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Comparison of Different Orders Optimization Using Continuous Genetic Algorithm

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Abstract

Method to optimize different orders of digital filter and their comparison results are presented in this paper. An algorithm that operates on a complex continuous search space is constructed. The algorithm developed in this paper is different from the conventional method since here algorithm is applied on the continuous search space parameter instead on discrete search space parameter. Also new genetic operators are presented which combine crossover and adaptive mutation to improve solution quality of Genetic Algorithm (GA). A customized application layer called filter design algorithm (FDA) to handle specific format and properties of filter design problem has been develop for optimized GA.

Keywords: *Butterworth filter, Continuous genetic algorithm (CGA), Discrete Genetic Algorithm (DGA), Filter design algorithm (FDA), IIR digital filters, Optimization.*

1 Introduction

The main concern while designing digital filter is instability, magnitude and phase response. In this work, algorithm is proposed which can be used to design higher order Butterworth filter with a desired characteristic and response. The above proposed work is based on Genetic Algorithm (GA), GA is a parallel optimization technique that relies on the basics of evolution for optimizing a group of solutions [3]. The concept of evolution was popularized by Charles Darwin in the early 1900s [5]. The GA is merely a framework under the actual algorithm required for optimization and it must be customized for a given application [4]. The algorithm presented for optimizing continuous problems is called the Continuous Genetic Algorithm (CGA).

2 Previous Work

In this paper, digital IIR filter design has been proposed and implemented. Due to a multi-modal mean square error function, IIR design is more preferable than finite impulse response (FIR) design. To solve this, a global least mean square algorithm has been proposed to allow a global minimum search. This is similar to simulated annealing (SA) and is called stochastic approximation with convolution smoothing (SAS) and is realized by convolving the objective function with a noise probability density function. When combined with the least mean square (LMS) algorithm, it has shown success in adaptive IIR filtering [6]. Another approach for adaptive IIR filter design is also an extension of the LMS algorithm. The LMS algorithm is applied to multiple filters of different initial conditions to help reduce the probability of convergence to a local minimum. Whenever the rate of convergence slows or a filter becomes unstable, an embedded evolutionary computation is used to move the previous filter coefficients. This approach benefits from the directed search of the LMS algorithm and the ability to recover from instability [7]. Simulated results show improvements in global optimization compared to the LMS algorithm, pure SA, and pure GA, but the proposed technique are used to IIR design problem for investigation of abilities [8]. The main differences between them lie in the customization of the algorithm for use in the IIR design problem. The method of generating a sample population, performing crossover and mutation and evaluating the results [9]. The coefficient symmetry gives us a linear phase even in the case of an IIR filter [10].

3 Discrete Genetic Algorithm

The Discrete Genetic Algorithm (DGA) operates on a discrete set of chromosomes where, the chromosomes are generally considered to be binary encoded as single-bit genes. There are three operators of discrete genetic algorithm.

Selection

The purpose of selection is to drive population $P(g)$ towards the most promising areas of search space (S) while still maintaining enough variation in $P(g)$ to prevent premature solution convergence. The accepted notion of survival of the fittest, any selection strategy chosen for the DGA must have a higher probability of selecting the more fit elements of $P(g)$ to form the selection subset. There are three most common methods of selection *proportionate*, *rank-based*, and *truncation selection* each with their own set of advantages and disadvantages.

Crossover

The crossover genetic operator redistributes genetic material within *P*(*g*). Crossover is executed by combining or mixing two elements with probability to form one or more offspring that combined genetic material. The goal is to generate new elements that are fit than their parents, thereby contributing to the overall fitness and convergence of the population. Three most common methods of crossover are *single point, double point and uniform crossover.*

Mutation

During the optimization process it is sometimes necessary to remove desirable genetic material from the population to overcome local optima. Therefore, the mutation operator has been developed to introduce new or lost genetic material into the population. There are three most common methods of mutation are *uniform, normal and Breeder genetic algorithm (BGA).*

4 Continuous Genetic Algorithm

A customized application layer has been developed for the CGA to handle the specific format and properties of the filter design problem. These requirements include a method for mapping a filter to an element, calculate the fitness of the filter, develop an initial population of filters, and ensuring that all filters are realizable. First, digital IIR filters are designed and optimized to magnitude responses with known transfer functions such as a Butterworth bandpass filter. A complete flow chart of CGA is shown in Fig.1

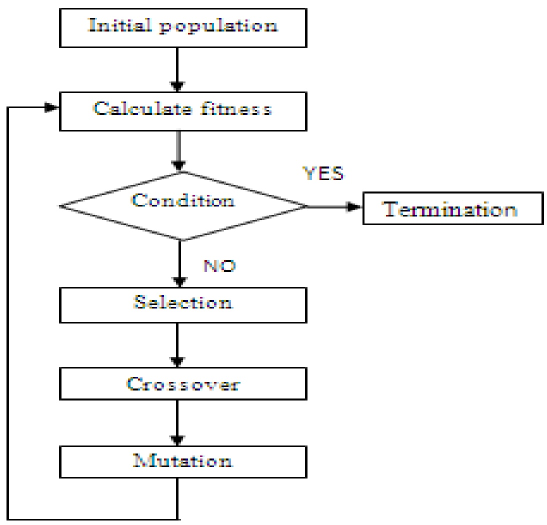


Fig.1: Flow chart of CGA

4.1 IIR Filter Properties

Let us begin by defining the transfer function $H(z)$ for a digital IIR filter as [1]

$$H(z) = \frac{N(z)}{D(z)} = \frac{\sum_{i=0}^{\alpha} c_i z^{-i}}{1+\sum_{i=1}^{\alpha} b_i z^{-i}} = k\ \frac{\prod_{i=1}^{\alpha}(z-z_i)}{\prod_{i=1}^{\alpha}(z-p_i)} \quad (1)$$

The coefficients of the polynomial form are b_i and c_i. The zeros and poles of the factored form are z_i and p_i respectively. The gain factor K is necessary for equivalence between the polynomial and factored forms. The order of $H(z)$ is determined by α.

Two properties of $H(z)$

- A causal, linear, time invariant (LTI) system with system function $H(z)$ is bounded input bounded output (BIBO) stable if and only if all the poles of $H(z)$ lie inside the unit circle. ($|p_i|<1$)
- A causal, stable, LTI system with system function $H(z)$ is real if and only if all complex poles and zeros of $H(z)$ have complex conjugate pairs or exist singularly on the real axis.

4.2 Mapping of the Filter

For the CGA to evolve and optimize digital IIR filters, a method for mapping a filter transfer function $H_n(z)$ to an element x_n is needed. Two straightforward methods for doing this include mapping either the coefficients of the polynomial form of $H_n(z)$ or the roots and gain of the factored form of $H_n(z)$ to the vectors of x_n. While both of these options are mathematically equivalent, polynomial coefficients b_i and c_i can have several orders of magnitude of dynamic range necessitating a very large search space S. Filter stability requires that all poles p_i of $H_n(z)$ are inside the unit circle, thus, limiting the search space S for p_i. Minimum-phase is accomplished by having all poles and zeros of $H_n(z)$ inside the unit circle. This relationship between vectors changes the way crossover and mutation can operate. For instance, if crossover generates an offspring with a complex vector $a_{n,m}$ the crossover operator must ensure that a complex vector $a_{n,k}$ that satisfies is also generated. Furthermore, if mutation modifies $a_{n,m}$ by then mutation must also modify $a_{n,k}$ *by* λ^*. Removing these vector dependencies allows previously discussed crossover and mutation strategies for the CGA to be used. This is done by removing the vector $a_{n,k}$ from x_n and interpreting every complex vector $a_{n,m}$ as two vectors $a_{n,m}$ and $a^*_{n,m}$. This simplification act does not come without consequence. It requires the number of poles and the number of zeros of $H_n(z)$ to be even, and poles and zeros are no longer able to exist singularly on the real axis. However, the reduction in the size of M and the ability to use regular crossover and mutation operators serves as sufficient justification.

4.3 Fitness of the Filter

Thus, the fitness function of the CGA should be based on both the magnitude responses of the filter undergoing evaluation and the desired magnitude response. A frequency weighted squared error technique is proposed for this. The fitness of x_n is calculated by first mapping the vectors of x_n to the pole and zero pairs of H_n(z). Next, the magnitude response $H_n(e^{j\omega})$ of H_n(z) with a default gain of k=1 is evaluated for all frequency bins. The desired magnitude response of $H_d(e^{j\omega})$ must also be identified at these same frequency bins. To compensate for the use of unity gain in H_n(z),$H_n(e^{j\omega})$ is scaled byk_n, where $k_n$is chosen to minimize the error between $k_n H_n(e^{j\omega})$ and $H_d(e^{j\omega})$.This is achieved by forcing the average magnitude value of $k_n H_n(e^{j\omega})$ to equal the average magnitude value of $H_d(e^{j\omega})$.The equation for calculating $k_n$is,

$$k_n = \frac{\sum_{y=1}^{Y} |H_d(e^{j\Omega y})|}{\sum_{y=1}^{Y} |H_n(e^{j\Omega y})|} \tag{2}$$

Next, the squared error is calculated by squaring the difference between $K_n H_n(e^{j\omega})$ and $H_d(e^{j\omega})$ for all. The squared error values are then weighted by multiplying them with a weighting vector Q that assigns a weighting factor to each frequency bin. This enables certain frequency bins of the magnitude response to contribute more or less to the overall fitness of x_n. Finally, the weighted squared error values are summed and scaled to produce the fitness value of x_n. If $k_n H_n(e^{j\omega})$ is identical to $H_d(e^{j\omega})$, then the fitness value will be zero. The complete fitness function used by the FDA is,

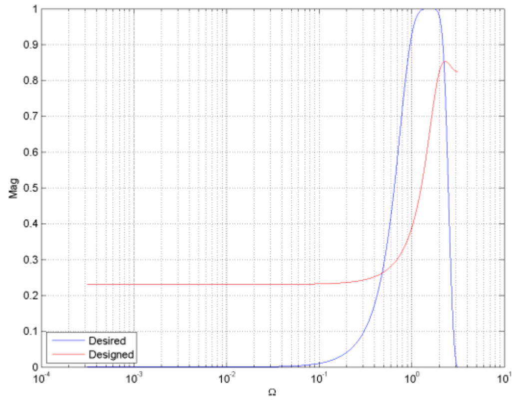
$$f(x_n) = \frac{1}{Y}\sum_{y=1}^{Y}\left[K_n\left|H_n\left(e^{j\Omega y}\right)\right| - \left|H_d\left(e^{j\Omega y}\right)\right|\right]^2 Q_y \tag{3}$$

Where Y is the total number of frequency bins, Ω_y is an element of Ω, and Q_y is an element of Q. Since only real filters are considered for H_n(z), Ω is generally only specified in the range of 0 to π. Any distribution of frequency points, such as linear for logarithmic spacing, within Ω can be applied with varying results on the outcome of the algorithm.

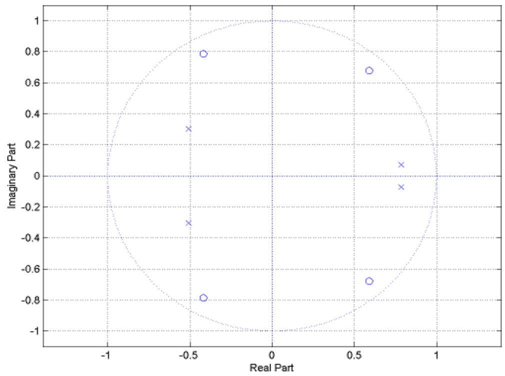
5 Simulation Results

The result comparison between 4th and 6th order Butterworth bandpass filter design are shown in Fig. 2 and 3 shows the variation in magnitude response, pole-zero and fitness curve between the different order of filter. In this simulation, the population size N = 100, element size $M = \alpha$ = 4 or 6 and probability of crossover

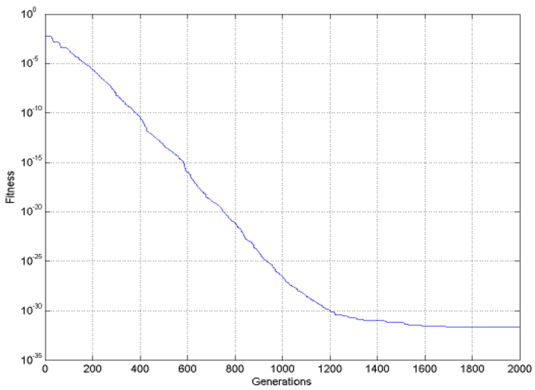
P_c= 0.7. The exit criteria are set to *gen_ max* = 2000 and *fit_ min* = 0, so that the FDA will search for the optimal solution for 2,000 generations. In this algorithm when the order of the filter is increased the complexity also increases. But when the lower order is selected then the result is less complex and better than the higher order filter.



(a) Magnitude response

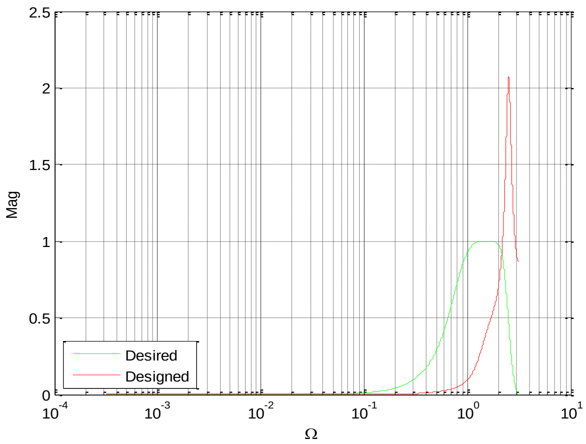


(b) Pole-zero diagrams

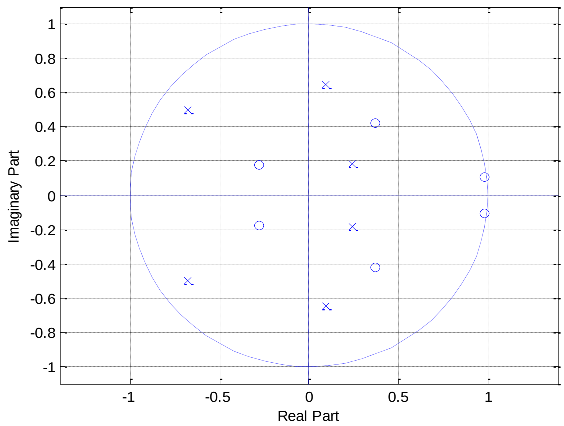


(c) Fitness curve

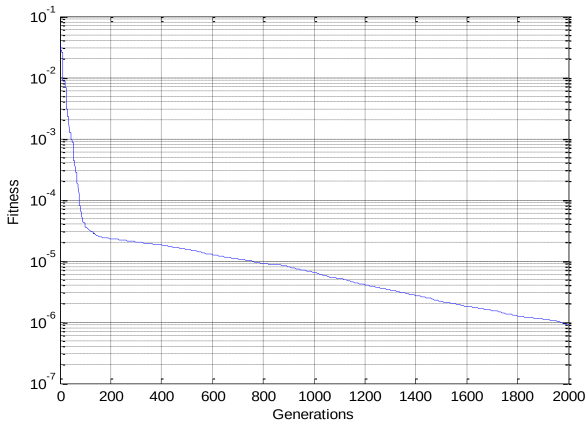
Fig.2: 4^{th} order Butterworth Bandpass Filter



(a) Magnitude response



(b) Pole-zero diagram



(c) Fitness curve

Fig.3: 6th order Butterworth Bandpass filter

6 Conclusion

In this paper, the design of a digital Butterworth IIR filter with an arbitrary magnitude response, pole-zero diagram and fitness curve are presented. An algorithm is used for continuous search parameter rather than discrete search parameter. This is due to the continuous valued coefficients or roots of a filter transfer function. The algorithm result shows that the best CGA filter is 4^{th} order filter as compared to 6^{th} order filter. Because when we increase the order of the filter the number of pole-zeros are increases and the pole-zero diagram become so complicated and error of the magnitude response also increases between the desired and designed filter. These results were analyzed to select the best CGA configuration for complex, continuous parameter optimization of the digital IIR filter. A proposed algorithm provides the better stability, robustness, minimum error and arbitrary magnitude response of the filter as compared to different existing algorithm.

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