



Performance Enhancement of MPM DoA Estimation Technique using Wavelet De-noising Filter

Amr H. Hussein , Sameh A. Napoleon* and Haidy A. Eldawy

Electronics and Electrical Communications Engineering Department, Faculty of Engineering, Tanta University, Tanta, Egypt.

*Corresponding author E-mail: s.napoleon@f-eng.tanta.edu.eg

Abstract

Direction-of-arrival (DoA) estimation is now an imperative part in many radar applications and localization techniques. There are numerous algorithms that have been studied in the previous decades for DoA, for example: MUSIC, ESPRIT, and Matrix Pencil Method (MPM), which are subspace super resolution methods. MPM is one of the most commonly used subspace based techniques. It is generally utilized for DoA estimation because of its effortlessness and high resolution contrasted with other subspace techniques. But, it suffers from performance deterioration under low Signal-to-Noise Ratio (SNR) conditions. This paper, explores the possibility of utilizing the wavelet de-noising technique to intercept the degradation in the performance of MPM under different SNR values. Wavelet De-noising is intended to remove noise or distortion from signals while retaining the original quality of the signal. The simulation results indicate that the Daubechies wavelet (db12) at 5 levels of decomposition is the most suitable wavelet for de-noising the signals under test. Also, the results show that the proposed wavelet de-noising matrix pencil method (WDMPPM) outperforms the traditional MPM.

Keywords: Direction of Arrival (DoA); wavelet de-noising; signal-to-noise ratio (SNR); wavelet de-noising matrix pencil method (WDMPPM)

1. Introduction

The challenging goal of signal de-noising process is to recover the desired signal from its noisy version. Direction-of-arrival estimation is one of the most studied themes in signal processing. DoA estimation has been a vital area of research because of its significance in applications such as: wireless communications, smart antenna, and radar systems. An important widespread example of DoA methods is the Matrix Pencil Method because of its effortlessness and high resolution contrasted with other methods [1]. In wireless communications, the received signals from different paths are highly correlated. MPM is a direct data domain method that can assess the angles of the sources when the arriving signals are highly correlated, unlike (Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) and Multiple Signal Classification (MUSIC) [1],[2]. However, it has two main problems relative to practical applications; firstly, its performance suffers from a progressive degradation when SNR is reduced [1] and secondly, its performance can't be considered stable. Overcoming these drawbacks can be achieved by using the de-noising theory in conjunction with the ordinary MPM. There are a lot of signal de-noising techniques that can be utilized for removing the noise from the contaminated signals. Independent Component Analysis (ICA) de-noising that is used to search about the components that are statistically independent [3]. Principal Component Analysis (PCA) is a method of de-noising that transforms a number of correlated variables into a (smaller) number of uncorrelated variables called principal components [4], or in other words it expresses the original signal in the smallest form possible by eliminating redundant data from the set. Adaptive filters are

used to minimize the noise portion and get the uncorrupted desired signal [5]. Wavelet based de-noising has been applied on some of DoA techniques such as: the MUSIC technique where in the case of correlated signals and/or at low SNRs, the multiple signal classification (MUSIC) method degrades and also fails to estimate closely spaced signal directions-of arrival (DoAs). So, an improved DoA estimation technique using wavelet de-noising denoted as (WMUSIC) in the context of correlated signals was introduced in [6]. The same idea for ESPRIT was presented in [7]. This paper is dedicated to explore the possibility of utilizing wavelet based de-noising filters to harness the performance degradations of the Matrix Pencil Method in case of low SNR scenarios. The wavelet de-noising algorithm of Donoho [8], [9] is applied to boost the SNR values at the output of each antenna element. Matrix Pencil is then applied on the de-noised data matrix for more accurate DoA estimation. The influence of the de-noising process on the performance of Matrix Pencil is analyzed by simulations. The simulation results reveal that de-noising process leads to a significant improvement in the performance of the Matrix Pencil estimator. The remaining parts of this paper are arranged as follows: Section 2 demonstrates the modeling of the received signal. In the section 3, the classical MPM is presented. The wavelet de-noising theory is explored in section 4. The proposed WDMPPM method is presented in section 5. In the section 6, the simulation results are performed for verifying the proposed WDMPPM. Finally, the conclusion about these results is introduced in section 7.

2. Received Signal Model

Consider N narrowband signals falling from different directions on a uniformly-spaced linear antenna array (ULA) with M sensors where $N \leq M$. The array sensors are spaced by $d = \lambda/2$ where λ denotes to the wavelength of the received multi-path signals. The output signal from the antenna array at an arbitrary time t can be expressed as in [10]:

$$X(t) = A(\theta)S(t) + n(t) \quad (1)$$

where $S(t)$ is a matrix of the signals that impinging on the array sensors, $n(t)$ is assumed to be an additive white Gaussian noise matrix with zero mean ($\mu = 0$) and variance of σ^2 , $X(t)$ is the $(M \times K)$ output matrix, K represents the number of samples, and $A(\theta)$ denotes to the steering matrix of the ULA. They can be expressed as;

$$\begin{aligned} X(t) &= [x_1(t), x_2(t), \dots, x_M(t)]^T \\ S(t) &= [s_1(t), s_2(t), \dots, s_N(t)]^T \\ n(t) &= [n_1(t), n_2(t), \dots, n_M(t)]^T \\ A(\theta) &= [a(\theta_1), a(\theta_2), \dots, a(\theta_N)]^T \end{aligned}$$

where

$$a(\theta_i) = [1, e^{-j\frac{2\pi}{\lambda}d \sin(\theta_i)}, \dots, e^{-j\frac{2(M-1)\pi}{\lambda}d \sin(\theta_i)}]^T$$

and

$$x_m(t) = \sum_{i=1}^N s_i(t) e^{-j\frac{2\pi(m-1)}{\lambda}d \sin\theta_i} + n_m(t)$$

where $i = 1, 2, 3, \dots, N$, $m = 1, 2, \dots, M$, $a(\theta_i)$ denotes to the steering vector for a certain angle of arrival θ_i , superscript T is the transpose and $x_m(t)$ denotes to the input vector to the m^{th} antenna elements [11].

3. The Traditional MPM DoA Estimation Technique

The traditional MPM algorithm can be implemented using the following steps:

Firstly, apply the Henkel matrix on Eq.1 to obtain the matrix Y directly from $X(t)$ as in [1]:

$$Y = \begin{bmatrix} X(0) & X(1) & \dots & X(L) \\ X(1) & X(2) & \dots & X(L+1) \\ \vdots & \vdots & \ddots & \vdots \\ X(M-L-1) & X(M-L) & \dots & X(M-1) \end{bmatrix}_{(M-L) \times (L+1)} \quad (2)$$

where the parameter $L = (M/2) - 1$ refers to the pencil parameter and it must be in the range $N \leq L \leq M - N$.

Secondly, getting the Eigen structure of the matrix Y by applying the singular value decomposition (SVD) technique on Eq.2 as shown in Eq.3 according to the literature [1]:

$$Y = U \Sigma V^{\mathcal{H}} \quad (3)$$

where U and V are the unitary matrices and Σ has the eigenvalues of Y on the main diagonal. Consequently, Y can be rewritten as:

$$Y = U_s \Sigma_s V_s^{\mathcal{H}} + U_n \Sigma_n V_n^{\mathcal{H}} \quad (4)$$

where \mathcal{H} expresses the Hermitian transpose. The suffix s and n are symbols that demonstrate the signal subspace and noise subspace,

respectively. Σ_s is a square diagonal matrix involving the eigenvalues η_i of Y such that $\eta_1 > \eta_2 > \dots > \eta_N > \eta_{N+1} = \eta_{N+2} \dots = \eta_M = \sigma_n^2$. This implies that the signal eigenvalues are practically greater than the noise floor σ_n^2 . The eigenvectors in the columns of U can be partitioned into $U_s = [u_1, u_2, \dots, u_N]$ for signal eigenvectors that correspond to the signal subspace and $U_n = [u_{N+1}, \dots, u_M]$ for noise eigenvectors that represent the noise subspace [12], [13]. In the same way, V can be separated into $V_s = [v_1, v_2, \dots, v_N]$ and $V_n = [v_{N+1}, v_{N+2}, \dots, v_M]$. $\Sigma_s = \text{diag}(\eta_1, \eta_2, \dots, \eta_N)$ and $\Sigma_n = \text{diag}(\eta_{N+1}, \eta_{N+2}, \dots, \eta_M)$. For the MPM technique, two new submatrices U_1 and U_2 are built from U_s as below [1]:

$$\begin{aligned} U_1 &= U_s : \text{With the last row is removed} \\ U_2 &= U_s : \text{With the first row is removed.} \end{aligned}$$

where U_1 and U_2 achieve the following equation:

$$U_2 - \beta U_1 = 0 \quad (5)$$

which is a general eigenvalue equation by solving it, the eigenvalues β are found. Eq.5 can be reduced to an ordinary eigenvalue problem utilizing the following method to obtain β :

Calculate $U_1^+ U_2$, where U_1^+ represents the Moore-Penrose pseudo-inverse of U_1 and can be expressed as:

$$U_1^+ = (U_1^{\mathcal{H}} U_1)^{-1} U_1^{\mathcal{H}} \quad (6)$$

Hence, the eigenvalues can be derived as follows:

$$\beta = \text{eig}((U_1^{\mathcal{H}} U_1)^{-1} U_1^{\mathcal{H}} U_2) \quad (7)$$

The estimated DoAs, θ_i , can be obtained using Eq.8 as shown

$$\theta_i = -\sin^{-1}\left(\frac{\lambda \text{Im}(\log \beta)}{2\pi d}\right) \quad (8)$$

4. Wavelet De-noising Theory

Removing noise from a signal is the key idea that can be achieved via Wavelet de-noising. Wavelet is a wave-like variation with the ability of representing a signal in the time-frequency plane. This variation or oscillation has amplitude that varies starting from a zero level, and then it increased or decreased incrementally, and finally back gradually to zero. Wavelets possess specific properties which fit them to digital signal processing. Moreover, they are considered as a mathematical tool for analyzing time-variant signals or transient phenomena. Wavelet based de-noising filters may be implemented via various methods. Discrete Wavelet Transform (DWT) is one of the commonly used methods. The wavelet theory is based on representing a general function using an infinite series expansion in terms of a basic mother wavelet function ψ [14]. The technique which is examined in this paper is *Wavelet de-noising by Thresholding*.

Let $x_m(t)$ be the received signals at each element of the antenna array that is contaminated by additive white Gaussian noise $n_m(t)$ with variance σ^2 as described in section 2. Hence $x_m(t)$ can be represented as:

$$x_m(t) = \sum_{i=1}^N s_i(t) e^{-j\frac{2\pi(m-1)}{\lambda}d \sin\theta_i} + \sigma n_m(t), \quad (m = 1, 2, \dots, M) \quad (9)$$

where the dimensions of $x_m(t)$ is $(1 \times K)$, and $f_m(t) = \sum_{i=1}^N s_i(t) e^{-j\frac{2\pi(m-1)}{\lambda}d \sin\theta_i}$ has the same dimensions of $x_m(t)$. The target is to find a function \hat{f}_m from the noisy matrix $x_m(t)$ that satisfies:

$$\hat{f}_m = \min_{\hat{f}_m} \|\hat{f}_m - f_m\|_2 \quad (10)$$

where \hat{f}_m is the estimation of x_m without noise. If y_{jcm} are the wavelet coefficients of x_m , then the transformation of Eq.9 in the wavelet domain can be expressed as in [15]:

$$y_{jcm} = w_{jc} + \sigma u_{jcm} \quad (11)$$

where $y_{jcm} = Wx_m^T$ are the wavelet coefficients of x_m , $w_{jc} = Wf_m^T$ which represents the uncontaminated wavelet coefficients of the function f_m , $u_{jcm} = Wn_m^T$, W denotes a $K \times K$ Discrete Wavelet Transform matrix, j refers to the decomposition level, and c represents the coefficient index in this level. The wavelet coefficients can be divided into approximations and detail coefficients. Some of these coefficients belonged to a distorted version of the matrix. So, to recover the function (\hat{f}_m) from the noisy matrix x_m , firstly obtaining its clear coefficients. These desired coefficients can be obtained by deleting the coefficients that have small magnitude as they represent pure noise. This process is called wavelet thresholding. Wavelet thresholding method is applied just to the detail coefficients d_{jcm} of y_{jcm} , and it isn't necessary to be applied on the approximation coefficients c_{jcm} , since the c_{jcm} represent 'low-frequency' terms that usually include important information of the data. Also, the approximation coefficients aren't sensitive to noise. The thresholding concept can be described as; the process of zeroing all the coefficients whose magnitude values are less than a certain threshold τ , and keeping or modifying the other coefficients [16]. Next, the thresholded wavelet coefficients values will be obtained in two methods. The first one is the *Hard Thresholding*, and its equation is as described as in [16]:

$$h(d_{jcm}) = \begin{cases} 0, & \text{if } |d_{jcm}| \leq \tau \\ d_{jcm}, & \text{if } |d_{jcm}| > \tau \end{cases} \quad (12)$$

while, the other one is the *Soft Thresholding*, and its equation is as described:

$$h(d_{jcm}) = \begin{cases} 0, & \text{if } |d_{jcm}| \leq \tau \\ d_{jcm} - \tau, & \text{if } d_{jcm} > \tau \\ d_{jcm} + \tau, & \text{if } d_{jcm} < -\tau \end{cases} \quad (13)$$

Hard thresholding nulls out all the coefficients values smaller than τ . If the magnitude of a coefficient is only somewhat less than τ , then this value is set to zero, while a coefficient whose magnitude is only slightly greater than τ is kept unchanged. So, hard thresholding creates discontinuities and it is not suitable for removing the noise. Soft thresholding or the kill follows the same manner of hard threshold, but, subtracts τ from the values larger than τ . Unlike hard thresholding, soft thresholding causes continuities in the resulting signal [16].

According to Donoho, David L's method found in [8], [9], and in [16], the threshold estimate τ for denoising the signal is given by:

$$\tau = \sigma \sqrt{2 \log(K)} \quad (14)$$

This threshold rule called universal threshold (*Visu Shrink*), where K is the number of samples. Also, the computed thresholds require knowledge of the noise variance σ which can be calculated as shown below [16], [17]:

$$\sigma = \frac{MAD}{0.6745} \quad (15)$$

where, $MAD = \text{median}(|d(j,c)|)$ is the median absolute deviation of detail coefficients of level j . Finally, estimate the desired signal using inverse discrete wavelet transform (IDWT).

5. Proposed Wavelet De-noising Matrix Pencil Method (WDMPM)

As illustrated in [1], [12], the Matrix Pencil algorithm can get good performance on DoA estimation when the environment exhibits high SNRs. On the other hand, as the SNRs decreases, the performance decreases dramatically. To resolve this issue at lower SNRs, we proposed de-noising the contaminated signals via the wavelet method to enhance the SNR of received signals. Next, the de-noised version is applied to the Matrix Pencil algorithm. The de-noising algorithm is implemented in three steps as shown in Figure 1.



Figure 1: Block diagram of Wavelet De-noising mechanism.

1. **Decomposition:** is the process of choosing the best suitable wavelet family, number of decomposition levels J , and then computing the decomposition coefficients at each level.
2. **Thresholding:** This step involves estimating noise variance σ from noisy signal using Eq.15, computing threshold values τ for each level using universal threshold (Visu Shrink) rule using Eq.14, and applying threshold to the coefficients at each level j . Next using soft/hard threshold method to discard small values that represent the additive noise and obtain the desired coefficients using Eq.12, or Eq.13.
3. **Reconstruction:** It is considered as the reverse process of decomposition. The approximation and detail coefficients at each level are up sampled by two, passed through the low pass and high pass filters and then added to recover the de-noised signal \hat{f}_m as indicated in [18]. This operation is continued through the same number of levels as in the decomposition operation to get the original signal.

These steps as discussed above are run on the output of each array sensor simultaneously. Once reconstructing the de-noised data matrix \hat{X} , the Matrix Pencil algorithm is applied on it to obtain the DoAs. The block diagram of the proposed WDMPM technique is shown in Figure 2.

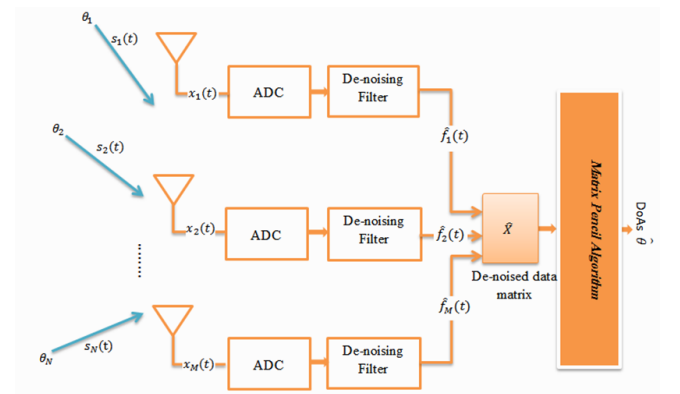


Figure 2: The block diagram of the proposed Wavelet De-noised Matrix Pencil Method (WDMPM).

6. Simulation Results

In this section, simulations are performed to assess the performances of traditional MPM versus the proposed WDMPM. Simulations are executed using a ULA of eight isotropic sensors ($M = 8$) with half wavelength $d = \lambda/2$ inter-element spacing, and $K = 16$

snapshots. Assume two sources that emit narrowband coherent signals at angles $30^0, 45^0$ with the same power. These signals are corrupted by additive white Gaussian noise. DWT with a wavelet family (*db12*) (Daubechies wavelets) is employed to de-noise the contaminated signals. The number of decomposition levels for the DWT is $J = 5$, and the decomposition coefficients are calculated at each level. Also, universal thresholds selection are used, combined with soft threshold type, and the variance of noise level is kept without scaling. DoAs are calculated within the SNR range (-10 to 5) dB for both MPM and WDMPM. Wavelet de-noising is applied on each sensor's output independently. The Mean Square Error (MSE) [19] is utilized as a performance measure as calculated in the following equation:

$$MSE = \frac{1}{N} \sum (\hat{X} - X)^2 \quad (16)$$

Monte Carlo Method is used to obtain 1000 independent runs for measuring the MSE. The results are shown in Figure 3. It is clear that the WDMPM performance is greatly better than the MPM on the whole range of the $SNRs$ considered. Examining MSEs at $SNR = -10$ dB, it is found that it is -2.019 dB for MPM, while it is -4.253 dB for WDMPM. In case of $SNR = -5$ dB the MSE for MPM is -2.915 , but -11.16 dB for WDMPM. As the SNR reaches 0 dB, the MSE reaches -4.437 dB for MPM while it becomes -25.93 dB for WDMPM. Finally, the MSE at $SNR = 5$ dB is -8.623 dB, and -30.87 dB for MPM and WDMPM respectively. Figure 4 summarizes the improvement in MSE for the WDMPM over the ordinary MPM method. It is clear that the proposed WDMPM technique outperforms the MPM at the same range of SNR . It is found that the MSE is enhanced by 2.234 dB at $SNR = -10$ dB while at $SNR = -5$ dB, the MSE enhanced by 8.245 dB. At $SNR = 0$ dB, the MSE is decreased by 21.493 dB in WDMPM under the ordinary MPM. Moreover, the MSE of the WDMPM reaches 22.247 dB below the MSE of MPM at $SNR = 5$ dB. These results show that the WDMPM has superior performance compared to the MPM through all over the SNR range from -10 to 5 dB as in Figure 3 and Figure 4.

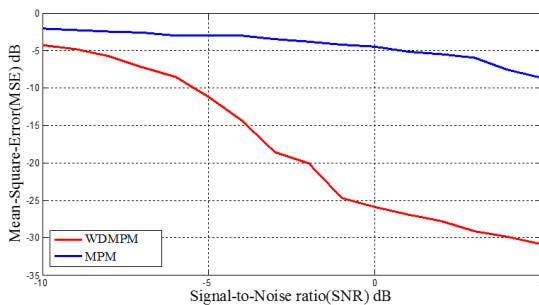


Figure 3: Comparison between MPM and WDMPM.

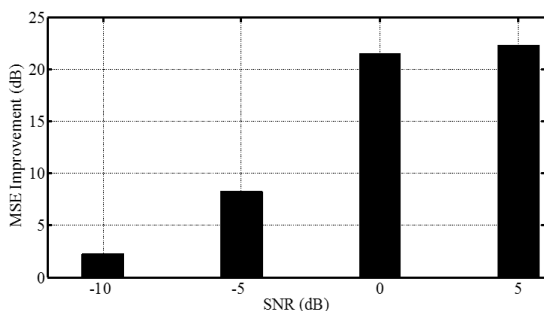


Figure 4: Improvement of MSE for WDMPM over MPM.

7. Conclusions

In this paper a new highly accurate DoA estimation technique is proposed. It relies on a smoothing algorithm based on the wavelet de-noising theory that is applied to the Matrix Pencil Method. It is known as the WDMPM. From the results, it is evident that the WDMPM's performance outperforms the traditional MPM technique. Testing both WDMPM and MPM under the same conditions shows a great enhancement in the MSE of the WDMPM over MPM. The improvement in MSE ranges from 2.234 dB to 22.247 dB over the corresponding SNR range of -10 dB to 5 dB. Furthermore, the MSE improvement reflects the increase in the stability of the WDMPM over the MPM.

References

- [1] Nuri Yilmazer, Tapan K Sarkar, and Magdalena Salazar-Palma. Doa estimation using matrix pencil and esprit methods using single and multiple snapshots. In *Electromagnetic Theory (EMTS), 2010 URSI International Symposium on*, pages 215–218. IEEE, 2010.
- [2] Muhammad Faisal Khan. *Improved Matrix Pencil Methods for Parameters Estimation of Plane Wave Signals*. PhD thesis, Pakistan Institute of Engineering & Applied Sciences, 2011.
- [3] Salvatore Calcagno, Fabio La Foresta, and Mario Versaci. Independent component analysis and discrete wavelet transform for artifact removal in biomedical signal processing. *American Journal of Applied Sciences*, 11(1):57, 2014.
- [4] Priyanka Khatwani and Archana Tiwari. A survey on different noise removal techniques of eeg signals. *International Journal of Advanced Research in Computer and Communication Engineering*, 2(2):1091–1095, 2013.
- [5] Gyanendra Singh, Kiran Savita, Shivkumar Yadav, and Vaibhav Purwar. Design of adaptive noise canceller using lms algorithm. *International Journal of Advanced Technology & Engineering Research (IJATER)*, 3(3):85–89, 2013.
- [6] Yanbo Xue, Jinkuan Wang, and Zhigang Liu. A novel improved music algorithm by wavelet denoising in spatially correlated noises. In *Communications and Information Technology, 2005. ISCIT 2005. IEEE International Symposium on*, volume 1, pages 515–518. IEEE, 2005.
- [7] XJ Mao and HH Pan. An improved doa estimation algorithm based on wavelet operator. *Journal of Communications*, 8(12), 2013.
- [8] David L Donoho and Jain M Johnstone. Ideal spatial adaptation by wavelet shrinkage. *Biometrika*, 81(3):425–455, 1994.
- [9] David L Donoho. De-noising by soft-thresholding. *Information Theory, IEEE Transactions on*, 41(3):613–627, 1995.
- [10] Jie Zhang, Dengshan Huang, Ping Huang, and Jun Kang. Estimation doas of the coherent sources based on svd. In *Signal Processing Systems (ICSPS), 2010 2nd International Conference on*, volume 3, pages V3–174. IEEE, 2010.
- [11] Hongliang Duan, Dengshan Huang, Liyang Zhou, Haihua Chen, and Jinglin Shi. A decorrelation algorithm based on virtual array extension. In *Communications and Networking in China (CHINACOM), 2012 7th International ICST Conference on*, pages 348–351. IEEE, 2012.
- [12] Nuri Yilmazer, Raul Fernandez-Recio, and Tapan K Sarkar. Matrix pencil method for simultaneously estimating azimuth and elevation angles of arrival along with the frequency of the incoming signals. *Digital Signal Processing*, 16(6):796–816, 2006.
- [13] Liwei Jing, Pei Liang, Cao Maoyong, and Sun Nongliang. Super-resolution time of arrival estimation for indoor geolocation based on iccc 802.11 a/g. In *Intelligent Control and Automation, 2008. WCICA 2008. 7th World Congress on*, pages 6612–6615. IEEE, 2008.
- [14] Ying-Chang Liang, Yonghong Zeng, Edward CY Peh, and Anh Tuan Hoang. Sensing-throughput tradeoff for cognitive radio networks. *Wireless Communications, IEEE Transactions on*, 7(4):1326–1337, 2008.
- [15] Jean-Paul G Gallaire and Akbar M Sayeed. Wavelet-based empirical wiener filtering. In *Time-Frequency and Time-Scale Analysis, 1998. Proceedings of the IEEE-SP International Symposium on*, pages 641–644. IEEE, 1998.
- [16] Rami Cohen. Signal denoising using wavelets. *Project Report, Department of Electrical Engineering Technion, Israel Institute of Technology, Haifa*, 2012.
- [17] Ramesh Kumar and Prabhat Patel. Signal denoising with interval dependent thresholding using dwt and swt. *International Journal of Innovative Technology and Exploring Engineering (IJITEE) ISSN*, 1:2278–3075, 2012.
- [18] Zhao Hong-tu and Yan Jing. The wavelet decomposition and reconstruction based on the matlab. In *Proc. of the Third Int. Symposium on Electronic Commerce and Security Workshops (ISECS 2010), China*, 2010.
- [19] Robert J Barsanti and Jordan Gilmore. Comparing noise removal in the wavelet and fourier domains. In *System Theory (SST), 2011 IEEE 43rd Southeastern Symposium on*, pages 163–167. IEEE, 2011.