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Research Paper

Hilbert space filling curve using scilab

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Abstract

Space filling curve is used widely for linear mapping of multi-dimensional space. This provides a new line of thinking for various applications in image processing, Image compression being the most widely used. The paper highlights the locality preserving property of Hilbert Space filling curve which is essential in numerous applications such as in image compression, numerical analysis of a large aray of data, parallel processing and so on. A simplistic approach for using Hilbert Space filling curve using Scilab code has been presented.

Keywords: *Hilbert Space Filling; Locality Preserving; Scilab Code.*

1. Introduction

Space filling curve are useful in many image-space algorithms which are based on the spatial coherence of nearby pixel [2]. The most popular Space filling curve is the Peano-Hilbert curve, which has been considered for numerous applications [3]. The advantage of these curves is that they exhibit an inherent tiling effect. This kind of curve has many important characteristics, such as locality, clustering and self-similarity [5].

In 1890, Peano and later Hilbert introduced the space filling curve. Consider a unit square; it is made of infinite points of zero dimension. Peano defined a curve which could pass through each one of the infinite points on the square. Hilbert defined a simpler curve, which was based on iterating, to infinity, the unit square into smaller and smaller squares [1]. The sequences of curves for different divisions of the unit square are called pseudo Hilbert curves.

Hilbert curves basically allows a two dimensional space to be mapped into a single dimension space. It does this while ensuring locality of the points in two dimensional square. This is illustrated in the example below. In this article we demonstrate in an elementary simplistic way the utility of the Hilbert curve for image processing. A Scilab code which would be useful for further experimentation is also included.

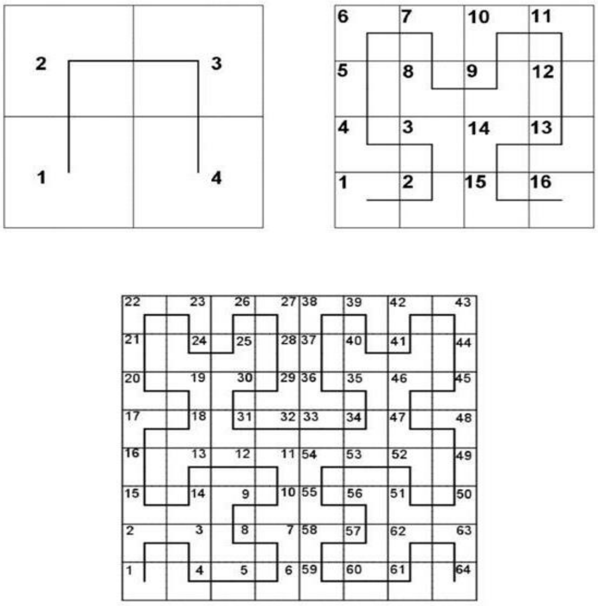

Fig. 1: First Three Stages in the Generation of Hilbert's Space Filling Curve.

2. Algorithm

The initial curve for a 2x2 division of the unit square is given by the 2x2 matrix a1. This matrix correlates with the subdividing of the square shown in figure.

$$a1 = \begin{matrix} 1 & 4 \\ 2 & 3 \end{matrix}$$

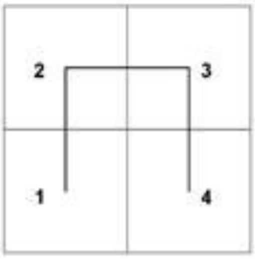

Fig. 2: Initial Pseudo Hilbert Curve.

The next iterated matrix is obtained by subdividing each of the 4 squares in figure 2 by 4 to obtain

```
      1    4    5 6
a2 =  2    3    8 7
     15   14    9 10
```

16 13 12 11

We notice that a2 is obtained from a1 by rotation and reflection. This is coded in the developed Scilab function hilspfill (a1, m). This function takes as input, the order of the iteration m, and the starting matrix a1. It outputs the next iterated Hilbert space filling matrix. The Scilab code is given in Appendix 1.

3. Image processing

In image processing it is easier to operate on the pixels, to modify or analyse certain portions of the image if it is mapped to a single vector. This, if done while preserving the locality of the image portions in contiguous sequences would enable effects of higher resolution to be easily handled.

Consider an image as below in figure 3.The diagonal portions, which are white have values 1 and black portions have value 0 .the corresponding image matrix is given by matrix B.

[1 1 1 1 0 0 0 0
1 1 1 1 0 0 0 0
1 1 1 1 0 0 0 0
B = 1 1 1 1 0 0 0 0
0 0 0 0 1 1 1 1
0 0 0 0 1 1 1 1
0 0 0 0 1 1 1 1
0 0 0 0 1 1 1 1]

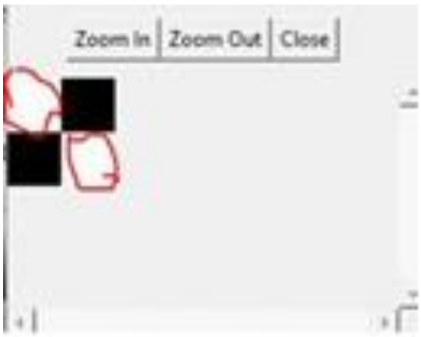

Fig. 3: Square Image with Diagonal Portions in White.

The comparison of the linear mapping of the Hilbert and snake curve is as shown in figure 4 and figure 5. This demonstrates the localizing property of the Hilbert space filling curve for image processing.

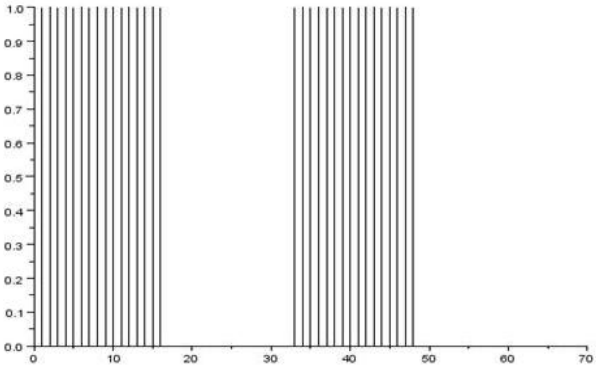

Fig. 4: Hilbert Curve Showing Contiguous Bands.

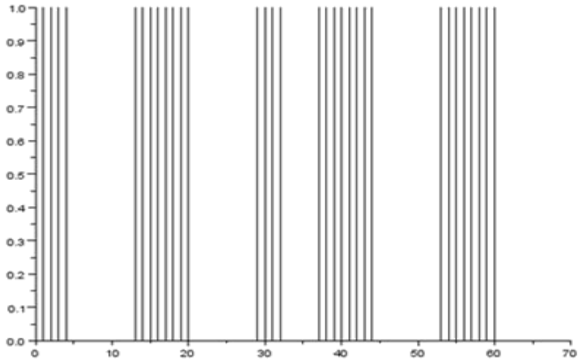

Fig. 5: Snake Curve Gives Disjointed Mapping.

A further advantage of Hilbert space filling curve is when resolution of the image is increased by increasing the order m. The linear mapping will still be confined to contiguous sequences for Hilbert space filling curves. So a processing of the image at one resolution will remain similar at a higher resolution. However in the snake curve the sequences are not consistent as shown in the figure. [4] discussed that the activity related status data will be communicated consistently and shared among drivers through VANETs keeping in mind the end goal to enhance driving security and solace. Along these lines, Vehicular specially appointed systems (VANETs) require safeguarding and secure information correspondences. Without the security and protection ensures, the aggressors could track their intrigued vehicles by gathering and breaking down their movement messages. A mysterious message confirmation is a basic prerequisite of VANETs.

4. Conclusion

The use of Hilbert space filling curve, which has a localizing property and its use in Image processing has been demonstrated. A Scilab code to iterate pseudo Hilbert space filling curve has been developed and serves as a reference for further work.

References

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Appendix

```
function[aa] =hilspfill(a1,m)
// This function outputs next order in matrix aa of Hilbert space
matrix
// given input a1 matrix and next order m
a1=a1'
//m order of Hilbert matrix
tm=2^m // dim h matrix
tm1=tm/2
tm2=tm*tm//total components
c1=[]
for i=1:tm1
c1(:,i)=a1(:,i); // first tm-1col of a1
end
con=max(a1)
a2=c1+con ;
a3=a2+con ;
a2=a2';
a3=a3';
g=flipdim(a3,1);
a4= flipdim(g',1) +con ;
aa=[a1 a2; a4 a3] ;
endfunction
// Run program starting from m=1 i.e. a1 matrix below
a1= [1 2 ;4 3]
m=2 //next order matrix ie 4,4
a2= hilspfill(a1,m) ;// gives following result
m=3
a3 =hilspfill(a2,m);
sa= size(a3)
sx=sa(1)*sa(2)
xlin= (1:sx) ;
eb=zeros(sa(1),sa(2));// eb is image matrix of size a3
eb(1:4,1:4)=1 ; // set some portions of eb to white
eb(5:8, 5:8)=1;
for i=1:sa(1)
for j=1:sa(1)
tm=a3(i,j);
xlin(tm)= eb(i,j);
end;
end;
figure(0);
title("hilbert space filling curve")
plot2d3(xlin) ;
imshow(eb);
// snake fill curve is generated for comparison
y1=(1:sa(1))
sn(1,:) =y1
for i=2:sa(1)
ic=i-1;
tmp= sn(ic,:) +sa(1) ;
sn(i,:)= flipdim(tmp,2) ;
end
sn
for i=1:sa(1)
for j=1:sa(1)
tm=sn(i,j);
ylin(tm)= eb(i,j); // snake filling curve
end;
end;
figure(1) ;
title("snake filling curve");
plot2d3(ylin)
```