Edition, McGraw Hill, New York, USA, 2004, page# 93.