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Research paper

A New 4-D Chaotic System with Hidden Attractor and its Circuit Implementation

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Abstract

In the chaos literature, there is currently significant interest in the discovery of new chaotic systems with hidden chaotic attractors. A new 4-D chaotic system with only two quadratic nonlinearities is investigated in this work. First, we derive a no-equilibrium chaotic system and show that the new chaotic system exhibits hidden attractor. Properties of the new chaotic system are analyzed by means of phase portraits, Lyapunov chaos exponents, and Kaplan-Yorke dimension. Then an electronic circuit realization is shown to validate the chaotic behavior of the new 4-D chaotic system. Finally, the physical circuit experimental results of the 4-D chaotic system show agreement with numerical simulations.

Keywords: *Chaos, chaotic systems,circuit simulation, hidden attractors, Lyapunov exponents*

1. Introduction

Chaos theory deals with nonlinear dynamical systems that are highly sensitive to initial conditions. Such nonlinear systems are characterized by the existence of a positive Lyapunov exponent. In the past five decades, it has been well-established that chaos can be applied in various disciplines, such as physics ([1]-[10]), Tokamak system [11], economy [12], ecology [13], random bit generators [14], chemical reactions ([15]-[19]), robotics ([20]-[21]), text encryption [22], image encryption [23], voice encryption [24], and secure communication systems ([26]-[28]).

Recently, a new classification has been made for chaotic attractors ([29]-[31]). According to this new classification, chaotic attractors are classified as (A) self-excited attractors, and (B) hidden attractors. A self-excited attractor has a basin of attraction which is excited from unstable equilibrium points. On the other hand, a hidden attractor has a basin of attraction which does not contain neighbourhoods of equilibrium points. Classical examples of self-excited attractors are Lorenz system [32], Chen system [33], Lu system [34], Liu system [35], etc. Some recent examples of self-excited attractors are Vaidyanathan systems ([36]-[37]), Zhu system [38], Sprott system [39], etc.

Hidden attractors include chaotic systems with no equilibrium points ([40]-[42]), chaotic systems with infinite number of equilibrium points ([43]-[44]), chaotic systems with stable equilibrium points ([45]-[46]) and chaotic systems with line equilibrium ([47]-[48]). A special case of the hidden attractors is a multi-stability and coexistence of attractors can be searched on ([49]-[52]). The Lyapunov chaos exponents of a chaotic system are determined using Wolf's algorithm [53]. In the literature, there is also good interest shown in building electronic circuit designs of chaotic systems and implementing them ([54]-[56]).

The main contribution of this work is the finding of a new 4-D chaotic system with hidden attractor. The chaotic and hyperchaotic systems have many engineering applications such as secure communication, encryption, cryptosystems, etc ([57]-[60]).

In Section 1, we derive a new 4-D chaotic system with no equilibrium points. Hence, the new chaotic system exhibits hidden attractor. In Section 2, the basic dynamical properties of the new 4-D chaotic system have been discussed in detail. In Section 3, a circuit implementation of the new 4-D chaotic system is shown to facilitate practical feasibility of the theoretical model. Section 4 concludes this work with a summary of the main results.

2. A new 4-D chaotic system with no equilibrium points

In this work, we propose a new 4-D chaotic system with two quadratic nonlinearities given by

$$\left\{ \begin{array}{rcl} \dot{x} & = & a(y-x)-w \\ \dot{y} & = & xz \\ \dot{z} & = & b-xy \\ \dot{w} & = & x \end{array} \right. \tag{1}$$

We show that the system (1) displays chaotic behaviour and hidden attractor when

$$a = 4, b = 40 \tag{2}$$

For numerical calculations, we take the initial conditions for the new 4-D system (1) as

$$x_0 = 0.3, y_0 = 0.3, z_0 = 0.3, w_0 = 0.3 \tag{3}$$

Lyapunov exponents of the new chaotic system (1) are determined using Wolf's algorithm [53] in MATLAB for the parameter values (2) and the initial conditions (3) as follows:

$$L_1 = 1.0714, L_2 = 0, L_3 = -0.0362, L_4 = -5.0352 \tag{4}$$

The Kaplan-Yorke dimension of the new chaotic system (1) is obtained as

$$D_{KY} = 3 + \frac{L_1 + L_2 + L_3}{|L_4|} = 3.2056 \tag{5}$$

The maximal Lyapunov exponent (MLE) of the new chaotic system (1) is $L_1 = 1.0714$.
Since the sum of the Lyapunov exponents of the new chaotic system (1) is negative, it is evident that the system (1) is dissipative. Thus, the system orbits of the new jerk chaotic system (1) are ultimately confined into a specific limit set of zero volume and the asymptotic motion settles onto a chaotic attractor.
For numerical simulation of the new chaotic system (1), we have used the classical fourth-order Runge-Kutta method in MATLAB.
The equilibrium points of the new chaotic system (1) are got by solving the equations

$$a(y - x) - w = 0 \tag{6a}$$

$$xz = 0 \tag{6b}$$

$$b - xy = 0 \tag{6c}$$

$$x = 0 \tag{6d}$$

From (6c) and (6d), we must have which is a contradiction to the chaotic case (2).
Hence, the chaotic system (1) has no equilibrium points. Hence, it displays hidden attractor.
Figs 1-4 show the 2-D projections of the hidden attractor of the new 4-D chaotic system (1) for the parameter values (2) and initial conditions (3). The time-evolution of the Lyapunov exponents of the system 1) is depicted in Fig. 5.
In order to investigate further the dynamics of system 1), Lyapunov exponent spectrum and the bifurcation diagram with $b\ \varepsilon[1, 40]$ are presented in Fig.6 and Fig.7, respectively. It is clear to see that the proposed system can generate chaos. In addition, the Poincare map of new chaotic system (1) is shown in Fig. 8, which also reflects the chaotic properties of system.

3. Circuit Realization of the New Chaotic System

The circuit electronic of a new 4D chaotic system (1) by MultiSIM is shown in Figs. 9-12) The chaotic circuit is composed of 6 operational amplifiers (TL082CD), 2 analog multipliers (AD633JN), 11 resistors, and 4 capacitors. In this study, a linear scaling is considered as follows:

$$\begin{cases} \dot{x} &= a(y - x) - \frac{w}{2} \\ \dot{y} &= xz \\ \dot{z} &= \frac{b}{4} - xy \\ \dot{w} &= 2x \end{cases} \tag{7}$$

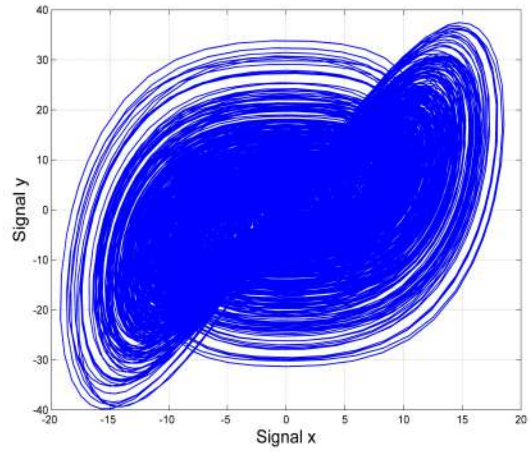

Figure 1: A two-dimensional view of the system (1) in (x, y)-plane. for $(a, b) = (4, 40)$

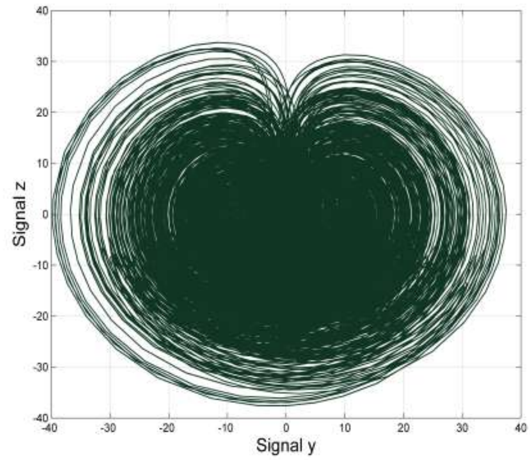

Figure 2: A two-dimensional view of the system (1) in (y, z)-plane. for $(a, b) = (4, 40)$

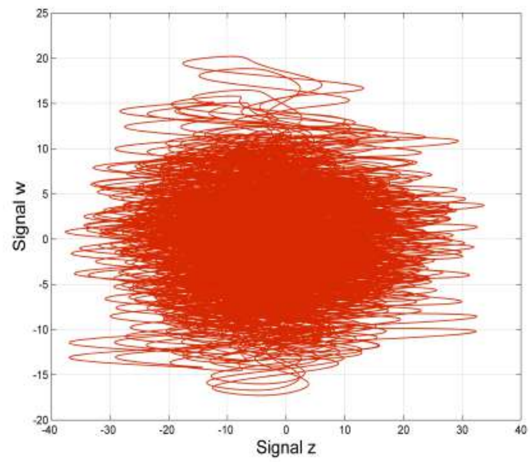

Figure 3: A two-dimensional view of the system (1) in (z, w)-plane. for $(a, b) = (4, 40)$

Chaotic differential equations of the new circuit are given below.

$$\begin{cases} \dot{x} &= \frac{1}{C_1R_1}y - \frac{1}{C_1R_2}x - \frac{1}{C_1R_3}w \\ \dot{y} &= -\frac{1}{C_2R_4}xz \\ \dot{z} &= \frac{1}{C_3R_5}V_1 - \frac{1}{C_3R_6}xy \\ \dot{w} &= \frac{1}{C_4R_7}x \end{cases} \tag{8}$$

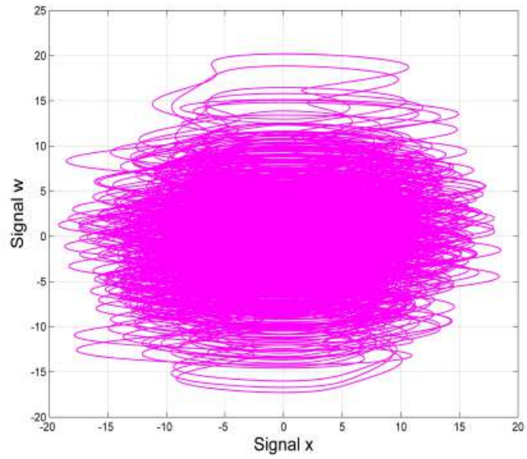

Figure 4: A two-dimensional view of the system (1) in (x,w)-plane. for $(a,b) = (4,40)$

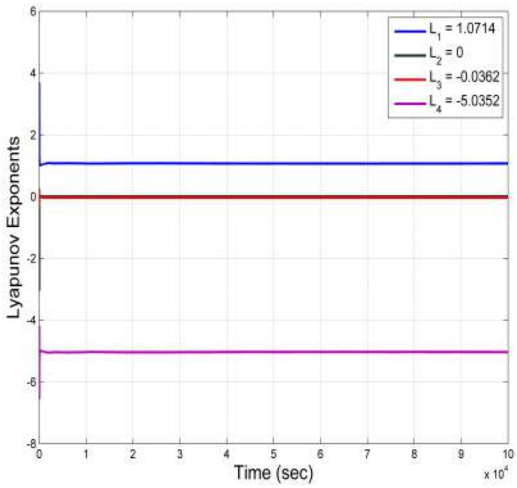

Figure 5: Lyapunov exponents of the new chaotic system (1) for $(a,b) = (4,40)$

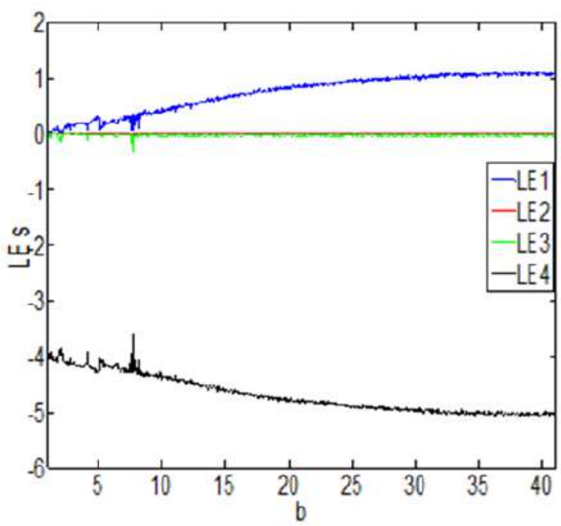

Figure 6: Lyapunov spectrum of system (1) when varying the parameter b

We choose $R_1 = R_2 = R_4 = R_6 = R_8 = R_9 = R_{10} = R_{11} = 100K\Omega, R_3 = 800K\Omega, R_5 = 40K\Omega, R_7 = 200K\Omega, C_1 = C_2 = C_3 =$

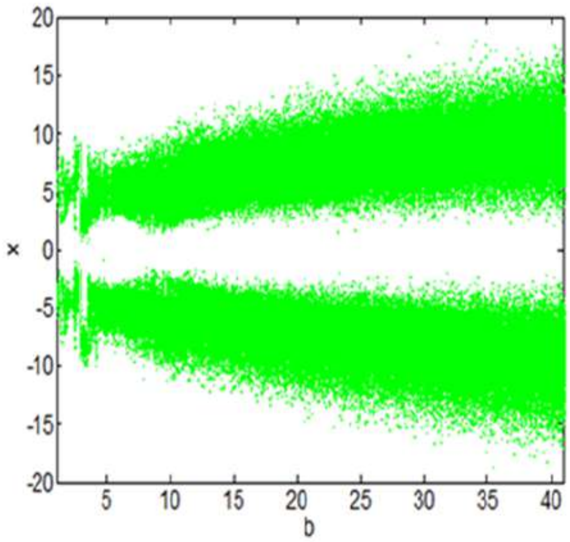

Figure 7: Bifurcation diagram of system (1) versus the parameter b

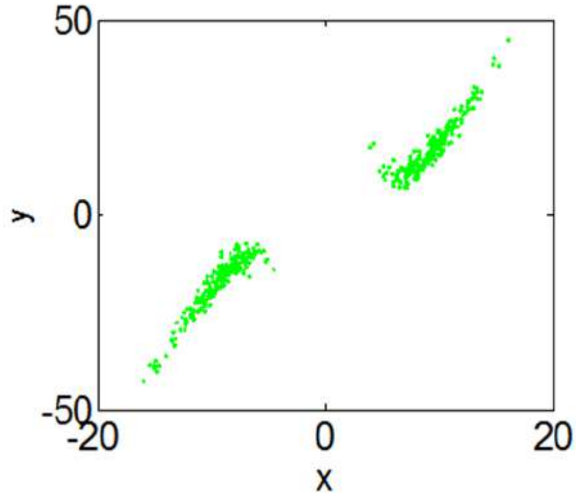

Figure 8: Poincare map of system (1) in the plane x versus y

$C_4 = 1nF$. The supplies of all active devices are $\pm 15Volt$. The designed electronic circuit is implemented in MultiSIM. The obtained results of the oscilloscope are displayed in Figs 13-16) which show the hidden attractors with different phase planes (*x, y*), (*y, z*), (*z, w*) and (*z, w*) respectively. Theoretical models (see Figs. 1-4) are similar with the MultiSIM results (see Figs. 13-16).

4. Conclusion

A new 4-D chaotic system with no equilibrium point is constructed and analyzed. The new 4D chaotic system has two quadratic nonlinearities and no equilibrium point. The dynamical properties of the new 4-D chaotic system have been reported by means of Lyapunov exponent spectrum and Kaplan-Yorke dimension. The obtained results confirm the complex dynamical behaviors. Finally, electronic circuit design of the new chaotic system has been implemented and validated using the MultiSIM software to verify the numerical simulations results. The output results of MultiSIM show good qualitative agreement with the MATLAB simulations of the new 4-D chaotic system.

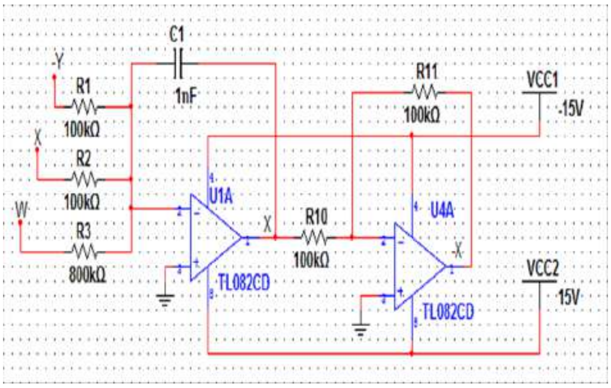

Figure 9: Circuit design of the new chaotic system (1) for X signal

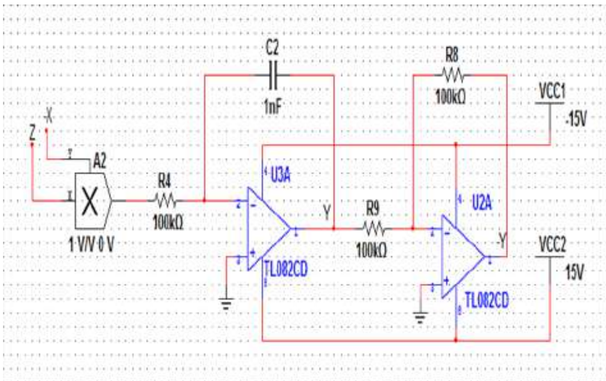

Figure 10: Circuit design of the new chaotic system (1) for Y signal

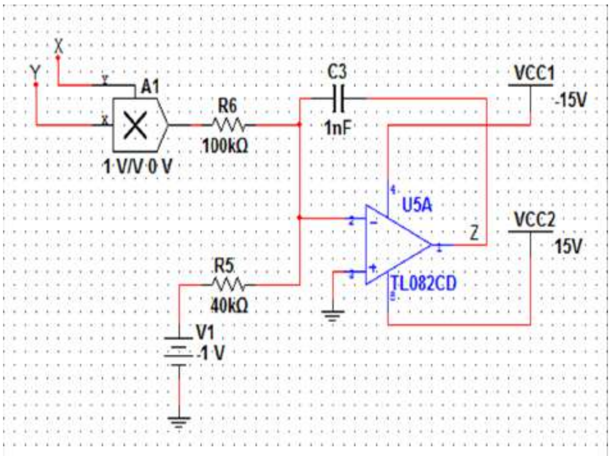

Figure 11: Circuit design of the new chaotic system (1) for Z signal

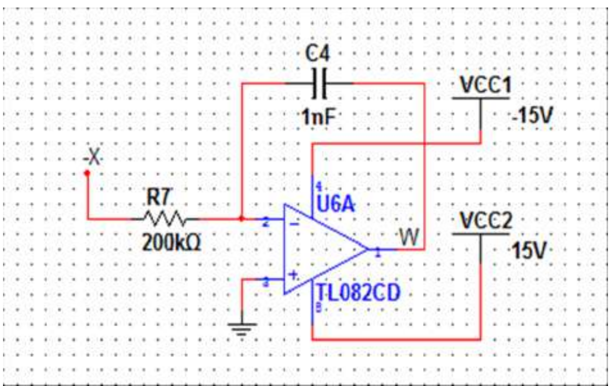

Figure 12: Circuit design of the new chaotic system (1) for W signal

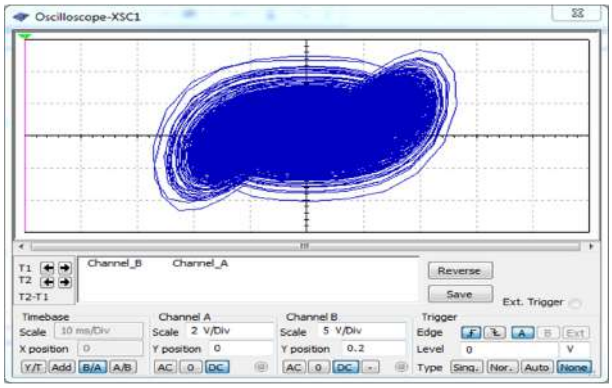

Figure 13: Multisim simulation of the new chaotic system (1) in (x, y)-plane

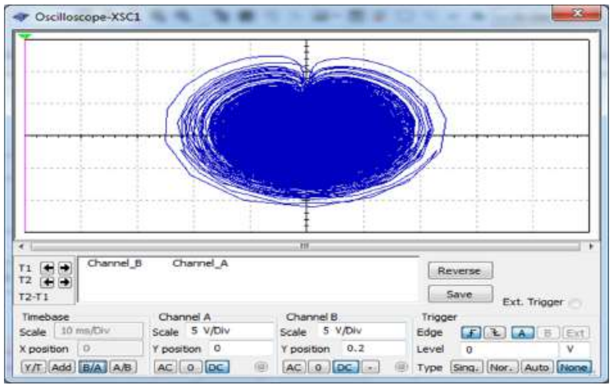

Figure 14: Multisim simulation of the new chaotic system (1) in (y, z)-plane

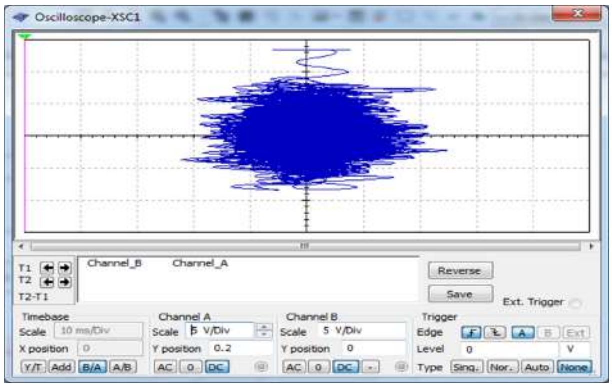

Figure 15: Multisim simulation of the new chaotic system (1) in (z, w)-plane

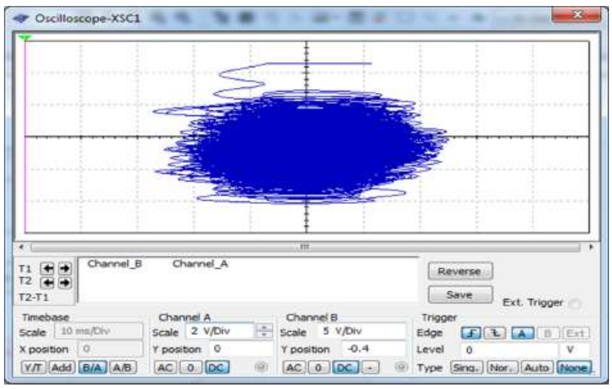

Figure 16: Multisim simulation of the new chaotic system (1) in (x, w)-plane

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