



# Replenishment policy for an economic production quantity model considering rework and multiple shipments

Usman Ghani<sup>1\*</sup>, Mubashir Hayat<sup>1</sup>

<sup>1</sup> Department of Mechanical Engineering, University of Engineering and Technology Peshawar, Jalozai Campus

\*Corresponding author E-mail: [usmanghani@uetpeshawar.edu.pk](mailto:usmanghani@uetpeshawar.edu.pk)

## Abstract

Determining the optimal replenishment lot size and shipment policy for a production setup has been of greater interest during the last few years. This paper derives the optimal replenishment lot size and shipment policy for an Economic Production Quantity (EPQ) model with rework of defective items. However, in a real life situation, multi-shipment policy is used in lieu of continuous issuing policy and generation of defective items is inevitable. The proposed research assume that all imperfect quality items are reworked to perfect quality items and then all perfect quality items are delivered to the customers. Mathematical modeling is used in this study and the long-run average production–inventory–delivery cost function is derived. Convexity of the cost function is proved by using the Hessian matrix equations. The closed-form optimal replenishment lot size and optimal number of shipments that minimize the long-run average costs for such an EPQ model are derived.

**Keywords:** Economic Production Quantity (EPQ) Model; Optimal Replenishment Lot Size; Shipment Policy; Imperfection in Production Process; Backorders.

## 1. Introduction

The traditional economic production quantity (EPQ) model have played an important role in the practical inventory management. The basic EPQ model assume that in the production setup, all the items produced are of good quality and are defect free. In reality, however, production process failures are always inevitable. Therefore, the items produced by manufacturers are usually of imperfect quality. Rosenthal and Lee (1986) first studied the influence of imperfect production process on the EPQ model. Salameh and Jaber (2000) developed an EPQ model to determine the optimal order size, where each order contains a percentage of imperfect items. Eroglu and Ozdemir (2007) extended Salameh and Jaber's model by allowing shortage to be backordered. Porteus (1986) analyzed the effect of an imperfect production process on the optimal production cycle. Tapiero (1987) discussed optimal quality inspection policies and the resulting improvements in the manufacturing cost. Hayek and Salameh (2001) proposed an EPQ model with the repairing of imperfect quality items. By considering capital investment in process quality improvement, Hou and Lin (2004) discussed the effect of process deterioration on the optimal production time. Chiu (2003) developed an EPQ model with a random defective rate, a remanufacturing process and backlogging. By using geometric programming, Leung (2007) proposed an EPQ model with a flexible and imperfect production process. Hou (2007) considered an EPQ model with imperfect production processes, in which both the process quality and the setup cost are dependent on capital expenditure. Sana (2010) presented an EPQ model for items with the imperfect quality, in which the amount of the defective items produced becomes more as the rate of production increases due to improper distribution of raw materials, machinery problems, and so on. Sana (2011) a three-tier production-inventory model of imperfective items was developed. Khan et al. (2010) and Wahab and Jaber (2010) further proposed an EOQ model for imperfective items with learning in inspection. Khan et al. (2011) presented a detailed review of EOQ/EPQ models with imperfect quality items. Recently, T. Hsu and F. Hsu (2013) built two economic production quantity models with imperfect production processes, inspection errors, planned backorders, and sales returns, whereas T. Hsu and F. Hsu (2013) developed an economic order quantity model with imperfect quality items, inspection errors, shortage backordering, and sales returns. Taheri-Tolgari et al. (2012) presented a discounted cash-flow approach for an inventory model for imperfect items under inflationary conditions with considering inspection errors and related defect sales return issues. Sarkar and Moon (2011) developed a production inventory model in an imperfect production system by considering stochastic demand and the effect of inflation, whereas Jaber et al. (2013) proposed an entropic version of an EOQ model with imperfect quality items. Khan et al. (2014) further provided a simple but integrated model for determining an optimal vendor–buyer inventory policy by accounting for quality inspection errors at the buyer's end and learning in production at the vendor's end. Jaber et al. (2014) developed economic order quantity models for imperfect items with buy and repair options.

Ertogral, Darwish et al (2007) found shipment lot size in a manufacturer-buyer setup for three different cases. In first case the model was developed without considering transportation cost. In second case the model considered transportation cost into account while developing the model whereas in third case the over-declaring of batch size is considered. Sajadieh and Jokar (2009) developed a marketing based inventory related model to find the appropriate values of decision variables to increase overall profit to organization. Chiu, Liu et



al (2011) found appropriate replenishment for EPQ considering multiple shipments and rework process. The model considered the failure of some of the reprocessed items during rework and made them scrap. Chiu, Lin et al (2013) examined a manufacturer-retailer combined setup with reprocessing and better items shipment to reduce inventory carrying cost. The model incorporated the ‘n + 1’ shipment policy and determined appropriate shipments and replenished items. Kropf and Sauré (2014) developed a model by considering a fixed cost per shipment. To conclude the size of fixed cost the model used a data of Switzerland border on performance level. Ekici, Altan et al (2016) developed a model by considering the order size restriction and the benefits of the order size consolidations. The model also observed the non-trivial pricing behavior of manufacturer as well as of suppliers by considering different settings. Giri, Chakraborty et al (2017) worked with consignment stock policy by considering a case in which a single type of item is delivered by a single producer to only one buyer in an un-equal size shipment. Sağlam and Banerjee (2017) formulated a mathematical model for integration of manufactured items size scheduling issue regarding transportation decisions. The model considered the both full and less than truck load shipments. The basic goal of the work was to minimize the overall cost on the manufacturer side.

The transportation have significant impact on economic activities of any organization. It means that special attention should be given to the shipment policies. In this direction, the proposed research extends the existing inventory models by developing a shipment policy for it.

The rest of the paper is organized as follows. Section 2 lists the notations and assumptions which are used in the model. Section 3 describes the model in details. Section 4 shows a numerical computation and sensitivity analysis to illustrate the managerial insights of the model. A brief conclusion and future recommendations are given in Section 5.

## 2. Model preparation

The setup considers the production-inventory system, where a manufacturer produces products in a single stage production. Each batch of items produced contains a percentage of defective items. These defective parts are reprocessed in the same production cycle to make perfect quality products. These good quality products are then shipped to the retailers. If the manufacturer chooses to make an order, the following points are to be considered:

- 1) What is the optimal ordering quantity?
- 2) What is the optimal shipment size?

The existing models do not consider a shipment policy for a case of imperfection in process. Therefore, in this regard it is presented a basic EPQ model with imperfect quality and a shipment policy. The model consider the following notation and assumptions:

### 2.1. Notation used in the model

Notation	Description
A	Demand rate (units per time)
B	Production rate (units per time, $B > A$ )
C	Batch size (units)
D	Size of backorders (units)
E	Manufacturing cost of a product (\$ per unit)
F	Manufacturer production setup cost (fixed cost, \$ per setup)
G	Retailer ordering cost (fixed cost, \$ per order)
H	Manufacturer inventory carrying cost per product per unit of time
I	Retailer inventory carrying cost per product per unit of time
J	Backorder cost per product per unit of time (linear backorder cost)
K	Backordering cost per product (fixed backorder cost)
L	Shipment lot size (decision variable)
M	Number of shipments (decision variable)
$\Delta X$	Difference between inventory carrying cost of manufacturer and retailer
TC	Total cost per unit of time

### 2.2. Assumptions in the development of model

- i) Demand and production rates are constant and known over horizon planning.
- ii) Production rate is greater than demand rate ( $B > A$ ).
- iii) The products are 100% screened and the screening cost is ignored.
- iv) All the defective products are reworked to make perfect quality products.
- v) Only single type of item is considered in the development of model.
- vi) Inventory carrying cost is considered for average inventory.
- vii) Production and rework is done in the same production system at same production rate.

## 3. Model formulation

As the model considers the shipment of finished products from the manufacturer to the retailers therefore, the model is composed of the different types of costs which are given in table 1.

**Table 1:** Types of the Costs Used in the Model

Type of Cost	Description	Mathematical Form
Setup cost of manufacturer	The annual setup cost for manufacturer is $A/C$ times per cycle cost	$\frac{FA}{C}$
Ordering cost of retailer	Total ordering cost of the retailer is equal to ordering cost times the number of orders	$\frac{MGA}{C}$
Inventory carrying cost of manufacturer	The inventory carrying cost of the manufacturer is maximum inventory times the manufacturer inventory carrying cost per product per unit of time	$H \left[ \frac{AL}{B} + \frac{(B-A)C}{2B} \right]$
Inventory carrying cost of	The inventory carrying cost of the retailer is equal to the average retailers inventory times the differ-	$\frac{\Delta XL}{2}$

retailer	ence between inventory carrying costs per product of manufacturer to retailer	
Fixed backordering cost	Fixed back-ordering cost for this system is equal to the DA/Q times the backorder cost per product	$\frac{KDA}{c}$
Linear backordering cost	Linear back-ordering cost for this system is equal to the average backorder times the backorders cost per product per unit of time	JV
Production cost	The production cost per batch in a cycle is $EC[1+Y(R)]$ and the total production cost per year is given by $EC[1+Y(R)]\left(\frac{A}{c}\right)$	$EA[1+Y(R)]$

Therefore, the total cost function is given as follow: Total cost = manufacturer setup cost + retailer ordering cost + manufacturer inventory carrying cost + retailer inventory carrying cost + fixed backordering cost + linear backordering cost + production cost. Incorporating the values for these cost functions,

$$TC(L, M) = \frac{FA}{c} + \frac{MGA}{c} + H \left[ \frac{AL}{B} + \frac{(B-A)C}{2B} \right] + \frac{\Delta XL}{2} + \frac{KDA}{c} + \frac{JD^2Z}{2CY} + EA(2 - Z) \quad (1)$$

After simplification,

$$TC(L, M) = \frac{(F+MG)A}{ML} + H \left[ \frac{AL}{B} + \frac{(B-A)ML}{2B} \right] + \frac{\Delta XL}{2} + \frac{KDA}{ML} + \frac{JD^2Z}{2MLY} + EA(2 - Z) \quad (2)$$

The total cost consists of two decision variables i.e. shipment lot size and number of shipments. Therefore, for minimization of the total cost the second order Hessian Matrix is positive definite which implies all principle minors are positive. And for this problem the sufficient conditions are as follows:

$$\frac{\partial^2 TC(L, M)}{\partial L^2} > 0 \text{ and } \frac{\partial^2 TC(L, M)}{\partial L^2 \partial M^2} > 0$$

Taking partial derivative of equation (2) with respect to L and n respectively,

$$\frac{\partial TC}{\partial L} = -\frac{FA}{ML^2} - \frac{GA}{L^2} + \frac{HA}{B} + \frac{H(B-A)M}{2B} + \frac{\Delta X}{2} - \frac{KDA}{ML^2} - \frac{JD^2Z}{2ML^2Y} \quad (3)$$

And

$$\frac{\partial TC}{\partial M} = -\frac{FA}{M^2L} + \frac{H(B-A)L}{2B} - \frac{KDA}{M^2L} - \frac{JD^2Z}{2M^2LY} \quad (4)$$

From sufficient conditions,

$$\frac{\partial^2 TC}{\partial L^2} = \left( \frac{2}{ML^3} \right) \left[ (F + KD)A + \frac{JD^2Z}{2Y} \right] + \frac{2GA}{q^3}$$

$$\frac{\partial^2 TC}{\partial M^2} = \left( \frac{2}{M^3L} \right) \left[ (F + KD)A + \frac{JD^2A}{2Y} \right]$$

$$\frac{\partial^2 TC}{\partial L \partial M} = \left( \frac{1}{M^2L^2} \right) \left[ (F + KD)A + \frac{JD^2Z}{2Y} \right] + \frac{H(B-A)}{2B}$$

$$\left( \frac{\partial^2 TC}{\partial L \partial M} \right)^2 = \left( \frac{1}{M^2L^2} \right)^2 \left[ (F + KD)A + \frac{JD^2Z}{2Y} \right]^2 + \left[ \frac{H(B-A)}{2B} \right]^2 + 2 \left( \frac{1}{M^2L^2} \right) \left[ (F + KD)A + \frac{JD^2Z}{2Y} \right] \left[ \frac{H(B-A)}{2B} \right]$$

$$\frac{\partial^2 TC \partial^2 TC}{\partial L^2 \partial M^2} = \left( \frac{4}{M^4L^4} \right) \left[ (F + KD)A + \frac{JD^2Z}{2Y} \right]^2 + \left( \frac{4GA}{M^3L^4} \right) \left[ (F + KD)A + \frac{JD^2Z}{2Y} \right]$$

Therefore,

$$\frac{\partial^2 TC \partial^2 TC}{\partial L^2 \partial M^2} - \left( \frac{\partial^2 TC}{\partial L \partial M} \right)^2 = \left( \frac{4}{M^4L^4} \right) \left[ (F + KD)A + \frac{JD^2Z}{2Y} \right]^2 + \left( \frac{4GA}{M^3L^4} \right) \left[ (F + KD)A + \frac{JD^2Z}{2Y} \right] - \left( \frac{1}{M^2L^2} \right)^2 \left[ (F + KD)A + \frac{JD^2Z}{2Y} \right]^2 - \left[ \frac{H(B-A)}{2B} \right]^2 - 2 \left( \frac{1}{M^2L^2} \right) \left[ (F + KD)A + \frac{JD^2Z}{2Y} \right] \left[ \frac{H(B-A)}{2B} \right]$$

$$\text{Let } U = \left( \frac{1}{M^2L^2} \right) \left[ (F + KD)A + \frac{JD^2Z}{2Y} \right]$$

$$T = \left( \frac{4GD}{M^3L^4} \right) \left[ (F + KD)A + \frac{JD^2Z}{2Y} \right]$$

$$S = \left[ \frac{H(B-A)}{2B} \right]$$

Therefore,

$$\frac{\partial^2 TC \partial^2 TC}{\partial L^2 \partial M^2} - \left( \frac{\partial^2 TC}{\partial L \partial M} \right)^2 = 4U^2 + T - U^2 - S^2 - 2US$$

$$\frac{\partial^2 TC \partial^2 TC}{\partial L^2 \partial M^2} - \left( \frac{\partial^2 TC}{\partial L \partial M} \right)^2 = 3U^2 + N - S^2 - 2US \quad (5)$$

For minimization of the cost equation, the condition is  $3U^2 + T - S^2 - 2UO > 0$  i.e. if the expression  $3U^2 + T - S^2 - 2UO$  is greater than 0, then it means that sufficient condition of optimality criteria is satisfied. Therefore, it can be concluded that the cost equation is convex when the expression  $3U^2 + T - S^2 - 2UO > 0$  is satisfied. To obtain the optimum points for this case, the 1st order partial derivatives with respect to the variables are separately equal to 0. Therefore, for this case the necessary conditions are as follows:  $\partial TC(L, M)/\partial L = 0$  and  $\partial TC(L, M)/\partial M = 0$

Putting Eq (3) equal to zero

$$0 = -\frac{(F+MG)A}{ML^2} + \frac{HA}{B} + \frac{H(B-A)M}{2B} + \frac{\Delta X}{2} - \frac{KDA}{ML^2} - \frac{JD^2Z}{2ML^2Y}$$

After simplifications,

$$L^* = \sqrt{\frac{[2YA(F+MG+KD)+JD^2Z]}{[2MY\left(H\left(\frac{A}{B} + \frac{(B-A)M}{2B}\right) + \frac{\Delta X}{2}\right)]}} \tag{6}$$

Now putting Eq (4) equal to zero

$$0 = -\frac{FA}{M^2L} + \frac{H(B-A)q}{2B} - \frac{KDA}{n^2L} - \frac{JD^2Z}{2M^2LY}$$

After simplification,

$$M^* = \sqrt{\frac{[8Y^2BAH(F+KD)(B-A)+4YB]D^2ZH(B-A)}{2LYH(B-A)}} \tag{8}$$

Solving Eq (6) and Eq (7) simultaneously,

$$L = \frac{\sqrt{[8Y^2BAH(F+KD)(B-A)+4YB]D^2ZH(B-A)}}{2MYH(B-A)}$$

Subtracting Eq (8) from Eq (6),

$$0 = \sqrt{\frac{[2YA(F+MG+KD)+JD^2Z]}{2MY\left[H\left(\frac{A}{B} + \frac{(B-A)M}{2B}\right) + \frac{\Delta X}{2}\right]}} - \frac{\sqrt{[8Y^2BAH(F+KD)(B-A)+4YB]D^2ZH(B-A)}}{2MYH(B-A)} \tag{9}$$

After simplification,

$$M^* = \sqrt{\frac{[2YA(F+KD)+JD^2Z][B\Delta X+2HA]}{2YDHG(B-A)}} \tag{10}$$

Hence, incorporating the values of optimum replenishment lot size ( $L^*$ ) and shipments size ( $M^*$ ) in equation 2 we get the optimum total cost equation as:

$$TC(L^*, M^*) = \frac{(F+M^*G)A}{M^*L^*} + H\left[\frac{AL^*}{B} + \frac{(B-A)M^*L^*}{2B}\right] + \frac{\Delta XL^*}{2} + \frac{KDA}{M^*L^*} + \frac{JD^2Z}{2M^*L^*Y} + EA(2 - Z) \tag{11}$$

### 4. Numerical computation and sensitivity analysis

This section performs numerical computations and sensitivity analysis. The example uses the data of Sarkar, Cárdenas-Barrón et al (2014) and Ertogral, Darwish et al (2007) models. In order to check the effect of key parameter on overall cost for the model, sensitivity analysis are performed.

Example: For the proposed model, the different values are taken as follows:  $A = 300$  units/year,  $a = 0.03$ ,  $b = 0.07$ ,  $B = 550$  units/year,  $H = \$50$ /unit/year,  $J = \$10$ /unit/year,  $K = \$1$ /unit short,  $F = \$50$ /lot size,  $E = \$7$ /unit,  $D = 33$  units,  $I = \$60$ /unit/year and  $G = \$10$ /lot size. Then by using equation (9), the optimum solution is:

$M^* = 5.97$ , therefor

- 1) For  $M^* = 5$ ,  $L^* = 10.87$ units and  $TC(L, M) = \$4142.83$
- 2) For  $M^* = 6$ ,  $L^* = 9.61$ units and  $TC(L, M) = \$4136.14$

As total cost is minimum for  $M = 6$  and  $L = 9.61$  units so optimum solution is:

For  $M^* = 6$ ,  $L^* = 9.61$ units and  $TC(L, M) = \$4136.14$

**Table 2:** Sensitivity Analysis of Key Parameters

Parameter	Changes (%)	Total Cost (%)
B	-50	-33.43
	-25	+0.15
	+25	+1.4
	+50	+2.56

F	-50	-3.14
	-25	-1.57
	+25	+1.57
	+50	+3.14
G	-50	-3.77
	-25	-1.89
	+25	+1.89
	+50	+3.77
H	-50	-8.18
	-25	-4.09
	+25	+4.09
	+50	+8.18
I	-50	-3.49
	-25	-1.74
	+25	+1.74
	+50	+3.49
E	-50	-26.66
	-25	-13.33
	+25	+13.33
	+50	+26.66
A	-50	-31.47
	-25	-16.01
	+25	+17.38
	+50	+40.41
M	-50	+7.88
	-25	+1.31
	+25	+0.8
	+50	+2.65
L	-50	+11.68
	-25	+1.95
	+25	+1.16
	+50	+3.89

## 5. Conclusion and future recommendations

The classic EPQ model assumes a continuous inventory issuing policy for satisfying product demand and a perfect quality production for all items produced during the production process. However, in a real life situation, multi-shipment policy is used in lieu of continuous issuing policy and generation of defective items is inevitable. This paper investigates the aforementioned issues by incorporating a multiple delivery policy for an Economic Production Quantity (EPQ) model with rework of defective items. Mathematical modeling is employed here, and the long-run average production–inventory–delivery cost function is derived and proved to be convex. The closed-form solutions in terms of optimal replenishment lot size and optimal number of shipments to the problem are obtained.

Furthermore, a numerical example and sensitivity analysis were presented to point out specification of this research work. It can be concluded from sensitivity analysis that by increasing the values of fixed setup cost (F), fixed ordering cost (G), unit inventory carrying cost (H and I), fixed cost per backorder (K), linear backorder cost (J), unit production cost (C), and demand (A) increases the total cost and vice versa. But an interested result can be seen for replenishment lot size (L) and shipment size (M). -50% to +50% changes in replenishment lot size (L) and shipment size (M) show an increase in the overall cost of the system. Also it is observed that demand (A) and unit production cost (E) are the most sensitive parameters as compared to all other parameters of the model.

For future study, interesting topics may be included to examine the effects on the same decisions when shortage with backlogging, or a deviation (e.g. seasonal) of shipment rate to customers are under consideration. Some practical case studies can also be expected to show the effectiveness of the research results. The model considers only one type of items in a single stage production system. However, in reality there may be situations where a different type of items are produced in a multi-stage production system, therefore, the present research can be extended for production of multi-items in multi-stage production system.

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