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Research paper

Analytic solutions of the chiral nonlinear schrödinger equations investigated by an efficient approach

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Abstract

This paper studies the chiral nonlinear Schrödinger equations, describing a central role in the developments of quantum me-chanics, particularly in the field of quantum Hall effect, where chiral excitations are known to appear. More precisely, in this paper, we acquired new exact solutions of the chiral nonlinear (1+1) and (1+2)-dimensional Schrödinger equations by using the modified Kudraysov method. As outcomes, some of the new exact traveling wave solutions for the equations above is formally produced. All solutions are plotted in the view of three-dimensional (3D) and two-dimensional (2D) line shape through the MATLAB software for investigating the real significance of the studied equations. The periodic type of solitons is generated by employing modified Kudryashov method which is different from other studied methods.

Keywords:*Chiral Nonlinear (1+1)-Dimensional Schrödinger Equation; Chiral Nonlinear (1+2)-Dimensional Schrödinger Equation; Modified Kudryashov Method; New Exact Traveling Wave Solutions; Symbolic Computation.*

1. Introduction

Nonlinear partial differential equations (NPDEs) have been analyzed in various fields of nonlinear sciences using their exact solutions. Specifically, they have a wide range of applications in various fields, such as fluid mechanics, solid state physics, plasma physics, chemical physics, quantum field theory, mathematical biology, optical fiber, geochemistry, etc. which are interesting to the scientists and engineers for real-world purposes.In recent years, due to the advent of computational facilities, there have been spectacular advancements in solving the NPDEs by a number of significant analytic methods, such as Homogeneous balance method [1], (G'/G)-expansion method [2], Extendedtanh functionmethod [3], Extended F-expansion method [4], Jacobi elliptic function method [5], Transformed rational function method [6], Weierstrass elliptic function expansion method [7], Generalized Kudryashov method [8], Auxliary equation method [9], Modified Kudryashov method [10–13], Sine-Gordon equation expansion method [14,15], Extended sinh-Gordon equation method [16–18], Hyperbolic function method [19] and so on [20–25]. Recently, researchers easily designed and applied these methods for reducing the computational difficulty using computational software packages like Maple, Mathematica, MATLAB etc.
Under the investigation of new traveling wave solutions, we consider the two types of chiral nonlinear Schrödinger equations. Firstly, we consider the chiral nonlinear (1+1)-dimensional Schrödinger equation is given by [27], [34]:

$$i\psi_t + \psi_{xx} - i\sigma(\psi^*\psi_x)\psi = 0 \tag{1}$$

Where$\psi = \psi(x,t)$ is a complex-valued function. It is a special type of nonlinear evolution equation that arises in the vast areas of applied sciences, such as nonlinear optics, plasma physics, quantum mechanics, and so on, σ is a non linear coupling constant and the *symbol indicates the complex conjugate.
Secondly, we consider the chiral nonlinear (1+2)-dimensional Schrödinger equation is given by [32–35]:

$$i\psi_t + a\left(\psi_{xx} + \psi_{yy}\right) + i\left(b_1(\psi\psi_x{}^* - \psi^*\psi_x) + b_2\left(\psi\psi_y{}^* - \psi^*\psi_y\right)\right)\psi = 0 \tag{2}$$

where ψ is the complex function of x and t, a is the coefficient of the dispersion terms and b_1, b_2 are nonlinear coupling constants.
Equation (1) and (2) give chiral solitons which play a vital role in the context of quantum Hall effect, where chirp and chiral excitations are known to appear [26-35]. Several studies have been conducted on the models (1) and (2). Nishino et al. [27] solved the chiral nonlinear (1+1)-dimensional Schrödinger equation and constructed two types of the progressing wave solutions such as bright and dark soliton train. Recently, bright and dark soliton solutions have been investigated by Bulutet al. [34] of the chiral nonlinear (1+1) and (1+2)-dimensional Schrödinger equations with the aid of extended sinh-Gordon equation method. With the aid of soliton perturbation theory,

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Biswas et al. [30] studied the perturbation of soliton due to the chiral nonlinear Schrödinger equation. Younis et al. [33] studied the chiral nonlinear (1+2)-dimensional Schrödinger equation analytically, with perturbation term and a coefficient of Bohm potential. As consequences, soliton-like solutions, triangular type solutions, single and combined non-degenerate Jacobi elliptic function like solutions are derived by using an extended fan method along with their constraint conditions. A trial solution technique is applied to chiral nonlinear (1+2)-dimensional Schrodinger's equations [32]. Soliton and singular soliton solutions are obtained by using the trial solution technique. Recently, Rana and Javid [35] carried out the optical dark and singular solitons for chiral nonlinear (1+2)-dimensional Schrodinger's equation by using two distinct integration schemes namely, extended direct algebraic and extended trial equation methods. Up to now, to the best of our knowledge, no scholar has studied the chiral nonlinear (1+1) and (1+2)-dimensional Schrödinger equations to look for new physical significance through the modified Kudryshov method.
The main aim of this paper is to produce new exact traveling wave solutions of the chiral nonlinear (1+1) and (1+2)-dimensional Schrödinger equation using an efficient method which is known as the modified Kudryashov method.
The rest of this paper is arranged as follows: Sections 2 indicate the description of the modified Kudryashov method. We will apply the above mentioned for acquiring new exact traveling wave solutions of the chiral nonlinear Schrödinger equation in Section 3.
Finally, section 4 provides a concluding remark about the results generated.

2. Outline of the modified Kudryashov method

The modified Kudryashov method is considered as a robust problem-solving technique that has received considerable attention to look for new exact solutions of nonlinear differential equations used in mathematical physics. First, we present a brief description of the modified Kudryashov method [10] to look for new exact solutions of a given nonlinear partial differential equation. For this aim, we assume a nonlinear partial differential equation as

$$F(u, u_x, u_t, u_{xx}, u_{xt}, u_{tt}, \dots) = 0 \tag{3}$$

where the function $u = u(x,t)$ is unknown and F is a polynomial. The main steps are as follows:
Step 1: Introducing the transformation $u(x,t) = U(\xi)$ where$\xi = kx + \omega t$, varies according to Eq. (3). This reduces to the following nonlinear ordinary differential equation

$$P(U, U', U'', \dots) = 0 \tag{4}$$

where P is a polynomial of U and its derivatives such that the superscripts indicate the ordinary derivatives with respect to ξ.
Step 2: Let us assume that the solution $U(\xi)$ of the nonlinear Eq. (4) can be presented as

$$U(\xi) = A_0 + \sum_{i=1}^{N} A_i Q^i(\xi), A_N \neq 0 \tag{5}$$

in which the constants A_i $(i = 0,1 \dots N)$ are determined later, and N is a positive integer which can be determined by the means ofbalancing principle on Eq. (5), and $Q(\xi)$ satisfies the following new equation

$$Q'(\xi) = \left(Q^2(\xi) - Q(\xi)\right)\ln(A) \tag{6}$$

with the exact solution $Q(\xi) = \frac{1}{1+dA^{\xi}}$, where $A \neq 0,1$.
Step 3: By substituting Eq. (5) along with Eq. (6) into Eq. (4) and using some mathematical operations, we get a system of algebraic equations in parametersA_i $(i = 0,1 \dots N)$, k, and ω. By setting the obtained values in Eq. (5), finally generates new exact solutions for the Eq. (3).

3. Mathematical analysis

3.1. Solution of the nonlinear chiral (1+1)-dimensional schrödinger equation

By considering the following transformation:

$$\psi(x,t) = U(\xi)e^{i\theta}, \xi = c(x + vt), \theta = kx + wt \tag{7}$$

By utilizing the transformation of Eq. (7) into the (1+1) dimensional nonlinear chiral Schrödinger equation (1) can be reduced to a nonlinear ordinary differential equation as below

$$c^2U'' + 2k\sigma U^3 - (w + k^2)U = 0, v = -2k \tag{8}$$

Using the homogeneous balance principle, we find $N = 1$.Then the solution of the Eq. (8) takes the form

$$U(\xi) = A_0 + A_1Q(\xi) \tag{9}$$

By substituting Eq. (9) along with its second derivative into Eq. (8), and equating the like powers of $Q(\xi)$, then we get in the following system of equations:

$$2\ln(A)^2c^2A_1 + 2k\sigma A_1^3 = 0,$$

$$-3\ln(A)^2c^2A_1 + 6k\sigma A_0A_1^2 = 0,$$

$\ln(A)^2c^2A_1 + 6k\sigma A_0^2A_1 - k^2A_1 - wA_1 = 0,$

$2k\sigma A_0^3 - k^2A_0 - wA_0 = 0.$

By solving the above system, we receive the following different solution sets:

Set –I: $w = -\frac{1}{2}\ln(A)^2c^2 - k^2, A_0 = -\frac{1}{2}\frac{c\ln(A)}{\sqrt{-k\sigma}}$, and $A_1 = -\frac{\ln(A)\, c\sqrt{-k\sigma}}{k\sigma}$.

Set-I possesses the following exact solution of the chiral nonlinear (1+1)-dimensional Schrödinger equation is extracted:

$$\psi_1(x,t) = \left(-\frac{1}{2}\frac{c\ln(A)}{\sqrt{-k\sigma}} - \frac{\ln(A)c\sqrt{-k\sigma}}{k\sigma(1+dA^{c(-2kt+x)})}\right)e^{i\left(kx+t\left(-\frac{1}{2}\ln(A)^2c^2-k^2\right)\right)} \quad (10)$$

Set –II: $w = -\frac{1}{2}\ln(A)^2c^2 - k^2, A_0 = \frac{1}{2}\frac{c\ln(A)}{\sqrt{-k\sigma}}$, and $A_1 = \frac{\ln(A)\, c\sqrt{-k\sigma}}{k\sigma}$.

Set-II possesses to the following exact solution of the chiral nonlinear (1+1)-dimensional Schrödinger equation is determined:

$$\psi_2(x,t) = \left(\frac{1}{2}\frac{c\ln(A)}{\sqrt{-k\sigma}} + \frac{\ln(A)c\sqrt{-k\sigma}}{k\sigma(1+dA^{c(-2kt+x)})}\right)e^{i\left(kx+t\left(-\frac{1}{2}\ln(A)^2c^2-k^2\right)\right)} \quad (11)$$

The three-dimensional (3D) and two dimensional (2D) plots of the Eq. (10), and Eq. (11) have been demonstrated in Fig.1 and Fig.2, respectively. The shape of the plots of the Eq. (10), and (11) seems periodic solitons behavior which is different from others [27],[34].

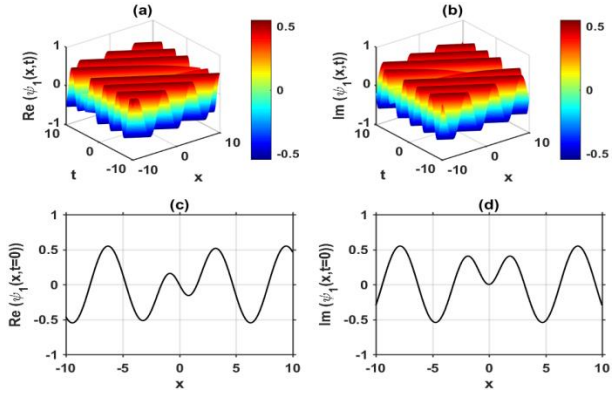

Fig. 1: A)-B) 3D Illustrations of the Eq. (10) for the Choosing Arbitrary Parameters $C = 1, A = 3, K = -1, D = 1, \Sigma = 1$ Within $-10 \le X \le 10, -10 \le T \le 10$, and (C)-(D) 2D Line Illustrations of (A)-(B) at $T = 0$, Respectively.

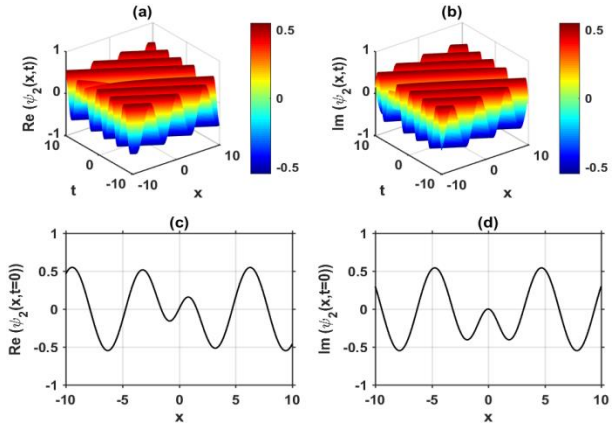

Fig. 2: A)-B) 3D Illustrations of the Eq. (11) for the Choosing Arbitrary Parameters $C = 1, A = 3, K = -1, D = 1, \Sigma = 1$ Within $-10 \le X \le 10, -10 \le T \le 10$, and C)-D) 2D Line Illustrations of A)-B) at $T = 0$, Respectively.

3.2. Solution of the nonlinear chiral (1+2)-dimensional Schrödinger equation

By introducing the following transformation:

$$\psi = U(\xi)e^{i\theta}, \xi = \alpha x + \beta y - vt, \theta = px + qy + wt \quad (12)$$

By employing the transformation Eq. (12) into the nonlinear chiral (1+2)-dimensional Schrödinger Eq. (2) can be converted to a nonlinear ordinary differential equation as follows

$$a(\alpha^2 + \beta^2)U'' + 2(pb_1qb_2)U^3 - (a(p^2 + q^2) + w)U = 0, v = 2a(\alpha p + \beta q) \quad (13)$$

Using the homogeneous balance principle, we find $N = 1$. Then, the solution of Eq. (13) takes the form

$$U(\xi) = A_0 + A_1 Q(\xi) \tag{14}$$

By substituting Eq. (14) along with its second derivative into Eq. (13), and equating the like powers of $Q(\xi)$, then we get in the following system of equations:

$$2\,ln(A)^2\,a\alpha^2 A_1 + 2\,ln(A)^2\,a\beta^2 A_1 + 2pA_1{}^3 b_1 + 2qA_1{}^3 b_2 = 0,$$

$$-3\,ln(A)^2\,a\alpha^2 A_1 + 6pA_0 A_1{}^2 b_1 + 6qA_0 A_1{}^2 b_2 - 3\,ln(A)^2\,a\beta^2 A_1 = 0,$$

$$-ap^2 A_1 + ln(A)^2 a\beta^2 A_1 - aq^2 A_1 + 6qA_0{}^2\,A_1 b_2 + ln(A)^2 a\alpha^2 A_1 + 6pA_0{}^2 A_1 b_1 - wA_1 = 0,$$

$$2pA_0{}^3 b_1 + 2qA_0{}^3 b_2 - ap^2 A_0 - aq^2 A_0 - wA_0 = 0.$$

By solving the above system, we receive the following different solution sets:

Set–I:$w = -\frac{1}{2} ln(A)^2 a\alpha^2 - \frac{1}{2} ln(A)^2 a\beta^2 - ap^2 - aq^2$, $A_0 = \frac{\sqrt{-(pb_1+qb_2)a(\alpha^2+\beta^2)}\,ln(A)}{2pb_1+2qb_2}$, and$A_1 = \frac{a\,ln(A)(\alpha^2+\beta^2)}{\sqrt{-(pb_1+qb_2)a(\alpha^2+\beta^2)}}$.

Set-I corresponds the following exact solution of the nonlinear chiral (1+2)-dimensional Schrödinger equation is derived:

$$\psi_1(x,y,t) = \left(\frac{\sqrt{-(pb_1+qb_2)a(\alpha^2+\beta^2)}\,ln(A)}{2pb_1+2qb_2} + \frac{a\,ln(A)(\alpha^2+\beta^2)}{\sqrt{-(pb_1+qb_2)a(\alpha^2+\beta^2)}\left(1+dA^{-2ta(p\alpha+q\beta)+\alpha x+\beta y}\right)}\right) \times e^{i\left(px+qy+t\left(-\frac{1}{2}ln(A)^2 a\alpha^2 - \frac{1}{2}ln(A)^2 a\beta^2 - ap^2 - aq^2\right)\right)} \tag{15}$$

Set-II: $w = -\frac{1}{2} ln(A)^2 a\alpha^2 - \frac{1}{2} ln(A)^2 a\beta^2 - ap^2 - aq^2$, $A_0 = -\frac{\sqrt{-(pb_1+qb_2)a(\alpha^2+\beta^2)}\,ln(A)}{2pb_1+2qb_2}$, and $A_1 = -\frac{a\,ln(A)(\alpha^2+\beta^2)}{\sqrt{-(pb_1+qb_2)a(\alpha^2+\beta^2)}}$.

Set-II corresponds the following exact solution of the nonlinear chiral (1+2)-dimensional Schrödinger equation is explored:

$$\psi_2(x,y,t) = -\left(\frac{\sqrt{-(pb_1+qb_2)a(\alpha^2+\beta^2)}\,ln(A)}{2pb_1+2qb_2} - \frac{a\,ln(A)(\alpha^2+\beta^2)}{\sqrt{-(pb_1+qb_2)a(\alpha^2+\beta^2)}\left(1+dA^{-2ta(p\alpha+q\beta)+\alpha x+\beta y}\right)}\right) \times e^{i\left(px+qy+t\left(-\frac{1}{2}ln(A)^2 a\alpha^2 - \frac{1}{2}ln(A)^2 a\beta^2 - ap^2 - aq^2\right)\right)} \tag{16}$$

The three-dimensional (3D) and two dimensional (2D) plots of the Eq. (15), and Eq. (16) have been demonstrated in Fig.3 and Fig.4, respectively. The shape of the plots seems periodic solitons behavior which is different from others [32]-[35].

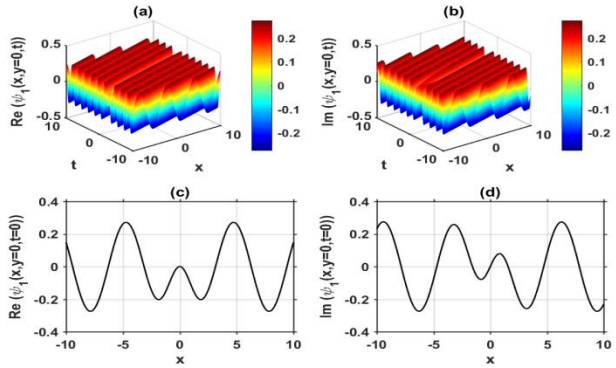

Fig. 3: A)-B) 3D Illustrations of the Eq. (15) for the Choosing Arbitrary Parameters $p = 1, b_1 = 1, q = 1, b_2 = 1, \alpha = 1, \beta = 1, A = 3, d = 1, a = 1, y = 0$ within$-10 \le x \le 10, -10 \le t \le 10$, and C)-D) 2D Line Illustrations of (A)-(B) at $y = 0, t = 0$, Respectively.

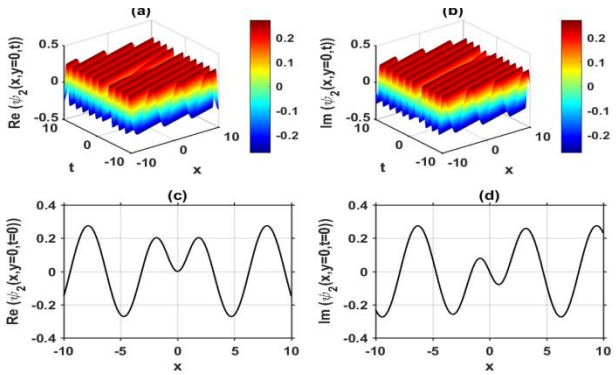

Fig.4:A)-B) 3D Illustrations of the Eq. (16) for the Choosing Arbitrary Parameters $p = 1, b_1 = 1, q = 1, b_2 = 1, \alpha = 1, \beta = 1, A = 3, d = 1, a = 1, y = 0$ within$-10 \le x \le 10, -10 \le t \le 10$, and C)-D) 2D Line Illustrations of A)-B) at $y = 0, t = 0$, Respectively.

4. Conclusions

In this paper, the nonlinear chiral Schrödinger equations which describe the edge of the fractional quantum Hall effect is studied. The modified Kudraysov method is applied as a robust technique to solve the nonlinear chiral (1+1) and (1+2)-dimensional Schrödinger equations. As outcomes, a number of new exact traveling wave solutions for the nonlinear chiral Schrödinger equations are formally derived. It should be stated the reliability of the results reported in this paper is examined by putting each new solution back into its corresponding equation.

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