



# Reference flipped-differential chaos shift keying scheme for chaos- based communications

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## Abstract

In this paper, a new differential coherent chaos-based communication system is proposed and named Reference Flipped-DCSK (RF-DCSK). By utilizing the low correlation value between a chaotic segment and its flipped version, each transmitted signal will be the sum of the information bearing signal and the mirrored version of the reference signal. This enhances the bandwidth efficiency of DCSK by doubling bits rate. Additionally, noise performance is improved at typical values of the spreading factor. According to Gaussian approximation method, theoretical expression for Bit-Error-Rate (BER) is derived. The proposed scheme is simulated and compared with Differential Chaos Shift Keying (DCSK), Correlation Delay Shift Key (CDSK) and High Efficiency -Differential Chaos Shift Keying (HE-DCSK) schemes in Additive White Gaussian Noise (AWGN) environment. Effect of spreading factor is studied. Results show that the proposed scheme clearly outperforms other systems.

**Keywords:** Chaotic Communication; Differential Chaos; Correlation Delay; Crosscorrelation; Noncoherent.

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## 1. Introduction

Many researchers have been motivated to explore the possibility of using chaos signals in spread spectrum communications due to their attractive properties. Chaotic signals are non-periodic with low cross correlation and impulse like autocorrelation values. In addition, chaotic signals are resistant to multipath fading.

Chaos based communication systems can be classified as a coherent, noncoherent, and differentially coherent. However, when the channel condition is very poor to synchronize, a differential coherent detection is preferred due performance. In DCSK [2], each symbol is transmitted by two chaotic segments; the first segment is served as a reference signal while the other segment is served as an information-bearing signal. Both segments are sent separately and successively. Therefore, data rate is halved but BER is better compared with other chaos systems. To overcome low attainable data rate problem, CDSK is proposed in [3]. In CDSK, each transmitted signal will be the sum of the current reference signal and the information bearing signal of the previous symbol. Thus, data rate is improved but noise performance is less than DCSK due to a large number of cross terms at the correlator output.

Many chaotic schemes have been proposed to enhance the performance of the differentially coherent systems. Usage of single reference segment for multiple bits transmission to improve bandwidth efficiency is proposed in [4-7]. In these systems, noise performance and data rate are increased, but schemes are complex and require additional synchronization circuits. In conventional DCSK, reference signal and information bearing signal are sent through consecutive time slots that require a difficult to implement radio frequency delay line at the receiver. RF delay line removal is suggested in [8] and developed in [9] by sending the reference signal and the information signal on the same time slot and separate them using Walsh code. The system adds more delay elements at the transmitter and requires lengthy Walsh code. Another system which is based on signal separation using initial condition modulation is examined in [10]. Multicarrier Modulation for DCSK (MC-DCSK) signal is explored in [11]. In spite of efficient utilization of the transmitted energy, system implementation in (MC-DCSK) includes bank of narrow band modulator that needs a high degree of accuracy in the design to maintain subcarrier synchronization. Space-Time Block Code –DCSK (STBC-DCSK) is tested in [12].

STBC-DCSK enhances the noise performance of DCSK compared to standard DCSK. However, data rate and the average energy for each bit are similar to that in DCSK with additional STBC encoder at the transmitter. Permutation of chaotic samples is used to build M-ARY modulation scheme for DCSK and enhance the security as in [13][14] respectively. First experimental DCSK is implemented in [15]. Another hardware implementation using FPGA is analyzed in [16].

In this paper, a new proposed differential coherent chaos-based communication scheme is analyzed. Utilizing the low correlation value between each chaotic segment with its flipped version, each reference signal is flipped and added with information bearing signal within the same bit duration. In section 2, transmitter and receiver structure are discussed. Theoretical expression for BER performance of RF-DCSK is derived in section 3. Comparison between BER of RF-DCSK and other chaos systems including (DCSK, CDSK, and HE-DCSK) is performed by computer simulations in section 4.

## 2. System description

At the transmitter, two synchronous chaos generators are used to emit chaotic samples simultaneously as shown in Fig.1. Within each bit duration  $l$ , the first source generates a chaotic sample  $x_i$  at each  $i$ th instant and with the length of  $M$ , while the second source generates a flipped version of the same chaotic sample  $x'_i$ , where  $x'_i = x_{M-i \bmod(M)+1}$  to be used as a reference signal. This means that the second generator create future samples in the reverse direction for  $x_i$ . For example, if the first source generates a segment  $\{x_{11} x_{12} \dots x_{20}\}$  to modulate a second information bit (i.e.  $l = 2$ ) at spreading factor  $M=10$ , a mirrored version of the same segment  $\{x'_{11} = x_{20}, x'_{12} = x_{19}, \dots x'_{20} = x_{11}\}$  is generated from the second source simultaneously. Moreover, synchronization unit will maintain the synchronization process between both sources. Each information bit  $b_l$  is modulated with the chaotic segment from first source. Therefore, the transmitted signal  $s_l$  for the  $l$ th information bit can be written as

$$s_l = b_l x_i + x'_i$$

$$= b_l x_i + x_{M-i \bmod(M)+1} \quad (l-1)M < i \leq M \tag{1}$$

Where  $M$  represents the spreading factor which is defined as the number of chaotic samples for each bit duration and  $b_l \in \{-1,1\}$

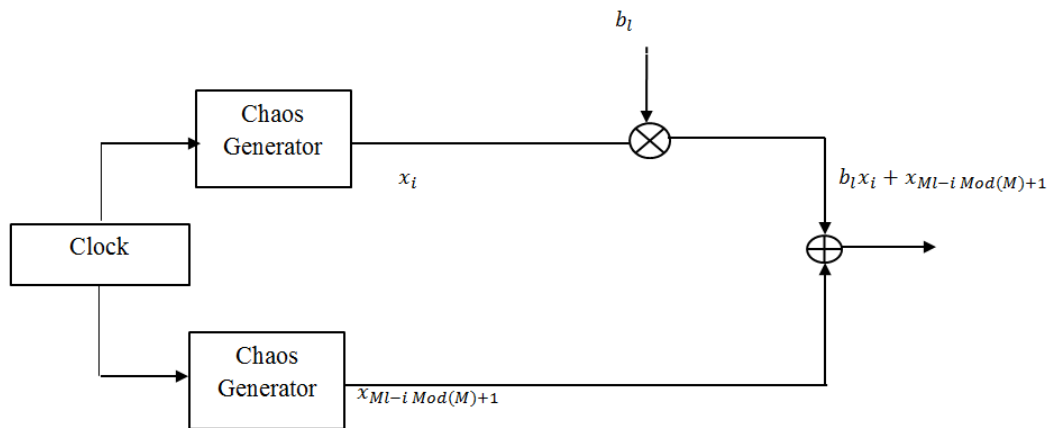


Fig. 1. Rf-Dcsk Transmitter Diagram

### 2.1. Receiver description

Receiver of RF-DCSK is shown in Fig.2. Each received segment is stored and flipped. Storing and flipping can cause a delay which will be processed by the synchronization unit. Here, we will assume a perfect synchronization is maintained between the received segment and its flipped version. To decode the information bit, each received signal  $r_i$  is multiplied by its flipped version  $r'_i$  and averaged over one bit duration, where  $r'_i = r_{M-i \bmod(M)+1}$ . Assume the channel is AWGN, and then the received signal can be written as  $r_i = s_i + \psi_i$ , where  $\psi_i$  is the noise sample with  $E(\psi_i) = 0$  and variance  $\sigma_\psi^2$ . The correlator output  $Z_l$  can be written as

$$Z_l = \sum_{i=(l-1)M+1}^{lM} r_i r'_i$$

$$Z_l = \sum_{i=(l-1)M+1}^{lM} r_i r_{M-i \bmod(M)+1}$$

$$Z_l = \sum_{i=(l-1)M+1}^{lM} (s_i + \psi_i)(s_{M-i \bmod(M)+1} + \psi_{M-i \bmod(M)+1})$$

$$Z_l = \sum_{i=(l-1)M}^M (b_l x_i + x_{M-i \bmod(M)+1} + \psi_i)(b_l x_{M-i \bmod(M)+1} + x_i + \psi_{M-i \bmod(M)+1}) \tag{2}$$

It can be simply shown that  $\sum_{i=(l-1)M+1}^{lM} x_{Ml-i \bmod(M)+1} x_i = \sum_{i=(l-1)M+1}^{lM} x_i x_{Ml-i \bmod(M)+1}$ , and similarly for all other symmetric terms in (2). For further simplification and without loss of generality, equation (2) can be rewritten for a single bit (i.e.  $l = 1$ ). Thus, the correlator can be found as

$$Z = b \sum_{i=1}^M (x_i^2 + x_{M-i+1}^2) + 2 \sum_{i=1}^M (x_i x_{M-i+1}) + 2b \sum_{i=1}^M (x_i \psi_{M-i+1}) + 2 \sum_{i=1}^M (x_i \psi_i) + \sum_{i=1}^M (\psi_i \psi_{M-i+1}) \quad (3)$$

First part in (3) represents the bit energy which have fluctuated value due to the chaotic nature of the source. It is interesting to note that the signal energy which is used at the correlator with one bit duration is more than that in DCSK CDSK, and HE-DCSK [3][6]. This increases the distance between the signal elements at the correlator output. Remaining terms are a combination of signal-signal (intrasignal), signal-noise and noise-noise cross terms. The cross terms are zero mean random quantity that contributes negatively or positively to the correlator output. Information bit is decoded by comparing the correlator output  $Z$  with zero according to the following rule

$$\tilde{b} = \begin{cases} -1 & Z < 0 \\ 1 & Z \geq 0 \end{cases}$$

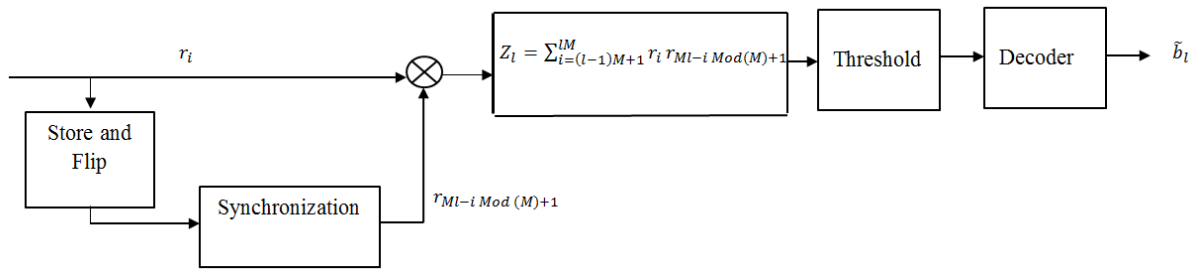


Fig. 2: Receiver Structure of RF-DCSK

### 3. Performance evaluation

Gaussian Approximation (GA) [3] method is used to evaluate BER performance of the system. The method is used to provide an accurate theoretical estimate at large value of spreading factor. To simplify the analysis, equation (3) can be rewritten as in the following form

$$Z = bE_b + b\alpha + \beta$$

Here,  $E_b$  represent average transmitted energy for each bit value and it is given by

$$E_b = ME(x_i^2 + x_{M-i+1}^2) = 2ME(x_i^2).$$

Where  $E(\cdot)$  is the expected value operator. Define  $\alpha = \sum_{i=1}^M (x_i^2 + x_{M-i+1}^2) - E_b$  and

$$\beta = 2 \sum_{i=1}^M (x_i x_{M-i+1}) + 2b \sum_{i=1}^M (x_i \psi_{M-i+1}) + 2 \sum_{i=1}^M (x_i \psi_i) + \sum_{i=1}^M (\psi_i \psi_{M-i+1}) \quad (4)$$

To process with the estimation, we shall assume the following:

- 1) Since  $x_i$  is stationary, it can be always shown that for that the correlation between  $x_i$  and its mirrored version  $x_i'$  over large spreading factor  $M$  is very small (i.e.  $E(x_i \cdot x_i') = E(x_i \cdot x_{Ml-i \bmod(M)+1}) \approx 0$ ). Sample of correlation plot between the chaotic segment and its mirrored version at  $M=100$  is shown in Fig. 3.
- 2) Chaotic sample  $x_i$  and its flipped version  $x_i'$  which can be generated at any instant is statistically independent from the noise sample  $\psi_j$  for any  $(i,j)$ .
- 3)  $\psi_i$  is statically independent from  $\psi_j$  for any  $i \neq j$ .

Thus, on the basis of previous assumptions for  $x$  and  $\psi$ ,  $\alpha + \beta$  and then  $Z$  in (3) tend to have a Gaussian distribution [17]. Therefore, it sufficient to calculate the mean and the variance of the correlator output  $Z$  to characterize the performance

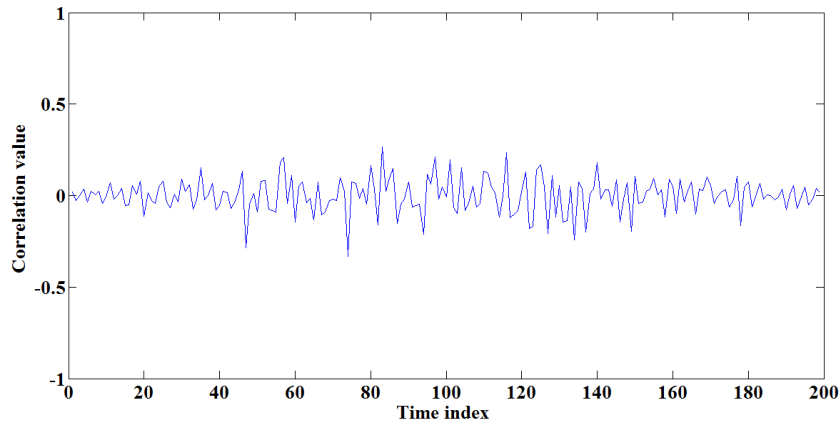


Fig. 3: Correlation Function Versus Time Index of the Chaotic Segment with Its Mirrored Version at Spreading Factor M=100.

Average values of the correlator can be found as

$$E(Z/b=1) = ME(x_i^2 + x_{M-i+1}^2) = E_b$$

$$E(Z/b=-1) = -1 * ME(x_i^2 + x_{M-i+1}^2) = -E_b$$

signal energy at the end of the correlation time is twice as in DCSK, CDSK [3] (i.e.  $E_{b-DCSK} = E_{b-CDSK} = ME(x_i^2)$ )

To compute the variance of the correlator  $\sigma_Z^2$

$$\sigma_Z^2 = \sigma_{\alpha+\beta}^2 = \sigma_\alpha^2 + 2Cov(\alpha\beta) + \sigma_\beta^2$$

It can be easily verified that  $Cov(\alpha\beta) = 0$ . Thus,  $\sigma_Z^2$  can be simplified to

$$\sigma_Z^2 = \sigma_{\alpha+\beta}^2 = \sigma_\alpha^2 + \sigma_\beta^2$$

$\alpha$  is highly depending on the chaotic source. For symmetric Tent map which has a uniform distribution between [-1, 1] and it is given by the equation

$$x_{n+1} = 1 - 2|x_n|$$

$$\sigma_\alpha^2 = Var(\sum_{i=1}^M(x_i^2 + x_{M-i+1}^2) - E_b) = Var(\sum_{i=1}^M(x_i^2 + x_{M-i+1}^2)) = \sum_{i=1}^M Var(x_i^2 + x_{M-i+1}^2)$$

$$= \frac{2E_b^2}{5M}$$

Since  $\psi_i$  is statistically independent from  $x_j$  for any (i, j) and  $\psi_j$  for any (i, j), and considering the symmetry in the first and last term in Equation (4). Equation (4) can be rewritten as

$$\beta = 4 \sum_{i=1}^{M/2} (x_i x_{M-i+1}) + 2b \sum_{i=1}^M (x_i \psi_{M-i+1}) + 2 \sum_{i=1}^M (x_i \psi_i) + 2 \sum_{i=1}^{M/2} (\psi_i \psi_{M-i+1})$$

Therefore,

$$\sigma_\beta^2 = 16 \cdot \frac{M}{2} \sigma_x^2 \sigma_x^2 + 4M \sigma_x^2 \sigma_o^2 + 4M \sigma_x^2 \sigma_o^2 + 2M \sigma_o^2 \sigma_o^2$$

$$\sigma_\beta^2 = 8M \sigma_x^2 \sigma_x^2 + 8M \sigma_x^2 \sigma_o^2 + 2M \sigma_o^2 \sigma_o^2$$

Putting noise variance  $\sigma_o^2 = \frac{N_o}{2}$  where  $N_o$  is the noise power spectral density. Hence,

$$\sigma_\beta^2 = \frac{2E_b^2}{M} + 2E_b N_o + M \frac{N_o^2}{2}$$

$$\sigma_Z^2 = \frac{2E_b^2}{5M} + \frac{2E_b^2}{M} + 2E_b N_o + M \frac{N_o^2}{2} = \frac{12E_b^2}{5M} + 2E_b N_o + M \frac{N_o^2}{2}$$

Under the assumption that  $Pr(0) = Pr(1)$ , BER can be calculated as

$$BER = 0.5P(\alpha + \beta < -E_b) + 0.5P(-\alpha + \beta > E_b) = \frac{1}{2} erfc\left(\frac{E_b}{\sqrt{2 \sigma_{\alpha+\beta}^2}}\right) = \frac{1}{2} erfc\left(\frac{E_b}{\sqrt{2 \sigma_Z^2}}\right)$$

BER Expression can be simplified to

$$BER_{RF-DCSK} = \frac{1}{2} \operatorname{erfc} \left( \sqrt{\frac{E_b}{4N_o} \left( 1 + \frac{12 E_b}{10 M N_o} + \frac{M N_o}{4 E_b} \right) - 1} \right) \tag{5}$$

Where  $\operatorname{erfc}(x)$  is the complimentary error function and it is given by

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-t^2} dt$$

According to [3] and [6]. Theoretical expression for BER of the DCSK, CDSK and HE-DCSK schemes are given by

$$BER_{HE-DCSK} = \frac{1}{2} \operatorname{erfc} \left( \sqrt{\frac{E_b}{4N_o} \left( \frac{9}{8} + \frac{9 E_b}{10 M N_o} + \frac{9 M N_o}{32 E_b} \right) - 1} \right)$$

$$BER_{DCSK} = \frac{1}{2} \operatorname{erfc} \left( \sqrt{\frac{E_b}{4N_o} \left( 1 + \frac{2 E_b}{5 M N_o} + \frac{M N_o}{2 E_b} \right) - 1} \right)$$

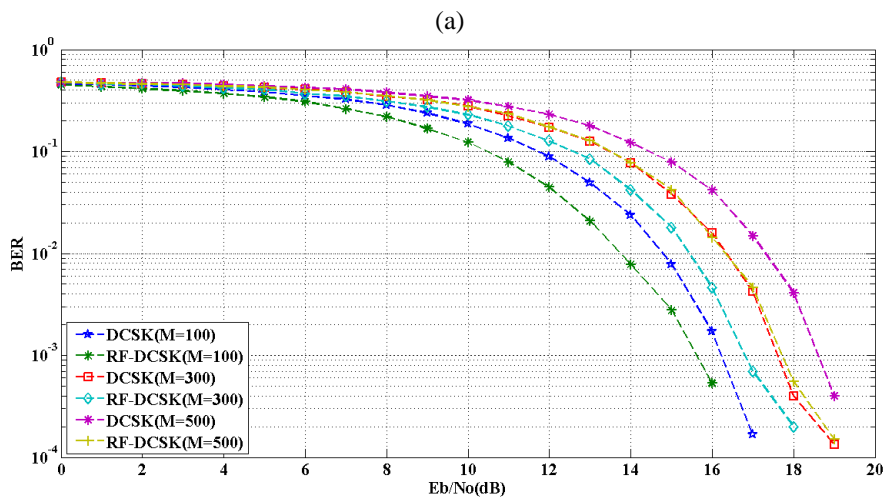
$$BER_{CDSK} = \frac{1}{2} \operatorname{erfc} \left( \sqrt{\frac{E_b}{4N_o} \left( 2 + \frac{19 E_b}{10 M N_o} + \frac{M N_o}{2 E_b} \right) - 1} \right)$$

### 4. Simulation results

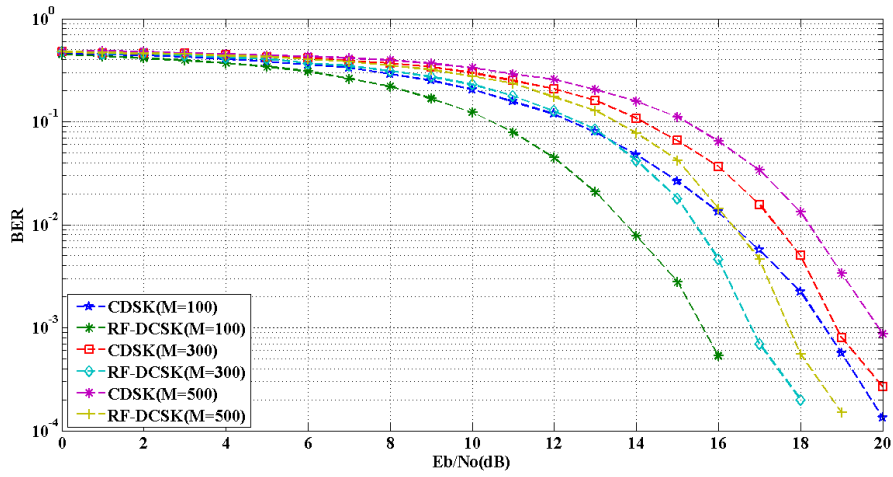
In this section, computer simulations are performed to evaluate the BER performance of and RF-DCSK versus DCSK, CDSK, HE-DCSK in AWGN environment and with different  $E_b/N_o$  levels. Effect of the spreading factor on the proposed system is studied. Furthermore, theoretical estimate for BER and simulation results for RF-DCSK is compared.

In the beginning, the RF-DCSK is compared with DCSK at  $M=100,200$  and  $300$  as shown in Fig 4.a. It can be observed that the noise performance for of RF-DCSK outperforms DCSK system by an average of 1 dB at BER of  $1 \times 10^{-3}$ . The reason of which is that the distance between the signal elements in correlator output of RF-DCSK is twice as in DCSK [3]. Then, the BER of the proposed scheme is compared with the CDSK at similar values of the spreading factors in DCSK case. Clearly, CDSK scheme performs worse than RF-DCSK by an average of 2dB at BER of  $1 \times 10^{-3}$  as shown in Fig 4.b. This is due two factors. First, the number of intrasignal terms in CDSK [3] is more to that in RF-DCSK. Second, the distance between signal elements at the correlate output of CDSK is half to that in RF-DCSK. In HE-DCSK, distance between signal elements is increased by 25% with respect to DCSK and CDSK. But this is not enough to exceed the noise performance of RF-DCSK as shown in Fig 4.c. which has a gain of less than 1 dB at BER  $1 \times 10^{-3}$ .

Effect of spreading factors is studied by testing the system with increasing value of  $M$  as illustrated in Fig. 5. Noise performance is enhanced typically between  $M=10$  and  $M=100$ , and then degraded with increasing value of  $M$ , which is consistent with equation (5). This trend occurs due to the increasing contribution of noise-noise cross terms in (3). When we increase  $M$ , keeping constant  $E_b/N_o$  at a fixed value, we increase  $N_o$  proportionally to  $M$ . Thus, while the useful signal in (3) increases linearly with  $M$ , and so does the standard deviation of  $2 \sum_{i=1}^M (x_i x_{M-i+1}) + 2b \sum_{i=1}^M (x_i \psi_{M-i+1}) + 2 \sum_{i=1}^M (x_i \psi_i)$ , the standard of deviation of  $\sum_{i=1}^M (\psi_i \psi_{M-i+1})$  grows faster. Additionally, there is a mismatch between theoretical estimate and simulation result at low value of the spreading factor. This is because of the fluctuated value of the bit energy at lower values of  $M$  that confirm the limitation of GA method. Excellent matching between theoretical estimate and simulation results can be noted in Fig. 6. at  $M=100$ ,  $M=300$  and  $M=500$  which positively support our derived equation in (5).



(b)



(c)

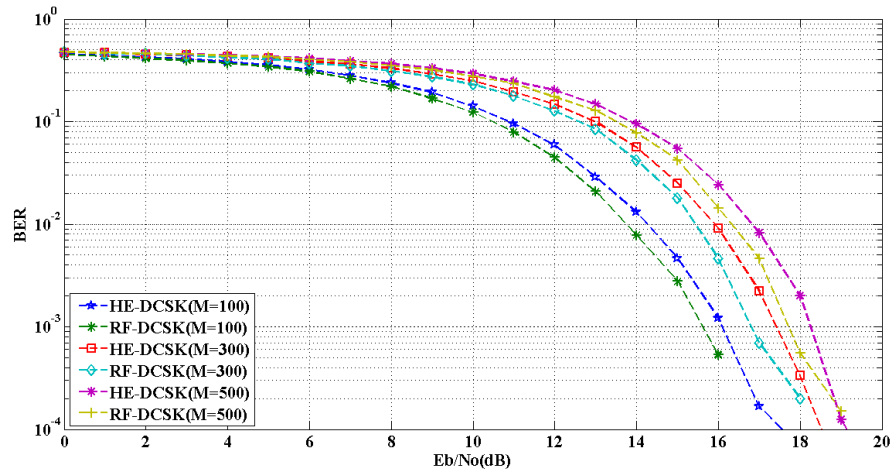


Fig. 4: BER Performance of RF-DCSK Versus. A) DCSK B) CDSK C) HE-DCSK

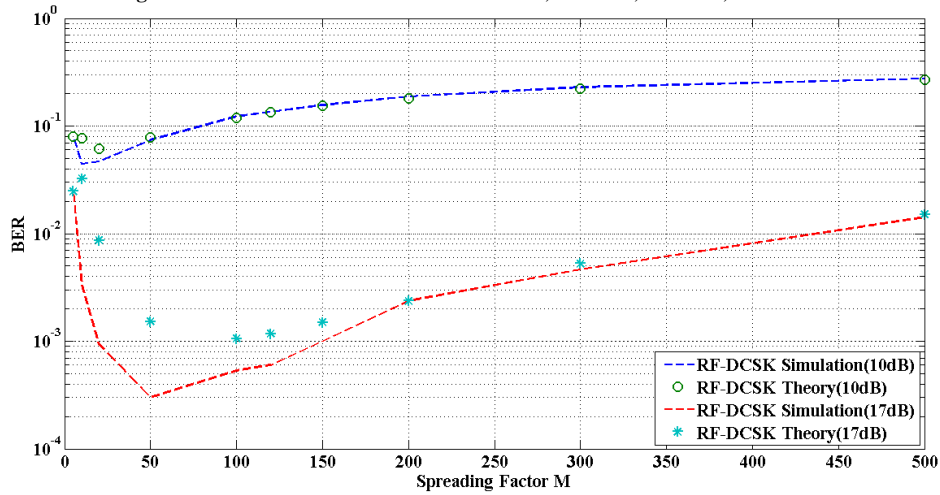


Fig. 5: BER Performance of RF-DCSK vs. Spreading Factor M

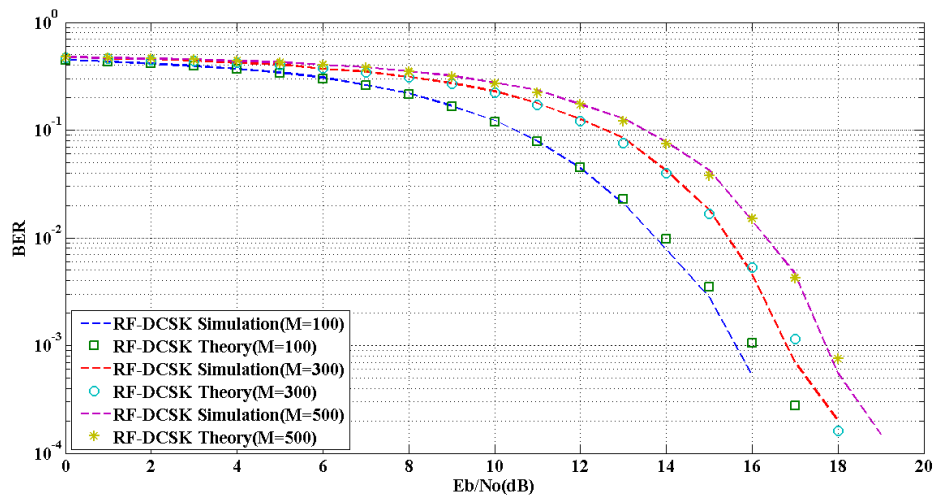


Fig. 6: BER Performance of RF-DCSK. Simulation Results and Theoretical Estimation At M=100

## 5. Conclusion

In this paper, a differential coherent chaos based communication scheme is proposed. At large value of spreading factors, which are normally used in spread spectrum communications, correlation between chaotic signal and its time-flipped version, is very low. Each transmitted signal will be the sum of mirrored version of the reference signal and the information bearing signal. Therefore, no extra time slot is required for a reference signal. At the receiver, simple correlator is used at to decode the information bits. System performance against AWGN has reasonable advantage compared to DCSK, CDSK, and HE-DCSK. DCSK.

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