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Research paper

Analysis of the wave solutions of the nonlinear evolution equations help of advanced exp(-φ(ξ))-expansion scheme

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Abstract

With the assistance of emblematic calculation programming, the current paper explores the specific voyaging wave arrangements from the general (2+1)- layered nonlinear development conditions by utilizing the advanced exp(-φ(ξ))-expansion with time-partial boundaries. As a result, the used technique is effectively utilized and recently created some precise voyaging wave arrangements. The recently created arrangements have been communicated regarding mathematical and exaggerated capabilities. The created arrangements have been inquired into their comparing condition with the guide of the emblematic calculation programming Maple. The elements of nonlinear wave arrangements are analyzed and shown by maple18 in 3D, and 2D plots, and form plots with explicit upsides of the mind boggling boundaries are plotted. The advanced exp(-φ(ξ))-expansion strategy is a solid treatment for looking through fundamental nonlinear waves that enhance an assortment of water waves in the long-frequency system that emerges in designing fields.

Keywords: *Time Fractional Derivative; The Advanced Exp(-Φ(Ξ))-Expansion Method; Exact Solution; The General ( )-Dimensional Nonlinear Evolution Equation; Mathematical Physics.*

1. Introduction

As of late, nonlinear halfway differential conditions (NPDEs) is extensively used to portray various critical marvels and dynamic techniques in various areas of science and designing, particularly in liquid mechanics, hydrodynamics, numerical science, dispersion process, strong state physical science, plasma physical science, brain physical science, substance energy and geo-optical filaments.
In this work, we will study the summed up (2+ 1) layered nonlinear advancement conditions in the structure

$$u_{xt} + a u_x u_{xy} + b u_{xx} u_y + u_{xxxy} = 0 \tag{1}$$

Recently, some special cases of Eq. (1) have been studied by several authors [1-4]. When setting a =4 and b = 2, Eq. (1) becomes the (2+1)-dimensional Calogero–Bogoyavlenskii–Schiff (CBS) equation:

$$u_{xt} + 4u_x u_{xy} + 2u_{xx} u_y + u_{xxxy} = 0 \tag{2}$$

When setting a = – 4 and b = – 2, Eq. (1) becomes the (2+1)-dimensional breaking soliton equation

$$u_{xt} - 4u_x u_{xy} - 2u_{xx} u_y + u_{xxxy} = 0 \tag{3}$$

When setting a = 4 and b = 4, Eq. (1) becomes the (2+1)-dimensional Bogoyavlenskii’s breaking soliton equation:

$$u_{xt} + 4u_x u_{xy} + 4u_{xx} u_y + u_{xxxy} = 0 \tag{4}$$

Numerous researchers arranged through NEEs to develop voyaging wave arrangement by carry out a few techniques. The strategies that are deep rooted in late writing, for example, the drawn out Kudryashov strategy [5], the changed basic condition technique [6], the new broadened (G′/G) development strategy [7,8], the Darboux change [9], the preliminary arrangement technique [10], the Exp-Capability Strategy [11], the numerous least difficult condition technique [12], $\exp(-\phi(\xi))$ - extension strategy [13-17], Pseudo illustrative model [18-20], etc.

The objective of this article is to apply the high level $\exp(-\phi(\xi))$ - development technique [21] to construct the exact journeying wave deals with any consequences regarding nonlinear headway conditions in logical material science through the time partial nonlinear changed Kawahara conditions.
The article is set up as seeks after: In segment 2, the high level $\exp(-\phi(\xi))$ - extension procedure has been discussed. In segment 3, we apply this plan to the nonlinear advancement conditions raised previously. In segment 4, addresses Results and Conversation and In area 5, closes are given.

2. Preliminaries and methods

2.1. Definition and some features of conformable fractional derivative

The conformable fractional derivative with a limit operator which was initially introduced by Khalil et al. [22].

Definition: $f:(0,\infty)\to\mathbb{R}$, *then, the conformable fractional derivative of* f *order* α *is defined as*

$D_t^{\delta}f(t)=\lim_{\varepsilon\to}(\frac{f(t+\varepsilon t^{1-\delta})-f(t)}{\varepsilon})$ for all $t>0, 0<\delta\le 1$.

Later, Abdeljawad [23] has also offered chain rule, exponential functions, Gronwalls inequality, integration by parts, Taylor power series expansions and Laplace transform for conformable derivative in fractional versions. The definition of conformable fractional derivative can easily overcome the difficulties of exiting modified Riemann-Liouville derivative definition [24].

Theorem 1: *Let* $\delta\in(0,1]$, *and* $f=f(t), g=g(t)$ *be α-conformable differentiable at a point* $t>0$, *then:*

i) $D_t^{\delta}(cf+dg)=cD_t^{\delta}f+dD_t^{\delta}g$, for all $c,d\in\mathbb{R}$.
ii) $D_t^{\delta}(t\gamma)=\gamma t^{\gamma-\delta}$, for all $\gamma\in\mathbb{R}$.
iii) $D_t^{\delta}(fg)=gD_t^{\delta}(f)+fD_t^{\delta}(g)$.
iv) $D_t^{\delta}(f/g)=\frac{gD_t^{\delta}(f)-fD_t^{\delta}(g)}{g^2}$.

Furthermore, if f is differentiable, then $D_t^{\delta}\,(f(t))=t^{1-\delta}\frac{df}{dt}$.

Theorem 2: *Let* $f:(0,\propto)\to R$ *be a function such that* f *is differentiable and α-conformable differentiable. Also, let* g *be a differentiable function defined in the range of* f. *Then*

$D_t^{\delta}fog)(t)=t^{1-\delta}g(t)^{\delta-1}g'(t)D_t^{\delta}(f(t))_{t=g(t)}$.

Where, prime denotes the classical derivatives with respect to t.

2.2. The advanced $\exp(-\phi(\xi))$-expansion method

In this section, we will précis $\exp(-\phi(\xi))$-expansion method step by step. Consider a nonlinear partial differential equation in the following form,

$$R(U,U_{xx},U_{xz},U_{xx},U_{xy},U_{xtt},\ldots\ldots)=0 \tag{5}$$

Where $U=U(x,y,z,t)$ is an unknown function, R is a polynomial of U, its different type partial derivatives, in which the nonlinear terms and the highest order derivatives are involved.
Step-1. Now we consider a transformation variable to convert all independent variable into one variable, such as $U(x,t)=u(\xi)$,

$$\xi=kx+ly+mz\pm Vt. \tag{6}$$

By implementing this variable Eq. (6) permits us reducing Eq. (5) in an ODE for $u(x,t)=u(\xi)$

$$P(u,u',u'',\cdots\cdots\cdot)=0 \tag{7}$$

Step-2. Suppose that the solution of ODE Eq. (7) can be expressed by a polynomial in $\exp(-\phi(\xi))$ as follows

$$u=\sum_{i=0}^{m}A_i\exp(-\phi(\xi))^i \tag{8}$$

Where the derivative of $\phi(\xi)$ satisfies the ODE in the following form

$$\varphi'(\xi)=\lambda\exp(-\varphi(\xi))+\mu\exp(\varphi(\xi)), \tag{9}$$

Then the solutions of ODE Eq. (9) are

Case I:
Hyperbolic function solution (when $\lambda\mu<0$):

$$\varphi(\xi)=\ln\left(\sqrt{\frac{\lambda}{-\mu}}\tanh(\sqrt{-\lambda\mu}(\xi+C))\right)$$

And

$$\varphi(\xi)=\ln\left(\sqrt{\frac{\lambda}{-\mu}}\coth(\sqrt{-\lambda\mu}(\xi+C))\right)$$

Case II:
Trigonometric function solution (when $\lambda\mu>0$):

$$\varphi(\xi)=\ln\left(\sqrt{\frac{\lambda}{\mu}}\tan(\sqrt{\lambda\mu}(\xi+C))\right)$$

And

$$\varphi(\xi)=\ln\left(-\sqrt{\frac{\lambda}{\mu}}\cot(\sqrt{\lambda\mu}(\xi+C))\right)$$

Case III:
When $\mu>0$ and $\lambda=0$

$$\varphi(\xi)=\ln\left(\frac{1}{-\mu(\xi+C)}\right)$$

Case IV:
When $\mu=0$ and $\lambda\in\Re$

$$\varphi(\xi)=\ln(\lambda(\xi+C))$$

Where C is integrating constants and $\lambda\mu<0$ or $\lambda\mu>0$ depends on sign of μ.

Step-3. By substituting Eq. (8) into Eq.(7) and using the ODE (9), collecting all same order of $\exp(\varphi(\xi))$ together, then we execute an polynomial form of $\exp(\varphi(\xi))$. Equating each coefficients of this polynomial to zero,yields a set of algebraic system.

Step-4. Assume the estimation of the constants can be gotten by fathoming the mathematical conditions got in step 4. Substituting the estimations of the constants together with the arrangements of Eq. (9), we will acquire new and far reaching precise traveling wave arrangements of the nonlinear development Eq. (5).

3. Application of the method

In this sub section, we will exert the advanced $\exp(-\phi(\xi))$-expansion strategy to solve the equation (1). Now Using the traveling wave variable $\xi=x+y-\omega\frac{t^{\delta}}{\delta}$, and integrating with respect to ξ reduces Eq. (1) to the following ordinary differential equation for $u=u(\xi)$.

$$-\omega u'+\left(\frac{a+b}{2}\right)(u')^{2}+u'''=0, \tag{10}$$

Where, primes denote the differentiation with regard to ξ . By balancing u''' and $(u')^2$, we obtain $N=1$. Therefore the advanced $\exp(-\phi(\xi))$ -expansion admits to solution of (8) in the form

$$U(x,y,t)=A_0+A_1 exp(-\varphi(\xi)) \tag{11}$$

Now, substituting Eq. (11) into Eq. (10), and equating the coefficients of the powers $\varphi(\xi)$ then we obtain a system of algebraic equations. Solving this system of equations for A_0, A_1 and ω, we obtain the following values:
Set-1:

$$\omega=-4\lambda\mu, A_0=A_0, A_1=\frac{12\lambda}{a+b}$$

Case-I: When $\lambda\mu<0$ we get following hyperbolic solution
Family-1

$$u_1(x,y,t) = A_0 + \frac{12\lambda}{(a+b)\sqrt{\frac{-\lambda}{\mu}}\, tanh\,(\sqrt{-\lambda\mu}\,(\xi+C))}$$

$$u_2(x,y,t) = A_0 + \frac{12\lambda}{(a+b)\sqrt{\frac{-\lambda}{\mu}}\, coth\,(\sqrt{-\lambda\mu}\,(\xi+C))}$$

Where, $\omega = -4\lambda\mu$ and $\xi = x + y - \omega\frac{t^{\alpha}}{\alpha}$.
Case-II: When $\lambda\mu > 0$ we get following trigonometric solution
Family-2

$$u_3(x,y,t) = A_0 + \frac{12\lambda}{(a+b)\sqrt{\frac{\lambda}{\mu}}\, tan\,(\sqrt{\lambda\mu}\,(\xi+C))}$$

$$u_4(x,y,t) = A_0 - \frac{12\lambda}{(a+b)\sqrt{\frac{\lambda}{\mu}}\, tan\,(\sqrt{\lambda\mu}\,(\xi+C))}$$

Where, $\omega = -4\lambda\mu$ and $\xi = x + y - \omega\frac{t^{\alpha}}{\alpha}$.
Case III:
When $\lambda = 0$ the executing value of $A_1 = 0$. So the solution cannot be obtained. For this purpose this case is rejected.
Case IV:

$$u_5(x,y,t) = A_0 + \frac{12}{(a+b)(\xi+C))}$$

Where, $\omega = -4\lambda\mu$ and $\xi = x + y - \omega\frac{t^{\alpha}}{\alpha}$.

4. Results & discussion

Around here, we will discuss the actual depiction of the obtained accurate and singular wave answer for the general (2+1)- layered nonlinear advancement condition. We address these arrangements in graphical and actually take a look at about the kind of arrangement. Presently we pictorial some get arrangements acknowledge by applied techniques for the general (2+1)- layered nonlinear development condition.

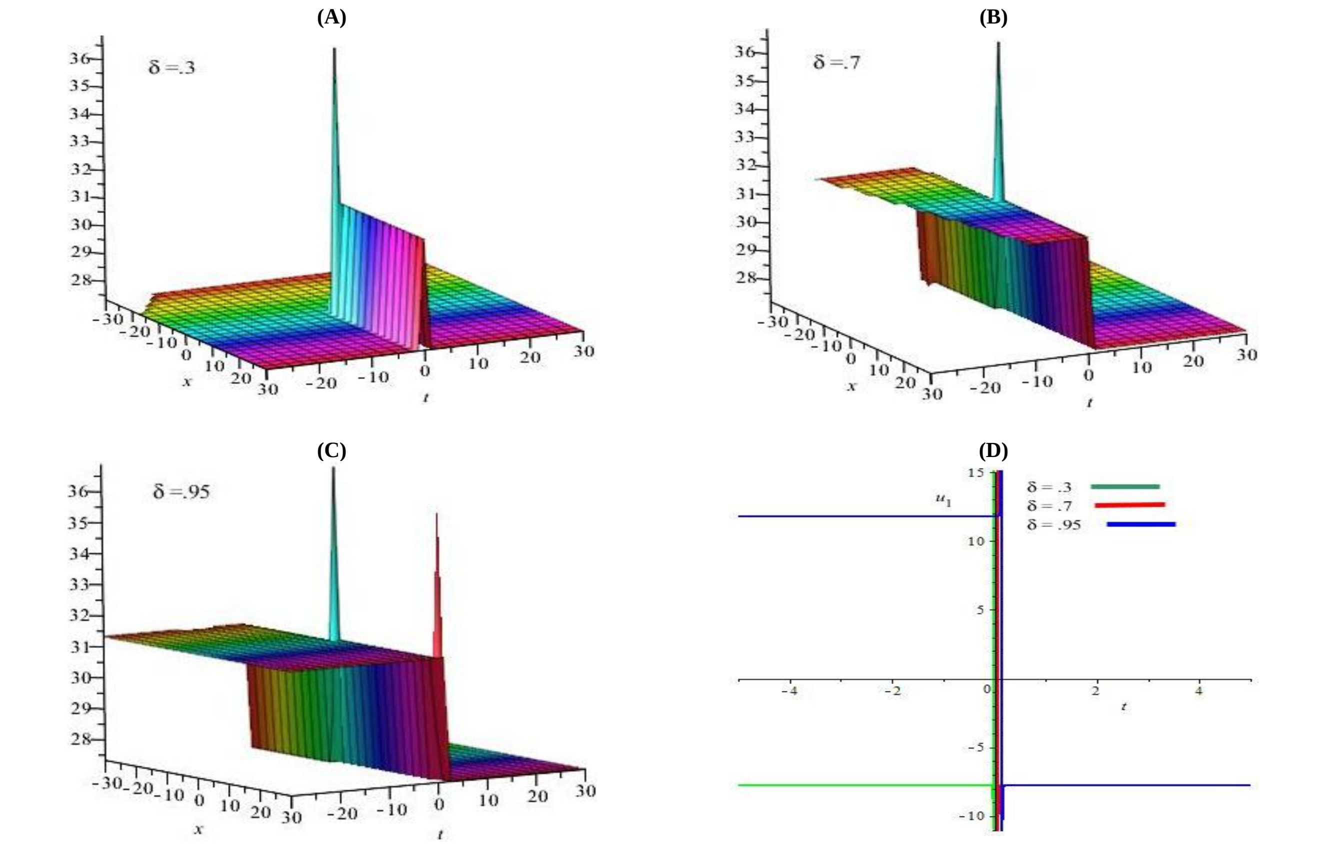

Fig. 1: Singular Kink Shape of $u_1(\xi)$, for $A_0 = 2, a = .002, b = 1, \lambda = 3, \mu = -2, C = 1, y = 2$. The Figure (A, B, C) Shows the 3D Plot and the Figure (D) Shows the 2D Plot for $x = 1$ with $\delta =$ 0.3,0 .7 *and* 0.95 Respectively.

(A) (B)

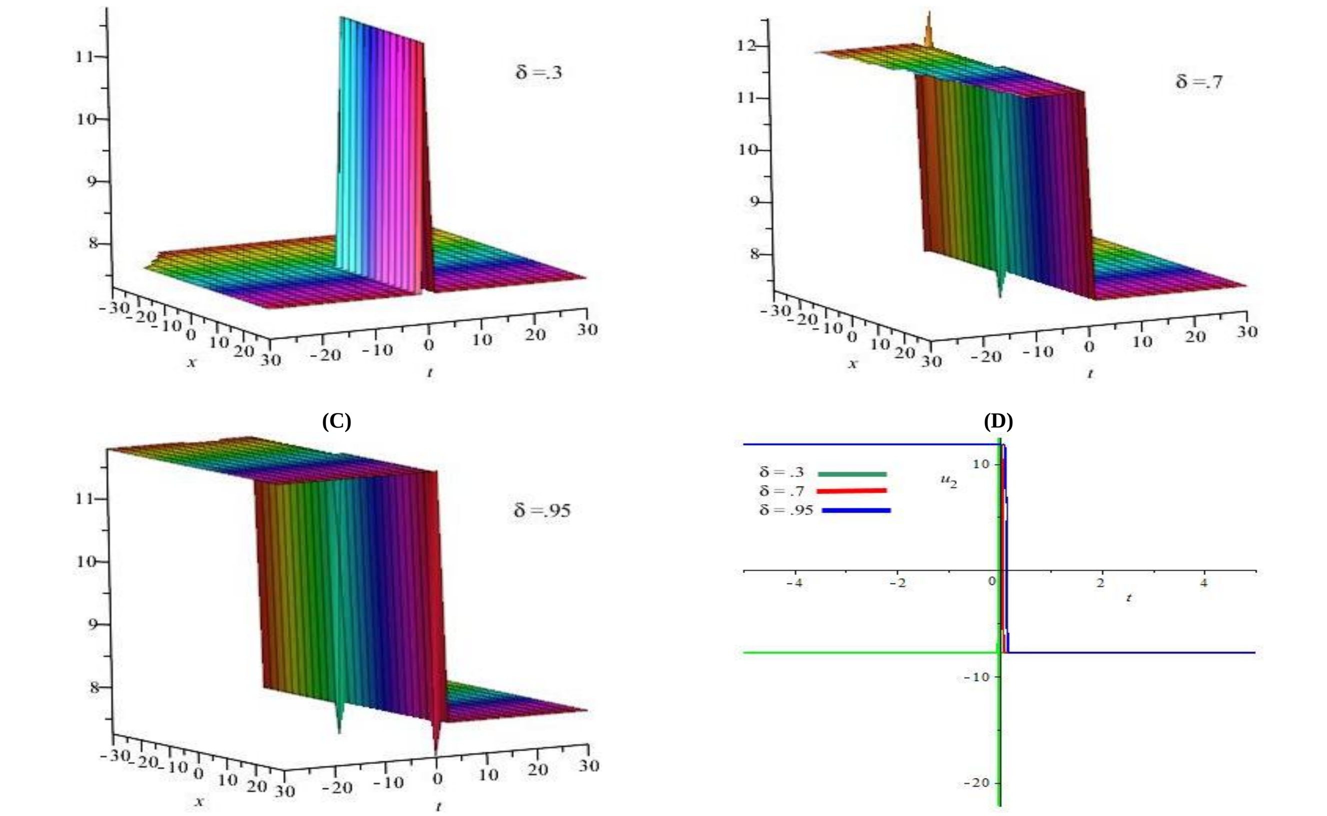

Fig. 2: Kink Shape of u_2 for $A_0 = 2, a = 2, b = 1, \lambda = 3, \mu = -2, C = 1, y = 2$. The Figure (A, B, C) Shows the 3D Plot and the Figure (D) Shows the 2D Plot for $x = 1$ with $\delta = 0.3, 0.7$ *and* 0.95 Respectively.

(A) (B)

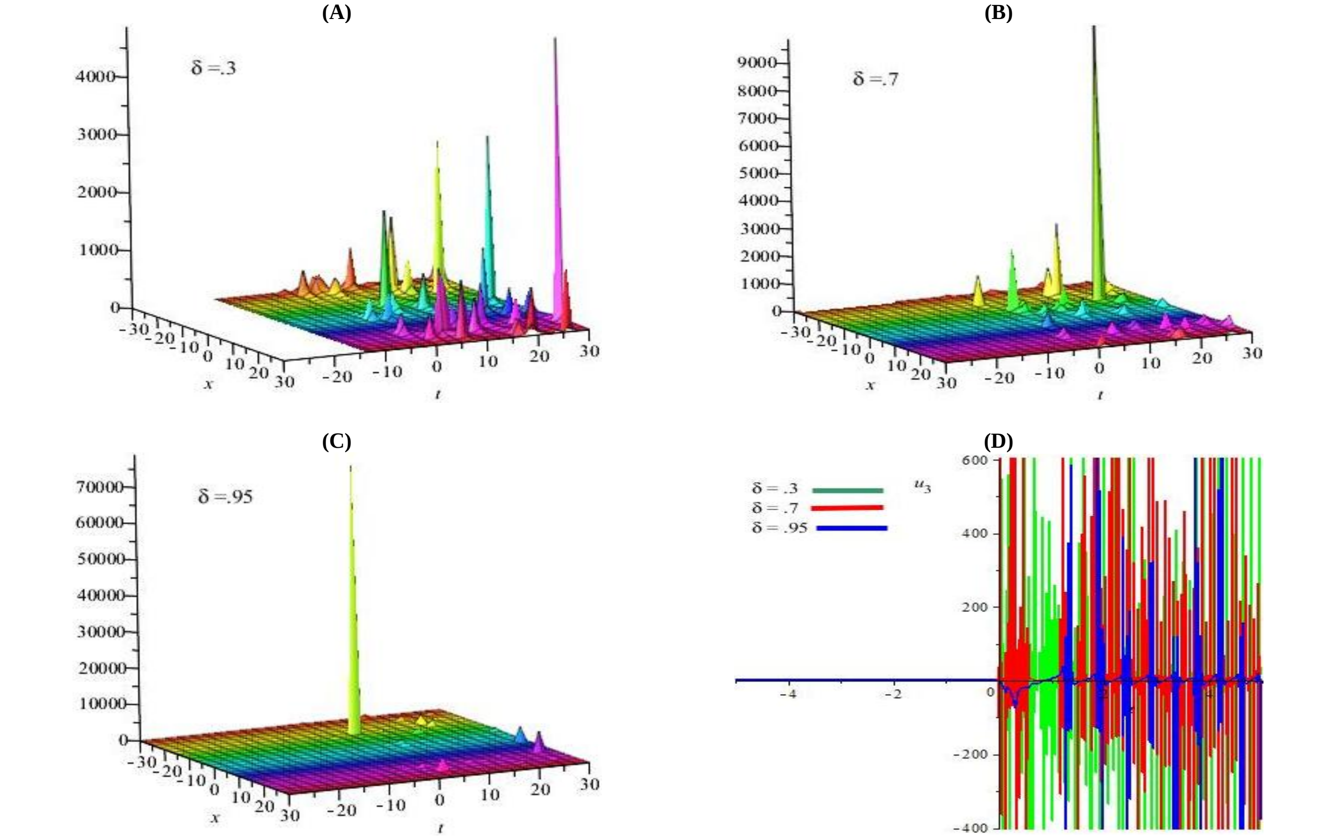

Fig. 3: Periodic Shape of u_3 for $A_0 = 2, a = 2, b = 1, \lambda = 3, \mu = 2, C = 1, y = 2$. The Figure (A, B, C) Shows the 3D Plot and the Figure (D) Shows the 2D Plot for $x = 1$ with $\delta = 0.3, 0.7$ *and* 0.95 Respectively.

(A) (B)

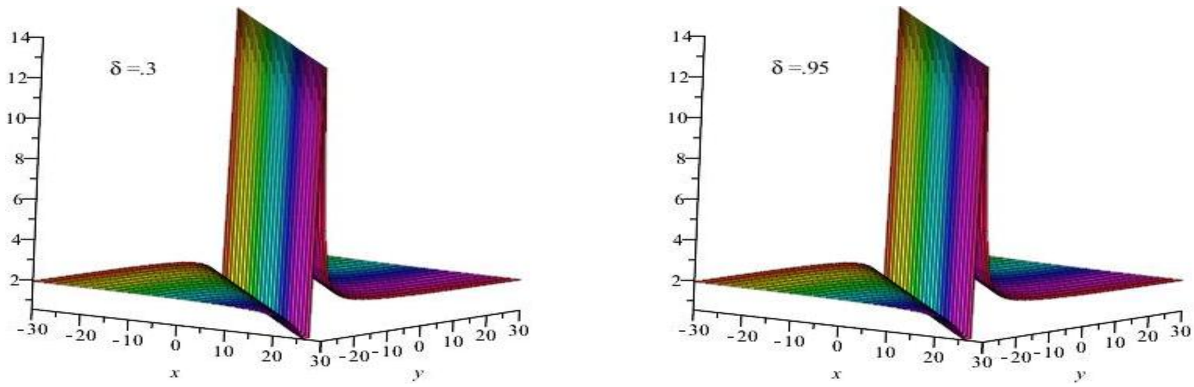

Fig. 4: Kink Shape of u_5 for $A_0 = 2, a = 2, b = 1, \lambda = 0, \mu = 2, C = 1,$. The Figure (A, B) Shows the 3D Plot with $\delta = 0.3$ *and* 0.95 Respectively.

5. Conclusion

In this portion, we have seen that two sorts of voyaging wave game plans similar to exaggerated and mathematical capacities with regards to the general (2+1)- layered nonlinear development condition is really found by using the high level $\exp(-\phi(\xi))$ - extension technique. From our results got in this paper, we wrap up the high level $\exp(-\phi(\xi))$ - development plot procedure is astonishing, strong and accommodating. The show of this strategy is reliable, essential and gives various new plans. The high level $\exp(-\phi(\xi))$ - development strategy enjoys more benefits: It is immediate and compact. Moreover, the courses of action of the proposed nonlinear improvement conditions in this paper have various expected applications in nuclear and particle material science. Finally, this procedure gives a momentous logical instrument to get progressively wide precise game plans of an enormous number of nonlinear PDEs in mathematical material science.

6. Conflict of interest

The authors declare that there is no conflict of interest regarding this paper.

References

[1] Al-Amr MO. Exact solutions of the generalized (2+1)-dimensional nonlinear evolution equations via the modified simple equation method. Comput. Math. Appl. 2015;69(5):390-397. https://doi.org/10.1016/j.camwa.2014.12.011.

[2] Najafi M, Najafi M, Arbabi S. New Exact Solutions for the Generalized (2+1)-dimensional Nonlinear Evolution Equations by Tanh-Coth Method. Int. J. Modern Theoretical Phys. 2013; 2(2):79-85. https://doi.org/10.14419/ijams.v1i2.685.

[3] Najafi M, Arbabi S, Najafi M. New application of sine-cosine method for the generalized (2+ 1) dimensional nonlinear evolution equations. Int. J. Adv. Math. Sci. 2013;1(2):45-49. https://doi.org/10.14419/ijams.v1i2.685.

[4] Darvishi MT, Najafi M, Najafi M. New application of EHTA for the generalized (2+ 1)- dimensional nonlinear evolution equations. Int. J. Math. Comp. Sci. 2010;6(3):132-138.

[5] Emrullah Yasar, Yakup Yıldırım and Abdullahi Rashid Adem, Perturbed optical solitons with spatio-temporal dispersion in (2 + 1)-dimensions by extended Kudryashov method Optik 158 (2018) 1–14. https://doi.org/10.1016/j.ijleo.2017.11.205.

[6] Harun-Or Roshid, Md. Mamunur Roshid, Nizhum Rahman and Mst. Razia Pervin, New solitary wave in shallow water, plasma and ion acoustic plasma via the GZK-BBM equation and the RLW equation, Propulsion and Power Research, 6(1) (2017)49–57. https://doi.org/10.1016/j.jppr.2017.02.002.

[7] H. O. Roshid, M. F. Hoque and M. A. Akbar, New extended (G'/G)-expansion method for traveling wave solutions of nonlinear partial differential equations (NPDEs) in mathematical physics, Italian. J. Pure Appl. Math. 33 (2014) 175-190.

[8] L. L. Feng andT.T. Zhang, Breather wave, rogue wave and solitary wave solutions of a coupled nonlinear Schrodinger equation, Appl. Math. Lett. 78 (2018) 133-140. https://doi.org/10.1016/j.aml.2017.11.011.

[9] Xu Shuwei and He Jingsong, The rogue wave and breather solution of the Gerdjikov-Ivanov equation, Journal of Mathematical Physics 53, (2012). https://doi.org/10.1063/1.4726510.

[10] A. Biswas, M. Mirzazadeh, M. Eslami, Q. Zhou, A. Bhrawy, M. Belic, Optical solitons in nano-fibers with spatio-temporal dispersion by trial solution method, Optik 127 (18) (2016) 7250–7257 https://doi.org/10.1016/j.ijleo.2016.05.052.

[11] Jalil manafian heris and isa zamanpour, Analytical treatment of the Coupled Higgs Equation and the Maccari System via Exp-Function Method, 33 (2013) 203-216.

[12] Y. M. Zhao, New Exact Solutions for a Higher-Order Wave Equation of KdV Type Using the Multiple Simplest Equation Method, Journal of Applied Mathematics, 2014 (2014), 1-13. https://doi.org/10.1155/2014/848069.

[13] Md. Rafiqul Islam and Harun-Or-Roshid, Application of $\exp(-\phi(\xi))$ -expansion method for Tzitzeica type nonlinear evolution equations, Journal for Foundations and Applications of Physics, 4 (1) (2017).

[14] N. Rahman, S. Akter, H. O. Roshid and M. N. Alam, Traveling Wave Solutions of The (1+1)-Dimensional Compound KdVB equation by $\exp(-\phi(\xi))$ –Expansion Method, Global Journal of Science Frontier Research,13 (8) (2014) 7-13.

[15] R. Islam, M. N. Alam, A. K. M. K. S. Hossain, H. O. Roshid and M. A. Akbar, Traveling wave solutions of nonlinear evolution equations via $\exp(-\phi(\xi))$ -expansion method, Global Journal of Scientific Frontier Research,13 (11) (2014) 63-71.

[16] N. Kadkhoda, H. Jafari, Analytical solutions of the Gerdjikov–Ivanov equation by using $\exp(-\phi(\xi))$ expansion method, Optik 139 (2017) 72–76. https://doi.org/10.1016/j.ijleo.2017.03.078.

[17] Amfilokhiev, VB, Voitkunskii, YI, Mazaeva, NP, Khodorkovskii, YS Flows of polymer solutions in the case of convective accelerations Tr. Leningr. Korablestroit. Inst. 96(1975)3-9.

[18] Md. Mamunur Roshid and Harun-Or Roshid, Exact and explicit traveling wave solutions to two nonlinear evolution equations which describe incompressible viscoelastic Kelvin-Voigt fluid, Heliyon 4 (2018). https://doi.org/10.1016/j.heliyon.2018.e00756.

[19] Omer Faruk Gozukızıl and Samil Akcagıl, The tanh-coth method for some nonlinear pseudoparabolic equations with exact solutions, Advances in Difference Equations, 2013 (2013)143. https://doi.org/10.1186/1687-1847-2013-143.

[20] Ak Turgut, Tugba Aydemir, Asit Saha and Abdul Hamid Kara, Propagation of nonlinear shock waves for the generalised Oskolkov equation and its dynamic motions in the presence of an external periodic perturbation, Pramana – J. Phys. (2018) 78-90. https://doi.org/10.1007/s12043-018-1564-7.
[21] Rahhman M.M., Aktar A. and Roy K.C. Analytical Solutions of Nonlinear Coupled Schrodinger–KdV Equation via Advance Exponential Expansion. American Journal of Mathematical and Computer Modelling, 3(3), 46-51, (2018) https://doi.org/10.11648/j.ajmcm.20180303.11.
[22] 22. R. Khalil, M. Al Horani, A. Yousef, M. Sababheh, A new definition of fractional derivative. J. Comput. Appl. Math. 264, 65 – 70, (2014) https://doi.org/10.1016/j.cam.2014.01.002.
[23] Abdeljawad T., On conformable fractional calculus. J. Comput. Appl. Math. 279, 57 – 69, (2015) https://doi.org/10.1016/j.cam.2014.10.016.
[24] G. Jumarie, Modified Riemann-Liouville derivative and fractional Taylor series of non-differentiable functions further results. Comput. Math. Appl. 51(9), 1367 – 1376, (2006) https://doi.org/10.1016/j.camwa.2006.02.001.