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Research Paper

Optimization of vibrational absorber rested on linear structures under arbitrary vibrations

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Abstract

Vibrational absorber is one of the common approaches for vibration control in structures. In this article, optimization criteria for mechanical systems under arbitrary vibrations is presented based on a multi-purpose method whose objective function vector collects the efficiency of arbitrary reliability and indexes of structure costs. The criterion is different from conventional criteria and standards used to design structures subjected to arbitrary vibrations and is based on minimizing the changes in displacement or response acceleration of the main structure, regardless of the required function for the failure. In this study, multi-purpose optimization approach to the design of vibrational an absorber is investigated to control non-uniform structural vibrations stimulating mechanical a mechanical structure based on an arbitrary acceleration process. It is performed based on bee optimization algorithm. In the following, a numerical example is shown for a simple vibrational absorber by this way. It shows the results of cost increase for decreased possibility of deconstruction, and therefore, allows appropriate decision making and selection based on need and cost.

***Keywords**: Absorber Optimization; Bee Algorithm; Random Vibrations; Linear Structures.*

1. Introduction

In the standard analysis of structures, different elements such as load severity or mechanical and geometrical properties are effective, which are usually considered as certain values. One of the most common methods to investigate the structures is considering load value as the only uncertain and random parameter [1]: like the situations where earthquake, wind or waves are the source of load. Standard random vibration's theory can be used in this state. In these conditions, traditional structural optimization can be used to design vibrational absorber.

The first optimization work on this field was designed by Nigam led to solve a constrained nonlinear problem [2]. In the past decades, various works have been used by probabilistic methods to involve uncertainty in structural optimization [3], [4]. Unlike traditional methods; objective function is minimized under probable constraints for reliability in standard mode of structural optimization design. It should be noted that, such works were done regardless of the effect of changing times.

The reliability for designing linear structures subject to dynamic random loads has been investigated by Papa Demetrio et al. [5]. At this work, systems safety was proposed as system efficiency index due to the structure displacement, and the inputs were uniform white-noise function. All the methods proposed in these articles are based on the optimization of objective function determining system response in a supported mode. In such optimizations, called single-purpose optimization, only one objective function that is able to describe system efficiency features is optimized [6], [7].

In this field, single-purpose method is usually used, but only a single criterion can merely indicate the structure efficiency in many practical applications; because various and sometimes conflicting criteria must be considered simultaneously. Thus, an optimization problem with more than one objective function is defined, meant that a problem with multi-purpose optimization. Such optimization is applied when elements include different structural prices and different efficiency indexes. Unlike single-purpose optimization, we will have a series of answers in this mode, and it will allow the project to choose the best possible mode among different answers. To do so, multi-purpose optimization

process has been used for optimum design of vibrational absorber for a linear structure with one freedom degree subject to random vibrations. For this purpose, two objective function vectors have been defined based on structural price and probable indexes of escape from failure. Finally, a numerical example will be investigated and selections of absorber features and possible modes are assessed.
Currently, several studies have been done on optimization methods, and many researches are performed on this basis. New Bee Algorithm, defined in 2005 by Pham et al for the first time, indicated good efficiency and high convergence speed for solving complicated optimization problems [8-11]. In this article, Bee algorithm has been used for multi-purpose optimization.

2. Analysis of linear system subject to random loads

Matrix differential movement equation of a system with n freedom degrees, which is motivated by a force vector $f(t)$ with zero mean Gaussian distribution function, is as follows:

$$M\ddot{y}(t)+C\dot{y}(t)+Ky(t)=f(t)\cdot \tag{1}$$

Where, M, C, and K denote mass, damping, and stiffness respectively. Also, $\ddot{y}$, $\dot{y}$, and y are acceleration, velocity, and displacement vectors. The Eq. (1) can be written by defining state space vector $z(t)=[y(t)\quad \dot{y}(t)]^{\mathrm{T}}$ as a first grade equation assuming Gaussian input to be zero. Random answer can be completely described by covariance matrix of state space $R(t)$. This matrix is obtained from Lyapunov covariance matrix.

$$\dot{R}(t)=AR(t)+R(t)A^{\mathrm{T}}+B(t) \tag{2}$$

Where

$$B(t)=\left\langle \hat{f}+z^{\mathrm{T}}\right\rangle+\left\langle z\ \hat{f}^{\mathrm{T}}\right\rangle\cdot \tag{3}$$

And system matrix and also input vector are as follows:

$$A=\begin{bmatrix}0 & 1\\ -M^{-1}K & -M^{-1}C\end{bmatrix},\ \hat{f}(t)=\begin{Bmatrix}0\\ M^{-1}f(t)\end{Bmatrix}. \tag{4}$$

3. Convergence criteria

Structural optimization problem can be formulated by selecting a set of design variables (design parameters that influence the structural characteristics). These variables, defined in a possible domain of Ωb, are shown by DVb. Optimized DV can minimize objective function by single-purpose optimization. Reliability theory is used to consider all uncertainty sources through a logical way. Unlike single-purpose optimization method, multi-purpose optimization method allows designer to have a set of possible answers that satisfy several indexes at the same time. This set of answers is known as Pareto range and Pareto optimization criteria, and provide a basic point in multi-purpose optimization problems.
Our issue is finding the values $b\subset\Omega\mathrm{b}$ where $b=[\omega_T,\ \xi_T]^{\mathrm{T}}$ is conditional upon minimizing the equation $\mathrm{OF}(b,T)=\{\gamma_m, P_f(b,T)\}$. This issue can be solved by numerical solution through multi-purpose optimization method. By defining general yield index $\mathrm{OF}_i(b)$, a kind of minimization based on multi-purpose optimization method is defined as follows:

$$\min\ \left\{\mathrm{OF}_1(\bar{b}),\ \mathrm{OF}_2(\bar{b}),\ \cdots,\mathrm{OF}_M(\bar{b})\right\}\cdot \tag{5}$$

If two answer candidates b_j and b_k are belonged to Ωb, then:

$$\begin{aligned}&\forall i\in\{1,\cdots,M\},\ \mathrm{OF}_i(b_j)\le\mathrm{OF}_j(b_k),\\ &\exists i\in\{1,\cdots,M\},\ \mathrm{OF}_i(b_j)\le\mathrm{OF}_i(b_k)\cdot\end{aligned} \tag{6}$$

Two objective functions are defined:

$$v(b_j) = \{\mathrm{OF}_1(b_j), \cdots, \mathrm{OF}_M(b_j)\} \tag{7}$$

And

$$v(b_k) = \{\mathrm{OF}_1(b_k), \cdots, \mathrm{OF}_M(b_k)\} \cdot \tag{8}$$

If there was no possible solution of v (b_k) dominant on v (b_j), then v (b_j) is known as an optimum Pareto solution. Unfortunately, Pareto optimization doesn't always give an only solution, but also a set of solution which cannot be processed analytically. Collecting all optimum Pareto solutions are known as Pareto optimal set.

4. Optimization algorithm used

Collective Intelligence is a sub-branch of artificial intelligence, which is built based on the collective behavior of, decentralized, self-organizing systems. Bee's algorithm is a searching algorithm that is based on team work, and was first coined back in 2005 by Pham et al. this algorithm is the simulation of the food searching behavior of bee groups. In the basic version of the algorithm, the algorithm performs a local search which is combined with a random search and can be used for combined optimization (when you want to optimize multiple variables simultaneously) or functional optimization.

Bee Algorithm is an optimization algorithm based on bee behavior to find the most optimal solution. They investigate each point in parametrical space - composed of possible responses - as a source of food. "Observer Bees" simulated workers - randomly simplify the space of answers, and report the quality of observed situations by the fitness function. Simplified solutions are ranked, and other "bees" are the new forces searching around to find the top rated sites (called "cemetery"). The algorithm selectively searches for other cemeteries to find the maximum point of the fitness function.

5. System state-space model

A standard method to model the vibrational absorber is modeling it by a mass, spring and damper mounted on the top of a multi-degree of freedom linear system [12], [13] [14]. The main goal is to reduce unwanted vibrations and thus the risk and the failure of the main structure. In this special case, the main stimulation exerted upon the structure is a uniform random force. Fig. 1 shows a simple model of the system under study.

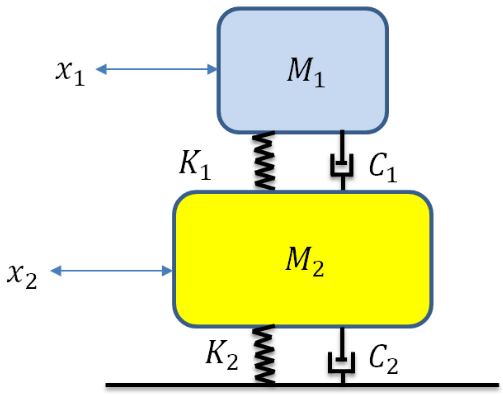

Fig. 1: Simple Model of the System under Study

Equilibrium equations of the system are as follows:

$$\ddot{X}(t) + C\dot{X}(t) + KX(t) = -Mr\ddot{X}_b \tag{9}$$

Where, M, C, and K are 2*2 matrixes of mass, damping, and stiffness, respectively. Also, $r = (1, 1)^{\mathrm{T}}$ and the vector $X = (x_1, x_2)^{\mathrm{T}}$; $\dot{X} = (\dot{x}_1, \dot{x}_2)^{\mathrm{T}}$; $\ddot{X} = (\ddot{x}_1, \ddot{x}_2)^{\mathrm{T}}$ indicates displacement, velocity, and acceleration of the main structure and vibrational absorber.

The characteristics of vibrational absorber are expressed by parameters m_a, k_a, and c_a; indicating the mass, stiffness, and damping of vibrational absorber respectively.

Introducing space-state vector $Z = (X, X_f, \dot{X}, \dot{X}_f)$, the matrix of system A is:

$$A = \begin{bmatrix} 0^{(n+2)\,(n+2)} & I^{(n+2)\,(n+2)} \\ -H_K & -H_C \end{bmatrix} \tag{10}$$

Where, n is the number of freedom degrees, I is a conformable identity matrix and sub-matrixes H_K and H_C are as follows:

$$H_K = \begin{bmatrix} & & & \omega_f^2 \\ & (M^{-1}K)^{(n+1)(n+1)} & & \cdots \\ & & & \omega_f^2 \\ 0 & \cdots & 0 & -\omega_f^2 \end{bmatrix} \quad (11)$$

And

$$H_C = \begin{bmatrix} & & & 2\xi_f\,\omega_f \\ & (M^{-1}K)^{(n+1)(n+1)} & & \cdots \\ & & & 2\xi_f\,\omega_f \\ 0 & \cdots & 0 & -2\xi_f\,\omega_f \end{bmatrix} \quad (12)$$

To describe the vibration acceleration, Kanayi and Tajimy model known as K-T model has been used. This model has found widespread application in analysis of structures' random vibrations due to the simple description of vibration from a dominant frequency. This is an interesting phenomenon of the life that many of natural disasters subjected to random vibrations have possible distribution called Gaussian Bell. Therefore, the model has been oscillated by using a simple and linear white noise filter with Gaussian distribution that encounters by the created vibration as a stationary phenomenon.
According to the above assumptions, final acceleration exerted on the structure base is equal to the sum of inertia force $\ddot{X}_f(t)$ resulted from K-T filter and white noise stimulation of time module $\phi(t)\;W(t)$:

$$\begin{cases} \ddot{X}_b(t) = \ddot{X}_f(t) + \phi(t)\;W(t) \\ \ddot{X}_f(t) + 2\xi_f\,\omega_f\,\dot{X}_f(t) + \omega_f^2 X_f(t) = -\phi(t)\;W(t) \end{cases} \quad (13)$$

Where, $X_f(t)$ indicates displacement response of K-T filter, $W(t)$ is stationary Gaussian zero mean white noise process, ω_f is natural frequency of filter, and ξ_f is damping coefficient of filter. There has been used different function for module $\phi(t)$ in different references. In this article, the module function used by Jenning has been used [15]:

$$\phi(t) = \begin{cases} (t/t_1)^2 & t < t_1 \\ 1 & t_1 < t < t_2 \\ \mathrm{e}^{-\beta\,(t-t_2)} & t > t_2 \end{cases} \quad (14)$$

Where, $t_d = t_2 = t_1$ is time interval and stimulation of peak toward a constant value. The parameters expressed in the above example are t_1=3s, t=15s, and β=0.4s.
Power Spectrum Density of constant S_0 is related to the standard deviation σ_x by the following relation:

$$S_0 = \frac{2\xi_f\;\sigma_{x_b}^2}{\pi(1+4\xi_f^2)\;\omega_f} \quad (15)$$

Mass ratio is as below:

$$\gamma_m = \frac{m_a}{\sum_{i=1}^{n_f} m_i}\,. \quad (16)$$

The possibility of structure failure assuming at early of vibration is obtained from Poisson's relation:

$$\begin{aligned} & r\;(b,\,x_{\mathrm{adm}},T) = 1 - P_f = 1\,, \\ & P_f\;(b,\,x_{\mathrm{adm}},T) = 1 - \exp\left[-2\int_0^T v\,(b,\,x_{\mathrm{adm}},T)\;\mathrm{d}T\right]. \end{aligned} \quad (17)$$

Where T is the duration of vibration of the system and v is the unconditioned mean crossing rate. The strategy used to optimize mechanical parameters of vibrational absorber is minimizing mass ratio and probability of structure failure.

6. Numerical results

Numerical results for optimal design of vibrational absorber are shown throughout this section. Mechanical characteristics as the system include 6000kg mass and spring stiffness of 5 MN/m^2. Optimal design of vibrational absorber contributes to the evolution of design vector that minimizes the efficiency of $OF(b,T)=\{\gamma_m, P_f(b,T)\}$ at the same time. By solving the main problem, a Pareto optimum front is obtained. Drawing objective functions whose non-dominant vectors are in the set of Pareto answers is called Pareto front. Fig. 2 shows Pareto function for the possibility of 8cm displacement.

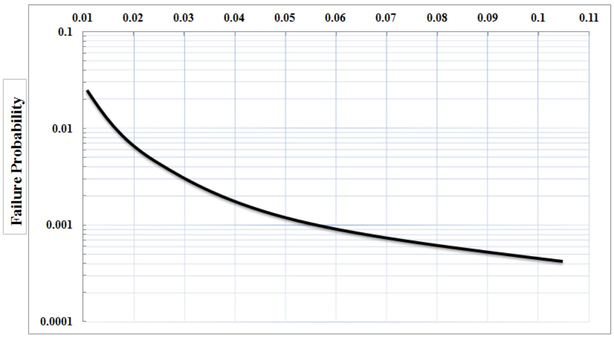

Fig. 2: Pareto Function for the Possibility of 8cm Displacement

As it can be seen, probability of failure Pf decreases as the mass ratio y_m (thus increases in costs of vibrational absorber) increases, and as required by the designer and the safety factor and also the amount of funds, vibrational absorber can be designed.

7. Conclusion

This study examined the efficiency of reliability based on the optimal design of linear elastic structures that are stimulated by random loads. Unlike traditional design methods which are based on minimizing the squares root of the system response; the reliability is considered based on the efficiency index in order to be more effective and more useful in engineering decisions. This method is performed by defining multi-purpose optimization criterion based on system reliability. As a case study, optimum design of the mechanical parameters of a vibrational absorber is studied for a structure with three degrees of freedom. Criteria for optimal design are based on minimizing the mass of the vibrational absorber on the performance reliability of system displacement, taking into account the allowable displacement. The results show an increase in costs for reducing the risk of structure failure and thus allow the designer to decide the right choice based on need and costs.

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