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Research paper

Performance assessment for MUSIC-based DoA estimators

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Abstract

Direction of arrival estimation is a popular technique used for localization and tracking. Many techniques were developed to detect it even in a multipath environment. In this paper we introduce an assessment for four techniques that are based on the MUSIC algorithm. They are the Toeplitz, modified virtual SVD (MV-SVD), Virtual array extension (VSS) and the Modified VSS (MVSS). Two assessment metrics are employed for evaluating these techniques under several conditions such as various SNR vales and numbers of samples. These metrics are RMSE and the success rate.

Keywords: *MUSIC, Toeplitz, MV-SVD, VSS, MVSS, Success Rate (SR)*

1. Introduction

Direction of Arrival (DoA) is not only a useful positioning technique but also finds a wide spread of applications such as channel sounding [1] and wireless sensor networks [2]. For indoor localization scenarios, multipath signals are dominant, which are considered as coherent signal as seen by the DoA estimator algorithm [3]. A famous and extensively studied technique for DoA estimation is the Multiple Signal Classification (MUSIC) [4-6]. It is a super resolution DoA estimation but for non-coherent signals [7]. To use it with coherent signals, many techniques were developed to its capabilities with such signals. These techniques include Forward Spatial Smoothing (FSS) [8], Forward-Backward Spatial Smoothing (FBSS) [9-12], Singular Value Decomposition (SVD) [13], DoA Estimation using the Toeplitz Technique [14], Modified Virtual SVD (MV-SVD) [15], Virtual Array Spatial Smoothing (VSS) [16], and Modified VSS (MVSS) [17]. These diverse techniques, used for DoA estimation, however they have to be compared and tested against each other under the same conditions to find out the strengths and drawbacks of each technique.

In this paper, assessments for those techniques are conducted using various metrics which are: available number of snapshots, (ANS), and success rate, (SR). SR is defined as a measure of the percentage of number of runs that results in successful estimates for all DoAs t to the total number of runs. This metric indicates the robustness and stability of an estimation technique

The paper is organized as follows: section 2 describes the signal model used throughout the paper. Section 3 is dedicated for introducing the DoA estimation algorithms. The system setup can be found in section 4. The simulation results and discussion are introduced in section 5. The conclusions for the paper are in section 6.

2. Signal model

Consider D coherent sources that produce a multipath signal and imping on an antenna array with number of elements M. The multipath signal is incident on the M antenna elements form the angles i , $i = 1,2, D$. The antenna elements have w_m potential weights, where $1 \leq m \leq M$. The received signals $x_m(k)$ are corrupted by additive white Gaussian noise with zero mean and variance ${\sigma_n}^2$, where k represents a time sample. The output of the array, $y(k)$ can be then expressed as,

$$y(k) = w^T x(k) \tag{1}$$

Where,

$$w = [w_1 \quad w_2 \quad \cdots \quad w_M]^T \tag{2}$$

And

$$x(k) = [a(\theta_1)\, a(\theta_2) \cdots a(\theta_D)].\begin{bmatrix} s_1(k) \\ s_2(k) \\ \vdots \\ s_D(k) \end{bmatrix} + \mathrm{n}(k)$$

$$= \mathbf{A}.\mathbf{s}(k) + \mathbf{n}(k) \tag{3}$$

Where $\mathbf{s}(k)$ is defined as the vector of incident signals at an instant k which include the snapshots, $s_i(k)$, $1 \leq i \leq D$. A is the steering matrix that is combined from the steering vectors, $a(\theta_i)$ for the M array sensors, and $\mathbf{n}(k)$ is the additive white Gaussian noise vector at each array element at an instant k. Fig.1 illustrates the modelling of the received signal $y(k)$.

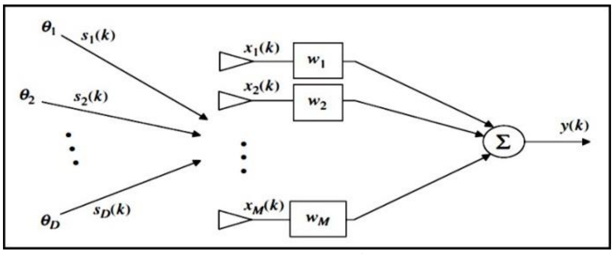

Fig. 1: Received Signal Model for Antenna Arrays.

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3. Direction of arrival estimation algorithms

There are many techniques for DoA estimation. Among them are the subspace techniques .This approach is based on obtaining the Eigen structure of the covariance matrix R_x of the received signal. The Eigen structure is partitioned into a signal subspace and noise subspace. One interesting feature in such a technique is that the steering vectors corresponding to the source directions are orthogonal to the noise subspace. Many algorithms were developed around this approach including the Multiple Signal Classification (MUSIC) algorithm. MUSIC can be used efficiently for DoA estimation of the non-coherent signals [4-6]. However, it fails if the signals impinging the array are coherent [11]. Such signal nature cause singularity in the covariance matrix R_x . To recover the capabilities of MUSIC many techniques were developed to remedy. Those include Forward Spatial Smoothing (FSS) [8], Forward-Backward Spatial Smoothing (FBSS) [9-12], Singular Value Decomposition (SVD) [13], Modified Virtual SVD (MV-SVD) [15], Virtual Array Spatial Smoothing (VSS) [16], and Modified VSS (MVSS) [17]. In general, subspace techniques feature super-resolution as compared to other techniques. In this section, the description of these algorithms is introduced.

3.1. The MUSIC algorithm

The covariance matrix can be estimated using the received signal samples by Eq. (4):

$$R_x = \frac{1}{k}\sum_{n=1}^{k} x(t)x^{\mathcal{H}}(t) \quad (4)$$

Where $\mathcal{H}$ is the Hermitian transpose operator.
The Eigen decomposition process is run on R_x to find Eigenvalues and Eigenvectors for the covariance matrix. These Eigenvalues and their corresponding Eigenvectors are then rearranged in a descending order. Using the noise variance, σ_n^2 as a threshold, the Eigenvalues that are greater than σ_n^2 and the corresponding Eigenvectors are separated to form a representation of the signal subspace, while the Eigenvalues that are below the threshold and corresponding Eigenvectors are separated to form the noise subspace. Since the noise subspace and the signal subspace are orthogonal, then the MUSIC spectrum can be calculated as in Eq. (5)

$$P_{MUSIC}(\theta)=\frac{1}{a(\theta)^{\mathcal{H}}\bar{E}_N\bar{E}_N{}^{\mathcal{H}}a(\theta)} \quad (5)$$

Where E_N is the matrix that includes the Eigenvectors that represent the noise subspace. Hence, at the DoAs, the $P_{MUSIC}(\theta)$ will peak and ideally reaches infinity [2], [3]. If the signals impinging on the array are coherent (such as the case of a multi-path signal), MUSIC fails to find the correct DoAs. There are many techniques to overcome this problem.

3.2. Forward-only smoothing (FS)

This techniques is proposed by [8] to restore the capabilities of MUSIC for the coherent signal problem .This can be achieved by splitting the array output to obtain L subarrays, where $L = M - p + 1$, and p is the number of array sensors in each subarray, $p < M$.this shown in Fig.2 The covariance matrix, R_i^{fs} , $1 \le i \le L$, for each subarray is then calculated as in Eq. (6). The total covariance matrix, R_{fs}, is simply the average of R_i^{fs} as shown in Eq. (7)

$$R_i^{fs} = x_L(t)x_L^{\mathcal{H}}(t) \quad (6)$$

$$R_{fs}=\frac{1}{L}\sum_{i=1}^{L} R_i^{fs} \quad (7)$$

Finally the MUSIC can be run using R_{fs}.

This algorithm can detect coherent signals up to $M/2$ sources but it has poor performance at low SNRs.

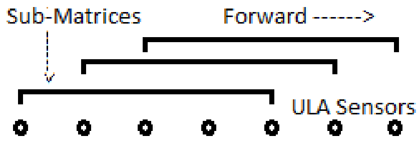

Fig. 2: Forward Spatial Smoothing.

3.3. Forward-backward spatial smoothing (FBSS)

This approach is found in [9-12]. It can improve the DoA detection over the FS. This can be done by calculating a forward covariance matrix, R_{fs} like the FS algorithm and a backward covariance matrix, R_{bs} like shown in Fig. 3. In this case the output of L^{th} forward sub-array is denoted by $x_{P,L}^f(t)$ with elements $\{x_L(t), \dots\dots, x_{L+P-1}(t)\}$, and the output of the L^{th} backward sub-array is denoted by $x_{P,L}^b(t)$ with elements $\{x_{M-L+1}^*(t), \dots, x_{M-P-L+2}^*(t)\}$.
The covariance matrix of the forward backward spatial smoothing (FBSS) can be calculated as indicated in Eq. (8).

$$R_x^{fb} = \frac{1}{2L}\sum_{i=1}^{L}\sum_{j}^{L}\{ x_{P,L}^f(t)x_{P,L}^{f\;\mathcal{H}}(t) + x_{P,L}^b(t)x_{P,L}^{b\;\mathcal{H}}(t)\}$$

$$R_x^{fb} = \frac{1}{2}\left(R_{fs} + R_{bs}\right) \quad (8)$$

The MUSIC algorithm is then run using R_x^{fb} to compute. the DoAs. This technique helped to increase the detection capabilities up to $2M/3$ DoAs. The drawback of FBSS is the poor performance at low. SNRs as for the FS [4].

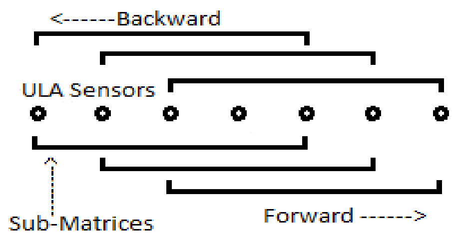

Fig. 3: Forward-Backward Spatial Smoothing.

3.4. Singular value decomposition (SVD)

The Singular value decomposition (SVD) can overcome the deterioration of performance at low SNRs for coherent signals as compared to the FS and FBSS techniques [13]. It employs the Eigenvectors corresponding to the largest Eigenvalues of the covariance matrix R_x, $e = [e_1, e_2, \dots\dots, e_M]$ to construct the matrix Y as follows;

$$Y = \begin{bmatrix} e_1 & e_2 & \dots & e_p \\ e_2 & e_3 & \dots & e_{p+1} \\ \dots & \dots & \dots & \dots \\ e_m & e_{m+1} & \dots & e_M \end{bmatrix} \quad (9)$$

Where $m > N$ and $m + p - 1 = M$
A singular value decomposition of Y is then calculated and the DoAs are estimated using the MUSIC algorithm.
According to the above discussions, (SVD) algorithm can be applied. But the resolution is not satisfactory especially at low SNRs. To improve the resolution, a modified SVD (MSVD) algorithm is proposed in [13], where Y_0 is calculated according to Eq. (10)

$$Y_0 = Y\,Y^{\mathcal{H}} \quad (10)$$

Just like the spatial smoothing technique in [9], [10], backward smoothing can be applied for effective utilization of data matrix, to find Y_1 according to Eq. (11)

$$Y_1 = \frac{1}{2}\left(Y_0 + J_m Y_0^{*} J_m\right) \quad (11)$$

The singular value decomposition of Y_1 is obtained and the estimate of DoAs is found using the MUSIC algorithm. the correlation of sources is not deduced or disrupted by using Y_1 in calculations,. It's efficient especially when the SNR is low. MSVD can detect up to $2M/3$ coherent signals.

3.5. DoA Estimation using the Toeplitz technique

This technique depends on building a Toeplitz matrix from the Eigenvectors of the data covariance matrix R_x [13]. These Eigenvectors are the corresponding to the largest Eigenvalues [18]. The Toeplitz matrix is constructed according to Eq. (12).

$$Y=\begin{bmatrix} r(0) & r(1) & \cdots & r\,(M-1) \\ r(-1) & r(0) & \cdots & r(M-2) \\ \cdots & \cdots & \cdots & \cdots \\ r(-M+1) & r(-M+2) & \cdots & r(0) \end{bmatrix}_{M\times M} \quad (12)$$

Where:

$r\,(k-1) = E\{e_{11}e_{1k}^{\mathcal{H}}\}, k = 1,2,\dots,M$

$r\,(-k+1) = E\{e_{1k}e_{11}^{\mathcal{H}}\}, k = 1,2,\dots,M$

It has high stability and resolution especially at low SNR. It can detect up to $M-1$ coherent sources [6].

3.6. Virtual array spatial smoothing (VSS)

Unlike the above discussed techniques, the Virtual array spatial smoothing (VSS) is based on extending the actual M sensor to virtual $(2M-1)$ sensors. The new data matrix from the extended array is found using:

$$x_{VSS}(t)=\begin{bmatrix} x'(t) \\ x(t) \end{bmatrix} \quad (13)$$

Where $x'(t)$ is the conjugate of $x(t)$. The FBSS algorithm is applied and MUSIC is applied on $x_{VSS}(t)$ to estimate the DoAs. This technique can result in a high resolution estimate of DoAs and features a higher stability compared to the above discussed techniques [16].

3.7. Modified VSS (MVSS)

This technique makes use of the idea behind the VSS. The real array samples, $x(t)$ is divided into two sub arrays, $x_1(t)$ and $x_2(t)$ that have the same dimensions, while the virtual array samples, $x'(t)$ is separated to equally dimensions sub arrays, $x_1'(t)$ and $x_2'(t)$. The virtual array extended matrix, $x_{MVSS}(t)$ is found to be best constructed according to Eq. (14)

$$x_{MVSS}(t) = \begin{bmatrix} x_1'(t) \\ x_1(t) \\ x_2(t) \\ x_2'(t) \end{bmatrix} \quad (14)$$

The $x_{MVSS}(t)$ is treated as the $x_{VSS}(t)$ in the VSS technique to find the DoAs. MVSS gives a better performance than the traditional VSS. This algorithm improved the estimation accuracy without increasing the number of the antenna array elements or changing any parameters in the ordinary VSS operation. [17]

3.8. Modified virtual –SVD (MV-SVD)

This algorithm is a combination of virtual array extension VSS, SVD and a modified version of the MUSIC. The procedure of this algorithm is as follows: (1) making virtual array on the receiving data $x(t)$ to get x_1 according to Eq. (13). Construct a new virtual data covariance matrix R_{x1} where:

$$R_{x1} = E\left[x_1 x_1^{\mathcal{H}}\right] \quad (15)$$

Apply SVD algorithm on R_{x1} to get matrix Y as in Eq. (9). To improve the resolution use MSVD to get Y_0 , Y_1 as in Eq. (10)and Eq. (11). Obtains the noise subspace from Y_1 using singular value decomposition. Estimate the DOA by using the modified $MUSIC$ equation: The modified MUSIC spectrum is used in spite of the original MUSIC spectrum to enhance the resolution of the DOA estimation process. In Eq. (16), as noticed from the denominator, the orthogonality between $a(\theta)$ and $V_n$will reduce it to a minimum, and hence will increase $P_{MUSIC}(\theta)$ which leads to high resolution in detecting the largest peaks of the MUSIC spectrum that correspond to the DOAs of the signals impinging on the array.

$$P_{MUSIC}(\theta)=\frac{a(\theta)^{\mathcal{H}} RA\, a(\theta)}{a(\theta)^{\mathcal{H}} \bar{E}_N \bar{E}_N^{\mathcal{H}} a(\theta)} \quad (16)$$

Where,

$RA = U_s B U_s^{\mathcal{H}}$ & $U_s = [e_1 e_2 \dots e_N]$ is the signal subspace.

$B = diagonal\,(\frac{1}{SS} - sigma \times I_N)$, $SS = diagonal\,(S_s)$ is signal's Eigen values, $sigma = \frac{trce\ (S_n)}{M-N}$.

MS-SVD advantages, it is used to estimate coherent and non-coherent signals .It has high stability and high resolution especially at low SNR and can detect $M-1$ signals [15].

4. Simulation setup

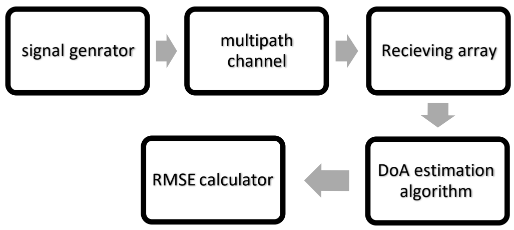

Fig. 4: Simulation Setup Model.

Fig. 4 introduces a block diagram for the simulation system used. The signal generator generates D signals. These signals enter the multipath channel and the AWGN is added. The receiving array collects the different incident signals. The DoA estimation algorithm block is responsible for applying the different algorithms for DoA to estimate RMSE & success rate. The root mean square error (RMSE) is calculated in the RMSE calculator block according to Eq. (17).

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{i=N}(\theta_{Ei} - \theta_i)^2} \quad (17)$$

Where θ_{Ei} is the estimated angle of the i^{th}sources , θ_i is the original angle of arrival of the i^{th}source .

5. Simulation results and discussion

5.1. Experiment 1: RMSE vs. SNR

In these simulations, a comparison is preformed between the algorithms to obtain the RMSE vs. SNR at $k = \{1000, 500, 100, 50, 10, 4, 2, 1\}$. Fig.5 and Fig.6 represent the results for these experiments.

In Fig.5a the k takes the values $1000, 100$. It can be shown that:

At $k = 1000$ and $k = 500$, the RMSE becomes nearly identical as SNR reaches 5 dB which can be considered the Marginal SNR point (MSP) for the tested four estimation algorithms.

As SNR decreases below $5\ dB$, the RMSE differs for the four estimation techniques. Moreover, the MVSS and VSS have nearly the same behaviour, while MV-SVD and Toeplitz behave nearly identical. It should be noted that the former has the worst performance while the later shows low RMSE. This means that MV-SVD and Toeplitz can perform accurately for low SNRs. At $SNR = -10\ dB$, RMSE$\approx 0.75^o to\ 0.8^o$ for Toeplitz and MV-SVD compared to RMSE$\approx 3^o to\ 3.5^o$ for VSS and MVSS at the same SNR.

To observe the effect of the number of samples on the RMSE, k is further set to 100 and to 50. The simulations also have indicated the same observation noted above but with two notes: Firstly, The MSP for the four algorithms is the same as for $k = 1000$ and $k = 500$. Secondly, As SNR decreases blow the MSP, RMSE differs. The MV-SVD and Toeplitz achieves the best performance in terms of RMSE as expected. The VSS and MVSS has also the same behaviour but with the worst performance as compared to MV-SVD and Toeplitz. At $SNR = -10dB$,RMSE $\approx 3^o$ for Toeplitz and MV-SVD while RMSE $\approx 11^o$ for MVSS and RMSE$\approx 12^o$for VSS.

In Fig.6 the k takes the values $2\ and\ 1$. It can be shown that for $k = 2$, the same notes were obtained, but with two differences: Firstly, The MSP for the four algorithms is shifted to $10dB$. Secondly, At $SNR = -10dB$, RMSE$\approx 12^o$ for Toeplitz and MVSS compared to RMSE$\approx 13^o$for MV-SVD and RMSE$\approx 14^o$for VSS at the same SNR.

At $= 1$, it is noted that, the four algorithms have the same behaviour after $SNR = 15\ dB$ which is the new MSP under the condition of $k = 1$. Blew this value the four algorithms have very close RMSE all over the test range. At $SNR = -10dB$, RMSE$\approx 21^o$ for MVSS, MV-SVD and VSS compared to RMSE$\approx 22^o$for Toeplitz at the same SNR.

In summation, as k dropes to small values (1 or 2), the RMSE is negatively impacted as k decreases.

As the k increases the MSP shifts to the right. Teoplitz and MV-SVD are better than MVSS and VSS at low SNRs.

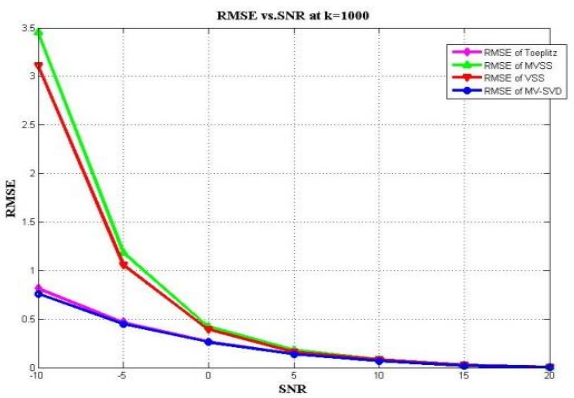

Fig. 5a: $k = 1000$.

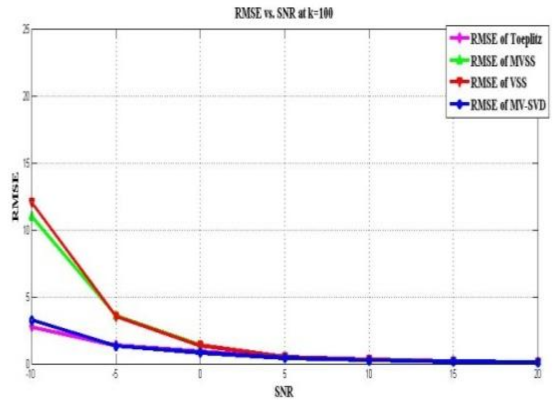

Fig. 5b: $k = 100$.

Fig. 5: Comparison between MV-SVD, VSS, MVSS and Toeplitz to Estimate RMSE vs. SNR at $k\ = 1000\ and\ K = 100$.

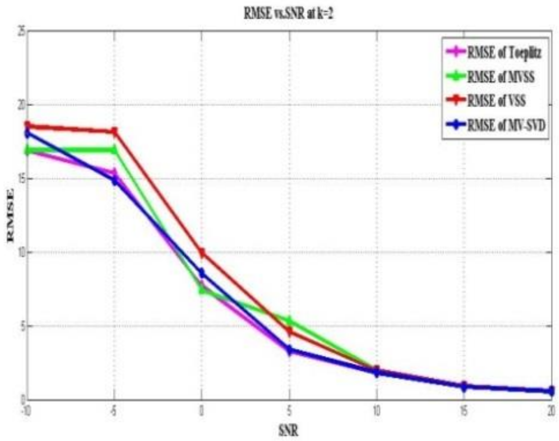

Fig. 6a: $k = 2$.

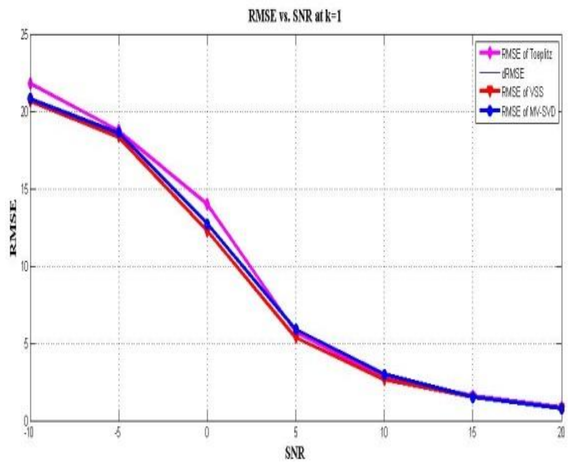

Fig. 6b: $k = 1$.

Fig. 6: Comparison between MV-SVD, VSS, MVSS and Toeplitz To Estimate RMSE Vs. SNR at $k = \ 2\ and\ k = 1$.

5.2. Experiment 2: success rate (SR)

In this experiment, the performance of the estimation algorithms is evaluated using the number of successfully estimating all the DoAs of the received signal at different SNRs if the estimation is run under the same conditions for only 100 runs. This measure is called the Success Rate (SR). SR is an indication to the number of times that the algorithm can estimate all the correct coherent signals without losing any angle. Success rate can be gotten by dividing the number of correct runs to the total number of runs. The results for these simulations can be found in Fig.7.

In Fig.7a, $k = 1000$, and the SR is 100% for all techniques and all SNRs. At $k = 500$ as in Fig.7b, all algorithms have SR=100% allover the SNRs except the VSS has SR=98% at SNR= -10 dB.

SR vs. SNR at $k = 100$ is depicted in Fig.7c, where the 100% SR is still obtained except at SNR=-10 dB. The MVSS has SR=89% and VSS has SR=92% and Toeplitz has SR=99%.
In Fig.7d, $k = 50$, reports a deterioration in SR at SNR below 0 dB. At SNR=-5 dB, SR=99% for VSS and SR=96% for MVSS and 100% for Toeplitz and MV-SVD. Moreover, at SNR=-10, SR=94% for MV-SVD, SR=75% for MVSS, SR=85% for VSS and Toeplitz has SR=92%.
Examining Fig.7e, it is noted that, SR=100% for all techniques for SNR=5 and above. For SNR=0 all techniques have SR=100% except VSS which has SR=98%. At SNR=-5, MV-SVD and MVSS have SR=88%, VSS has SR=86%, and Toeplitz has SR=84%. The deterioration in SR become evident at SNR=-10, as MV-SVD has SR=65%, MVSS has SR=79%, VSS has SR=70% and Toeplitz has SR=60%.
The SR for the assessed techniques at $k = 2$ and $k = 1$ are depicted in Fig.7f, Fig.7g respectively. The numerical representations are found in Table.1 and Table 2. Table.1 shows results for $k = 2$ and it is noted that all the values of SR above SNR= 5 are 100%. But in Table.2 for $k = 1$, the values which are at SNR =15 are with SR of 100% which is clear in the table.

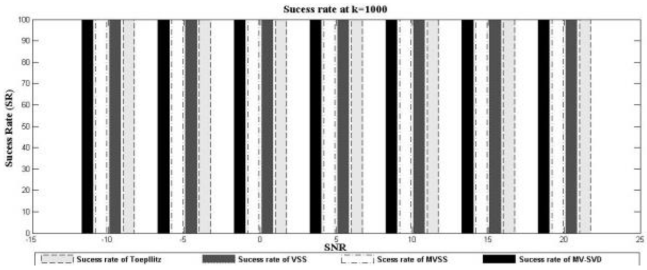

Fig.7a: $k = 1000$.

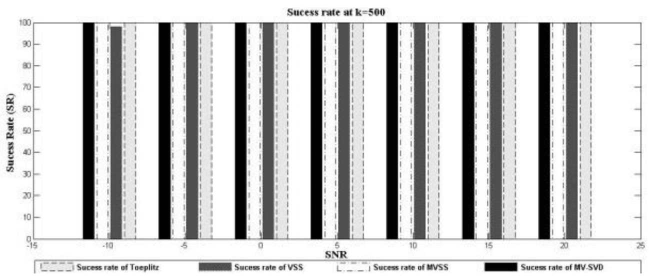

Fig.7b: $k = 500$.

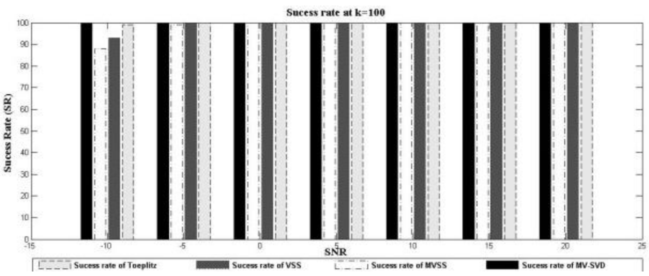

Fig.7c: $k = 100$.

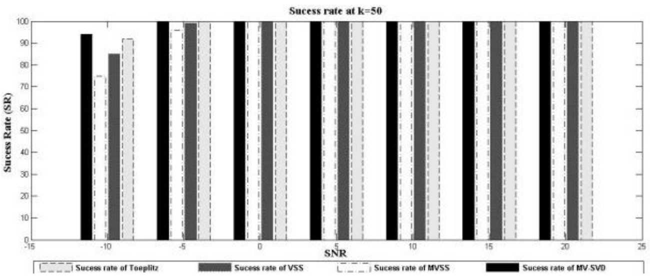

Fig.7d: $k = 50$.

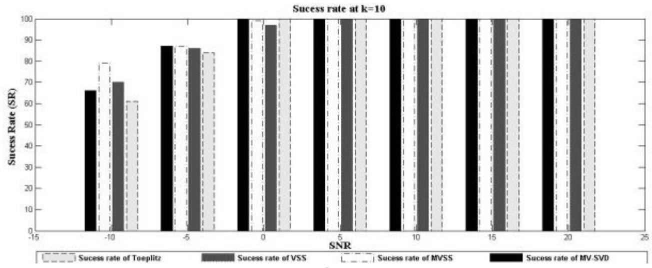

Fig.7e: $k = 10$.

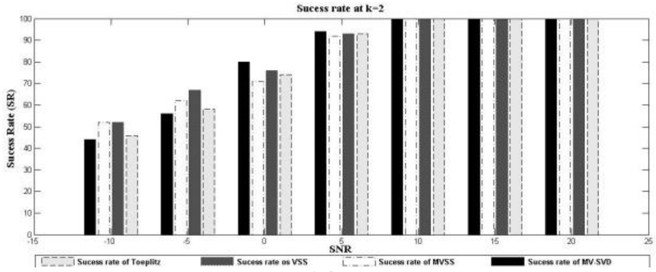

Fig.7f: $k = 2$.

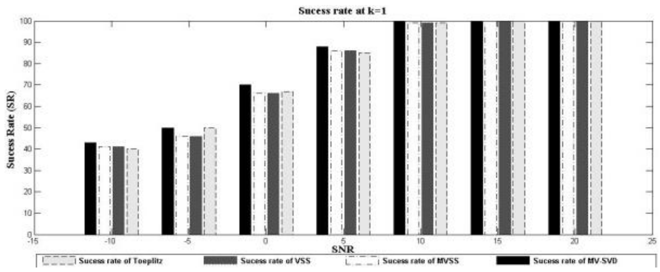

Fig.7g: $k = 1$.

Fig. 7: Success Rate (SR) for DoA Estimation vs. Number of snapshots, k.

Table 1: The Numerical Representations for $k = 2$

	SNR=5	SNR=0	SNR=-5	SNR=-10
MV-SVD	93	80	56	44
MVSS	90	70	62	52
VSS	91	75	67	52
Toeplitz	91	73	56	46

Table 2: The Numerical Representations for $k = 1$

	SNR=10	SNR=5	SNR=0	SNR=-5	SNR=-10
MV-SVD	100	87	79	59	42
MVSS	99	84	63	44	40
VSS	99	84	63	44	40
Toeplitz	100	82	64	50	39

From these results we can note that, if $k \geq 500$ the SR is always about 100% for all techniques. As k decreases, the SR decreases. To restore SR for a certain k value the SNR should be increased. The MV-SVD performs superior to other methods especially at low k, SNRs, where its SR is always the greater as noted for $SNR \leq -5$ at $k = 10$ and for $SNR \leq 5$ for $k = 1$, $k = 2$.

6. Conclusions

In this paper, an assessment for the direction of arrival estimation for coherent signals based on the MUSIC algorithms is introduced. Four MUSIC based techniques were examined. They are called, Toeplitz, MV-SVD, VSS and MVSS.
Two assessment metrics is deployed RMSE, and success rate. It is notes that as k dropes to small values (1 or 2), the RMSE is negatively impacted as k decreases.
It is also found that as k increases the MSP shifts to the right. Teoplitz and MV-SVD are better than MVSS and VSS at low SNRs
On measuring the success rate (SR) for the examined techniques, it is found that if $k \geq 500$ the SR is always about 100% for all techniques. As k decreases, the SR decreases. To restore SR for a

certain k value the SNR should be increased. The MV-SVD performs superior to other methods especially at low k and SNRs, where its SR is always the greater as noted for $\text{SNR} \leq -5$ at $k = 10$ and for $\text{SNR} \leq 5$ for $k = 1$, $k = 2$.

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