

# Performance enhancement of MPM using SNR boosting techniques

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## Abstract

Signal to noise ratio (SNR) boosting is one of the most important research areas in signal processing. The effectiveness of SNR boosting is not limited to a specific application rather, it is widely used in image processing, signal processing, cognitive radio, MIMO systems, digital beam forming, and direction of arrival (DOA) estimation ...etc. In this paper, the recursive least square (RLS) and wavelet based de-noising filters are exploited for SNR boosting in DOA estimation techniques for further performance enhancement. The matrix pencil method (MPM) as an effortlessness and high resolution DOA estimation technique is taken as a test case. That is because it suffers from performance deterioration under low SNR regimes. The simulation results reveal that the MPM based RLS de-noising filter outperforms the MPM based wavelet de-noising filter and the traditional MPM in terms of mean square error (MSE) especially at low SNR regimes.

**Keywords:** Direction of Arrival (DOA); Matrix Pencil Method (MPM); Recursive Least Square (RLS) Filter; and Uniform Linear Array (ULA).

## 1. Introduction

Direction of arrival estimation (DOA) for coherent and non-coherent signals is considered as one of the most important aspects recently. It has a noticeable impact on many communication systems such as, radar systems, smart antennas, mobile communications, and cognitive radio systems. Many algorithms had been investigated in this research area [1]. The most commonly used DOA estimation techniques are; Multiple Signal Classification (MUSIC), Root MUSIC, estimation of signal parameters via rotational invariance techniques (ESPRIT), and the Matrix Pencil Method (MPM). These techniques suffer from performance deterioration under low SNR regimes. The challenging goal of statistical estimation and signal processing is to recover the signal from its noisy version.

MUSIC is one of the earliest proposed and a very popular method for super-resolution direction finding, which gives the estimation of number of signals arrived, hence their direction of arrival. MUSIC is a technique based on exploiting the Eigen structure of input covariance matrix. Eigen vectors are easily obtained by either an Eigen decomposition of sample covariance matrix or a Singular Value Decomposition (SVD) of the data matrix. MUSIC makes the assumption that the noise in each channel is uncorrelated making the noise correlation matrix to be diagonal. The incident signals may be somewhat correlated creating a non-diagonal signal correlation matrix. However, under high signal correlation the traditional MUSIC algorithm breaks down and other methods must be implemented to correct this weakness [2].

MUSIC algorithm fails to obtain narrow and sharp peaks in case of coherent signals. An improved version of the MUSIC algorithm was presented in [3]. This improved algorithm achieves sharp peaks and makes the estimation process much accurate by increasing the inter element spacing, increasing the number of antenna sensors, number of snapshots and improving the incidence angle difference between the incoming signals.

In [4] an improved MUSIC algorithm based on wavelet de-noising filter was introduced. This technique improves the resolution of the detection process by de-noising the received signals at each antenna element in parallel. But, better performance can be obtained by applying more accurate de-noising filters.

In this paper, the recursive least square (RLS) and wavelet based de-noising filters are exploited for SNR boosting in DOA estimation techniques for further performance enhancement. The matrix pencil method (MPM) as an effortlessness and high resolution DOA estimation technique is taken as a test case. That is because it suffers from performance deterioration under low SNR regimes. The simulation results reveal that the MPM based RLS de-noising filter outperforms the MPM based wavelet de-noising filter and the traditional MPM in terms of mean square error (MSE) especially at low SNR regimes.

## 2. Traditional Matrix Pencil Method

Consider  $N$  narrowband signals falling from different directions on a uniformly-spaced linear antenna array (ULA) with  $M$  sensors where  $N \leq M$ . The array sensors are spaced by  $d = \lambda/2$  where  $\lambda$  denotes to the wavelength of the received multi-path signals. The output signal from the antenna array at an arbitrary time  $t$  can be expressed as in [6]:

$$X(t) = A(\theta) S(t) + n(t) \quad (1)$$

where  $S(t)$  is a matrix of the signals that impinging on the array sensors,  $n(t)$  is assumed to be an additive white Gaussian noise matrix with zero mean ( $\mu = 0$ ) and variance of  $\sigma^2$ ,  $X(t)$  is the ( $M \times K$ ) output matrix,  $K$  represents the number of samples, and  $A(\theta)$  denotes to the steering matrix of the ULA. They can be expressed as;

$$X(t) = [x_1(t), x_2(t), \dots, x_M(t)]^T$$

$$S(t) = [s_1(t), s_2(t), \dots, s_N(t)]^T$$

$$n(t) = [n_1(t), n_2(t), \dots, n_M(t)]^T$$

$$A(\theta) = [a(\theta_1), a(\theta_2), \dots, a(\theta_N)]^T$$

Where

$$a(\theta_i) = [1, e^{-j2\frac{\pi}{\lambda}d \sin(\theta_i)}, \dots, e^{-j(M-1)2\frac{\pi}{\lambda}d \sin(\theta_i)}]^T, \quad i = 1, 2, 3, \dots, N$$

And

$$x_m(t) = \sum_{i=1}^N s_i(t) e^{-j\frac{2\pi}{\lambda}d \sin(\theta_i)} + n_m(t), \quad (m = 1, 2, \dots, M)$$

Where  $a(\theta_i)$  denotes to the steering vector for a certain angle of arrival  $\theta_i$ , superscript T is the transpose and  $x_m(t)$  denotes to the input vector to the  $m^{\text{th}}$  antenna elements [7]. The traditional MPM algorithm can be implemented using the following steps:

Firstly, apply the Hankel matrix on Eq. (1) to obtain the matrix Y directly from X(t) as in [5]:

$$Y = \begin{bmatrix} X(0) & X(1) & \dots & X(L) \\ X(1) & X(2) & \dots & X(L+1) \\ \vdots & \vdots & \ddots & \vdots \\ X(M-L-1) & X(M-L) & \dots & X(M-1) \end{bmatrix}_{(M-L) \times (L+1)} \quad (2)$$

Where the parameter  $L = (M/2) - 1$  refers to the pencil parameter and it must be in the range  $N \leq L \leq M - N$ .

Secondly, getting the Eigen structure of the matrix Y by applying the singular value decomposition (SVD) technique on Eq. (2) as shown in Eq. (3) according to the literature [5]:

$$Y = U \Sigma V^H \quad (3)$$

Where U and V are the unitary matrices and  $\Sigma$  has the eigenvalues of Y on the main diagonal. Consequently, Y can be rewritten as:

$$Y = U_s \Sigma_s V_s^H + U_n \Sigma_n V_n^H \quad (4)$$

Where  $\mathcal{H}$  expresses the Hermitian transpose. The suffix s and n are symbols that demonstrate the signal subspace and noise subspace, respectively.  $\Sigma_s$  is a square diagonal matrix involving the eigenvalues  $\eta_i$  of Y such that  $\eta_1 > \eta_2 > \dots > \eta_N > \eta_{N+1} = \eta_{N+2} = \dots = \eta_M = \sigma_n^2$ . This implies that the signal eigenvalues are practically greater than the noise floor  $\sigma_n^2$ . The eigenvectors in the columns of U can be partitioned into  $U_s = [u_1, u_2, \dots, u_N]$  for signal eigenvectors that correspond to the signal subspace and  $U_n = [u_{N+1}, \dots, u_M]$  for noise eigenvectors that represent the noise subspace [8, 9]. In the same way, V can be separated into  $V_s = [v_1, v_2, \dots, v_N]$  and  $V_n = [v_{N+1}, v_{N+2}, \dots, v_M]$ .

$$\Sigma_s = \text{diag}(\eta_1, \eta_2, \dots, \eta_N), \text{ and } \Sigma_n = \text{diag}(\eta_{N+1}, \eta_{N+2}, \dots, \eta_M).$$

For the MPM technique, two new sub-matrices  $U_1$  and  $U_2$  are built from  $U_s$  as below [5]:

$$U_1 = U_s: \text{ With the last row is removed}$$

$$U_2 = U_s: \text{ With the first row is removed.}$$

Where  $U_1$  and  $U_2$  achieve the following equation:

$$U_2 - \beta U_1 = 0 \quad (5)$$

Which is a general eigenvalue equation by solving it, the eigenvalues  $\beta$  are found. Eq. (5) can be reduced to an ordinary eigenvalue problem utilizing the following method to obtain  $\beta$ :

Calculate  $U_1^+ U_2$ , where  $U_1^+$  represents the Moore–Penrose pseudo-inverse of  $U_1$  and can be expressed as:

$$U_1^+ = (U_1^H U_1)^{-1} U_1^H \quad (6)$$

Hence, the eigenvalues can be derived as follows:

$$\beta = \text{eig} \left( (U_1^H U_1)^{-1} U_1^H U_2 \right) \quad (7)$$

Find the estimated direction of arrivals  $\hat{\theta}_i$  directly by utilizing the following equation:

$$\hat{\theta}_i = -\sin^{-1}(\lambda * \text{Im}(\log \beta)) / (2\pi d) \quad (8)$$

### 3. Proposed DOA Estimation Techniques

As the traditional MPM suffers from performance degradation at low SNR regimes, the de-noising filters can be used to boost the SNR of the received signal. In this section, a new technique for accurate DOA estimation is introduced. It is based on a combination between the traditional MPM and de-noising filters. Two different de-noising techniques named RLS and wavelet are utilized. Figure (1) shows the block diagram of the proposed technique.

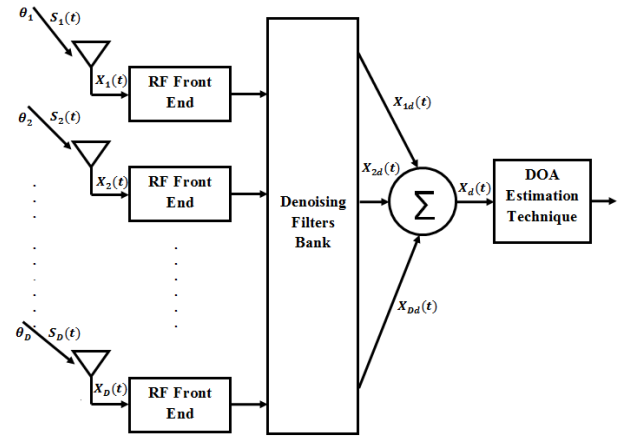


Fig. 1: Block Diagram of the Proposed DOA Estimation Technique.

In this system, the output noisy contaminated signals from each antenna element are applied separately to a de-noising filter. So that all the output signals from the antenna elements are de-noised in parallel. The outputs from the filters are combined together to form the de-noised data matrix  $X_d(t)$  where

$$X_d(t) = [x_{1d}(t) \ x_{2d}(t) \ \dots \ x_{M}(t)] \quad (9)$$

The data matrix X(t) is replaced directly by the de-noised data matrix  $X_d(t)$  in equation (2) of the traditional MPM to estimate the DOAs of the received signals.

### 4. De-noising Filters

In this paper, both wavelet de-noising filter and Recursive-Least-Squares (RLS) Adaptive filter are utilized to boost the SNR of the received signals. These filters are used to improve the accuracy of the DOA estimation process of the existing DOA estimation techniques. In addition, a comparison is performed between these two different types of de-noising techniques to clarify which type is recommended.

#### 4.1. Wavelet De-noising Filter

A wavelet is a small wave and in brief, a wavelet is an oscillation that decays quickly. Wavelet analysis is a new method for solving problems in physics, engineering, signal processing, and mathematics. Wavelets allow complex information as music, speech, images and patterns to be composed into elementary forms at different positions and scales and subsequently reconstructed with

high precision. Wavelet is also a powerful tool for removing noise from variety of signals. They can analyze the noise level separately at each wavelet scale and to adapt the de-noising algorithm accordingly. There are many cases in which the noise corrupts the signals in a significant manner, and it must be removed from the data in order to proceed with further data analysis. The process of noise removal is generally referred to as signal de-noising or simply de-noising [10].

**Wavelet transform:** A wavelet is a wave-like oscillation with an amplitude that starts out at zero, increases, and then decreases back to zero. Unlike the sines used in Fourier transform for decomposition of a signal, wavelets are generally much more concentrated in time. They usually provide an analysis of the signal which is localized in both time and frequency, whereas Fourier transform is localized only in frequency. Given a mother wavelet  $\Psi(t)$  (which can be considered simply as a basis function of  $L^2$ ), the continuous wavelet transform (CWT) of a function  $x(t)$  (assuming that  $x \in L^2$ ) is defined as

$$X(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} \Psi\left(\frac{t-b}{a}\right) x(t) dt \quad (10)$$

The scale parameter  $a$  corresponds to frequency information and the translation parameter  $b$  relates to the location of the wavelet function as it is shifted through the signal. So it corresponds to the time information in the transform. The integral in Eq. (10) can be seen as a convolution operation of the signal and a basis function  $\Psi(t)$  (up to dilations and translations). It should be emphasized that, unlike Fourier transform in which the basis function is  $e^{j\omega t}$ , there are many (in fact, infinite) possible choices of  $\Psi(t)$ . The transform which is used in the discrete wavelet transform (DWT) which transforms discrete (digital) signals to discrete coefficients in the wavelet domain. This transform is essentially a sampled version of CWT. Instead of working with  $a; b \in \mathbb{R}$ , the values of  $X(a; b)$  are calculated over a discrete grid:

$$a = 2^{-j}; b = k.2^{-j}; j; k \in \mathbb{Z} \quad (11)$$

This type of discretization is called dyadic dilation and dyadic position, respectively.

**Wavelet thresholding:** A common description of the de-noising problem is as follows. Suppose that there are  $n$  noisy samples of a function  $f$ :

$$y_i = f(t_i) + \sigma \varepsilon_i, i = 1, 2, \dots, n \quad (12)$$

Where the noise level  $\sigma$  may be known or unknown.

The orthogonality of DWT (assuming that orthogonal wavelets are used with periodic boundary conditions) leads to the feature that white noise is transformed into white noise. Hence, if  $y_{jk}$  (where  $j$  denotes the decomposition level and  $k$  is the index of the coefficient in this level) are the wavelet coefficients of  $y_i$ , the transform of Eq. (12) to the wavelet domain is:

$$y_{jk} = w_{jk} + \sigma \varepsilon_{jk} \quad (13)$$

Where  $w_{jk}$  are the (clean) wavelet coefficients of  $f(t_i)$ , consisting of the approximation and detail coefficients and  $\varepsilon_{jk}$  are iid  $N(0; 1)$ . That is, the wavelet coefficients of the observed signal can themselves be considered as a noisy version of the wavelet coefficients of the original signal.

The coefficients of the wavelet transform are usually sparse. That is, most of the coefficients in a noiseless wavelet transform are effectively zero. Therefore, we may reformulate the problem of recovering  $f$  as one of recovering the coefficients of  $f$  which are relatively "stronger" than the Gaussian white noise background. That is, coefficients with small magnitude can be considered as pure noise and should be set to zero. The approach in which each coefficient is compared with a threshold in order to decide whether

it constitutes a desirable part of the original signal or not, is called wavelet thresholding.

The thresholding of the wavelet coefficients is usually applied only to the detail coefficients  $d_{jk}$  of  $y$  rather than to the approximation coefficients  $c_{jk}$ , since the latter ones represent 'low-frequency' terms that usually contain important components of the signal, and are less affected by the noise. The thresholding extracts the significant coefficients by setting to zero the coefficients which their absolute value is below a certain threshold level, which is to be denoted by  $\lambda$ . This value can generally be a function of the decomposition/resolution level  $j$  and the index  $k$ , i.e.:  $\lambda = \lambda(j; k)$ , but usually it is function of  $j$  only:  $\lambda = \lambda(j)$ . In the latter case the threshold is called level-dependent threshold. The thresholded wavelet coefficients are obtained using either hard or soft thresholding;

The first one is called Hard Thresholding, and its equation is as follows:

$$\delta_{\lambda}^H(d_{jk}) = \begin{cases} 0, & \text{if } |d_{jk}| \leq \lambda \\ d_{jk}, & \text{if } |d_{jk}| > \lambda \end{cases} \quad (14)$$

The other one is called Soft Thresholding, and its equation is as follows:

$$\delta_{\lambda}^S(d_{jk}) = \begin{cases} 0, & \text{if } |d_{jk}| \leq \lambda \\ d_{jk} - \lambda, & \text{if } d_{jk} > \lambda \\ d_{jk} + \lambda, & \text{if } d_{jk} < -\lambda \end{cases} \quad (15)$$

Hard thresholding is usually pointed as wavelet thresholding. While soft thresholding is for the most part called wavelet shrinkage as it shrinks the high amplitude coefficients towards zero [10]. A popular estimate of the noise level  $\sigma$  is based on the last level of the detail coefficients, according to the median absolute deviation:

$$\sigma = \frac{\text{median}(\{|d_{j-1,k}|\}; k=0,1,\dots,2^{j-1}-1)}{0.6745} \quad (16)$$

The factor in the denominator is the scale factor which depends on the distribution of  $d_{jk}$ , and is equal to 0.6745 for a normally distributed data.

## 4.2. Recursive-Least-Squares Adaptive Filters

An adaptive filter may be understood as a self-modifying filter that adjusts its coefficients in order to minimize an error function. This error function, also referred to as the cost function, is a distance measurement between the reference or desired signal and the output of the adaptive filter. The input signal is given by  $x(n)$ , the reference signal or desired signal  $d(n)$  represents the desired output signal (that Usually includes some noise component), and  $y(n)$  is the output of the adaptive filter, and hence the error signal is defined as:

$$e(n) = d(n) - y(n) \quad (17)$$

Recursive-Least-Squares (RLS) adaptive filter is one of the most popular real time algorithms in system identification [11]. The concepts of RLS filters are that the coefficients are directly and continually adjusted on a step-by-step basis during the filtering operation. The three general steps of the RLS filtering are described below:

- Parameters:  $M$  is the filter order,  $\lambda$  is the exponential weighting factor,  $\delta$  is the value used to initialize  $P(0)$ , and  $P(n) = R^{-1}(n)$ .
- Initialization:  $w_0 = 0, P(0) = \delta^{-1}$ .
- Computation: for  $n = 0, 1, 2 \dots$  compute;

$$Z(n) = P(n-1) x^0(n) \quad (18)$$

$$g(n) = \frac{1}{\lambda + x^T(n)z(n)} z(n) \quad (19)$$

$$e(n) = d(n) - w_{n-1}^T x(n) \quad (20)$$

$$w(n) = w(n-1) + e(n)g(n) \quad (21)$$

$$P(n) = \lambda^{-1}[p(n-1) - g(n)z^H(n)] \quad (22)$$

There are four parameters which need to be initialized in RLS filter. Filter coefficient  $w(0)$ , input data  $x(0)$ , the matrix  $P(0)$  and the weighting factor  $\lambda$  that is used to discard the old data and placing more emphasis on recent data to enhance the tracing of time varying system.  $\lambda$  is chosen between 0.95 and 0.999 based on the concept of asymptotic sample length (ASL).

## 5. Simulation Results

In this section, the simulation results of the proposed DOA estimation techniques are introduced. Also, comparisons between the traditional MPM and the proposed algorithms are performed to verify the effectiveness of the proposed techniques. Consider two coherent signals incident on a dedicated ULA from the directions  $(30^\circ, 45^\circ)$ . The ULA consists of eight antenna elements with uniform elements spacing  $d = \lambda/2$ .  $K = 16$  snapshots are utilized.

### Test case (1):

The wavelet de-noising filter is applied on the noise contaminated received signals to boost the SNR value. The technique in this case is denoted as WDMPM. The filter parameters are adjusted as follow:

DWT with a wavelet family (db12) (Daubechies wavelets) is employed to de-noise the received signals. The number of decomposition levels for the DWT is taken as  $J=5$ , and the decomposition coefficients are calculated at each level. Also, universal thresholds selection are used, combined with soft threshold type, and the variance of noise level is kept without scaling. DOAs are calculated within the SNR range  $(-10$  to  $5)$  dB for both MPM and the proposed WDMPM. Wavelet de-noising is applied on each sensor's output independently.

The Mean Square Error (MSE) is utilized as a performance measure. It is calculated as follows:

$$MSE = \frac{1}{N} \sum_{i=1}^N (\hat{\theta}_i - \theta_i)^2 \quad (23)$$

where  $\hat{\theta}_i$  is the estimated angle and  $\theta_i$  is original incidence angle. The simulations are carried out over SNR range from  $-10$  dB to  $3$  dB. The mean square error (MSE) between the estimated and the original DOAs is measured for both traditional MPM and the proposed WDMPM as shown in figure 2. The MSE is reduced from  $-1.46$  dB to  $-2.75$  dB at SNR =  $-10$  dB while at SNR =  $3$  dB the MSE is reduced from  $-7.11$  dB to  $-25.94$  dB. These results indicate that the proposed WDMPM technique has much better performance than the traditional MPM.

### Test case (2):

The RLS de-noising is applied on the noise contaminated received signals to boost the SNR value. The technique in this case is denoted as RLSDMPM. The simulations are also carried out over the same SNR range from  $-10$  dB to  $3$  dB. The mean square error (MSE) is measured for both traditional MPM and the proposed RLSDMPM algorithm as shown in figure (2). The figure shows a significant enhancement in the system performance where the MSE is reduced from  $-1.46$  dB to  $-21.07$  dB at SNR =  $-10$  dB while at SNR =  $3$  dB the MSE is reduced from  $-7.11$  dB to  $-29.34$  dB.

The overall simulation results indicate that the MPM based RLS filter (RLSDMPM) performance greatly outstands the performance of the MPM based wavelet de-noising (WDMPM) and the traditional MPM especially at low SNR values. Furthermore, the MSE reduction reflects the high stability of the proposed algorithms under low SNR regimes.

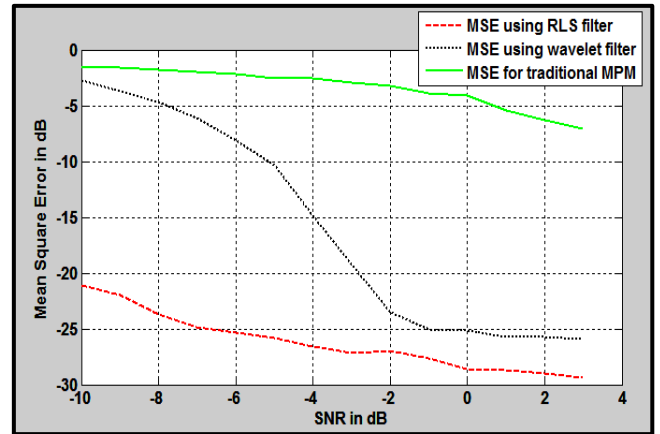


Fig. 2: Comparison between the Proposed MPM Based RLS Filter (RLS-MPM), MPM Based Wavelet De-Noising (WDMPM) and the Traditional MPM in Terms of MSE for Two Coherent Signals.

### Test case (3):

In this case, a comparison between the traditional MPM and the two proposed algorithms is performed to clarify the improvement obtained by the proposed techniques in terms of MSE and number of sources that can be detected. The proposed techniques mainly exploit de-noising methods to increase the ability of DOA algorithm to detect more number of sources.

Consider three coherent signals incident on the aforementioned ULA in test case (1) and test case (2) from the directions  $(20^\circ, 30^\circ, 45^\circ)$  using the same number of snapshots. The simulations are carried out over the same SNR range from  $-10$  dB to  $3$  dB. The MSE is measured for the traditional MPM, WDMPM and RLSDMPM techniques as shown in figure (3). It is clear that the RLSDMPM provides the lowest MSE within the range  $(-6.85$  dB to  $-7.8$  dB) for the three sources. However, in test case (1) and test case (2), the MSE of the traditional MPM for only two sources is bounded within the range  $(-1.6$  dB to  $-7.2$  dB) over the same SNR range as shown in figure (2). To summarize results, under a fair comparison of the different techniques, the RLS de-noising technique shows the best performance in terms of MSE with an increase in number of sources that can be detected compared to other reference methods.

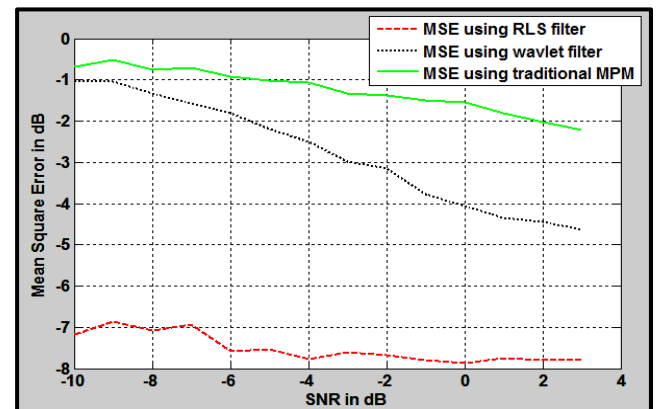


Fig. 3: Comparison between the Proposed MPM Based RLS Filter (RLS-MPM), MPM Based Wavelet De-Noising (WDMPM) and the Traditional MPM in Terms of MSE for Three Coherent Signals.

## 6. Conclusion

In this paper, the recursive least square (RLS) and wavelet based de-noising filters are exploited for SNR boosting in DOA estimation techniques for further performance enhancement. The simulations are carried out over SNR range from  $-10$  dB to  $3$  dB. The mean square error (MSE) between the estimated and the original DOAs is measured for both traditional MPM and the proposed

algorithms. The MSE in case of MPM based wavelet de-noising is reduced by 1.29dB at SNR = -10dB and 18.83dB at SNR = 3dB. While in case of MPM based RLS denoising is reduced by 19.61dB at SNR = -10dB and 22.23dB at SNR = 3dB. The simulation results indicate that the MPM based RLS filter performance greatly outstands the performance of the MPM based wavelet de-noising and the traditional MPM. Furthermore, the MSE reduction reflects the high stability of the proposed algorithms under low SNR regimes. In addition, the proposed RLSDMPM technique increases the number of detectable signals with lower MSE than the traditional MPM.

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