



# Decision making for stable inspection planning of deteriorating structures based on constraint reliability and uncertainties

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## Abstract

Life-time cost minimization is considered as the optimal criterion for planning of inspection, repair and maintenance of structures. However, most of the probabilities and the cost items related to the cost analysis generally contain inevitable uncertainties in actual cases. The appropriateness of inspection planning may be lost by several errors induced by such uncertainties. In this study, a cost minimization method with the constraint of reliability is developed in order to obtain stable inspection planning against the estimation errors of the parameters. In the analysis, the life-time cost optimization is carried out under the constraint that the failure probabilities of the members are controlled below the respective target values allowed for the members. First, initial target failure probabilities are assumed for each member. Then, the robustness of the inspection planning is investigated by adjusting the parameters within the range of uncertainties. The initial values of the target failure probabilities are altered until an acceptable result is obtained. The applicability of the proposed method is examined for a structure with several uncertain parameters. A sequential cost minimization method is employed to optimize the life-time cost. It is made clear that by using this approach, the stability of the life-time cost is maintained without losing the benefit of the cost minimization method.

**Keywords:** Crack, Fatigue, Inspection, Numerical Model, Reliability, Structure.

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## 1. Introduction

Structural safety and reliability against fatigue failure are important factors for ships and offshore structures. Structural reliability can be achieved by adopting some suitable inspection, repair and maintenance policies. On the other hand, sufficient safety can be achieved by well-balanced use of several safety items at design, fabrication, inspection and repair maintenance stages [1]. Each safety item has a certain cost. Therefore, it is of importance to minimize the total expected cost during the life time of the structure and to keep the structural reliability at an acceptable level [2]. From this viewpoint, life-time cost minimization can be considered as the optimal criterion for estimating the strategy of inspection and repair maintenance of structures [3].

For this current work, the estimation of the inspection and repair maintenance strategy was carried out based on three different viewpoints:

- 1) Standard reliability,
- 2) Cost minimization, and
- 3) Cost minimization with constraint of reliability.

From the reliability viewpoint, a target failure probability is assumed for the members of the structure. The inspection timing is decided when the failure probability of any member of the structure reaches the target failure probability. The inspection quality is decided so as to recover the failure probability and maintain the reliability at an acceptable level.

From the cost minimization viewpoint, the life-time cost minimization method is developed based on the sequential cost minimization method [1]. The method aims to find an appropriate inspection and repair maintenance strategy for fatigue deteriorating structures. The cost evaluation equations were developed for all available inspection methods. In the cost

evaluation equations, the following cost items are included; inspection cost, repair cost, risk of member failure, risk of catastrophic failure, and the loss due to the drop of operation rate caused by scheduled and accidental system downs. Further, the influence of inevitable uncertain parameters on the inspection and repair maintenance strategy is considered. The predicted strategy, based on this approach, can be regarded as optimal from the viewpoint of practical use.

From the viewpoint of cost minimization with the constraint of reliability, the life-time cost minimization method is achieved in order to obtain a stable and optimal inspection and repair maintenance plan against the estimated errors and uncertainties of the parameters. In the analysis, the life-time cost optimization is carried out with the constraint that the failure probabilities of the members are controlled below the respective target values allowed for the members. First, initial target failure probabilities are assumed for each member. Then, the robustness of the inspection planning is investigated by adjusting the parameters within the range of uncertainties. The initial values of the target failure probabilities are altered until an acceptable result is obtained [4].

In this study, the estimation of the inspection and repair maintenance strategy is carried out based on the above-mentioned three aspects. The applicability of the proposed method is numerically examined for a hypothetical structure with several uncertain parameters [5]. The sequential cost minimization method is employed to optimize the lifetime cost. It is made clear that by using this approach, the stability of the life-time cost is maintained without losing the benefit of the cost minimization method.

## 2. The inspection planning and the corresponding uncertain parameters

Structural safety during service can be achieved through enough considerations for several safety items related to inspection and repair maintenance. However, excessive assurance of safety is economically not accepted, since the frequent inspection and repair actions usually accompany the rise in operating cost. It is thought that a structure has an optimal reliability level depending on the specification, inspection and repair cost, risk against an unexpected failure, and so on. The best way to determine such a reliability level is to employ decision making on the basis of life-time cost minimization [6].

However, most of the probabilities and the cost items required in the analysis generally contain several uncertain parameters. Working load on member, such as accuracy of structural analysis, accuracy of construction, risk of failure, deterioration property of member, initial defect condition of member, inspection capability, the rate of inflation, etc., include uncertainties in the actual world. Therefore, it is doubtful whether the formulation of a cost minimization approach will produce an optimal strategy. So, traditionally, the experience and subjective judgment of experts are highly regarded for the inspection planning of ships and offshore structures [7].

Recently, new aspects have occurred in the field of structural maintenance. One is the development of important structures such as large-sized offshore structures, bridges and power plants. Risk against unexpected failure is quite large for these structures, and also an inspection and repair action accompanies large economic losses due to service suspension. In the ship-building industry, lately, reconsiderations on the risk against environmental pollution by the oil leakage from tankers have become necessary for several reasons. Further, the economic losses due to unexpected service suspension caused by member failure have become recognized as a serious problem in the operation management [8].

Another problem is the tendency to pursue the economic efficiency of structures. Life extension programs are considered for aging ships, power plants, chemical plants, etc. for economic reasons. Reliability assessments in deteriorating conditions are to be required for those structures. Further, improvements of maintenance strategy are necessary for aging structures to keep the reliability at an acceptable level [9]. Under such circumstances, inspection and maintenance planning relying on only experience and subjective judgment is becoming gradually difficult. On the other hand, the objective evaluation of structural safety and the objective judgment for the appropriateness of maintenance have been recognized as important in several fields of structural engineering. In order to make an objective judgment, the reliability analysis method, considering repeated inspections and the optimization [10] method of inspection planning, has to be developed. On the contrary, the efforts to decrease several uncertainties related to the planning are important. For example, the quantification of inspection capability and to grasp the deterioration property of a member is inevitable subjects. Fig. 1 shows the flow of the aforementioned [11].

In this study, a cost minimization method with the constraint of reliability is developed in order to estimate stable inspection planning versus the uncertain parameters. The following procedures are used to estimate the appropriate failure probability before making the final decision for the inspection planning. First, set up the constraint of reliability level based on the cost minimization approach for a given structural model containing uncertain parameters and estimation errors. Then, primary inspection planning is estimated by performing a cost minimization analysis under the constraint that failure probabilities of the members are controlled below the respective target values. Second, the life-time cost optimization is achieved to obtain a stable inspection plan against the estimated errors and inevitable uncertainties of parameters. Finally, the constraint of reliability is adjusted before taking the final decision making for the inspection strategy. Fig. 2 shows the flow chart of the above mentioned procedure to estimate an appropriate failure probability for the members of the structure.

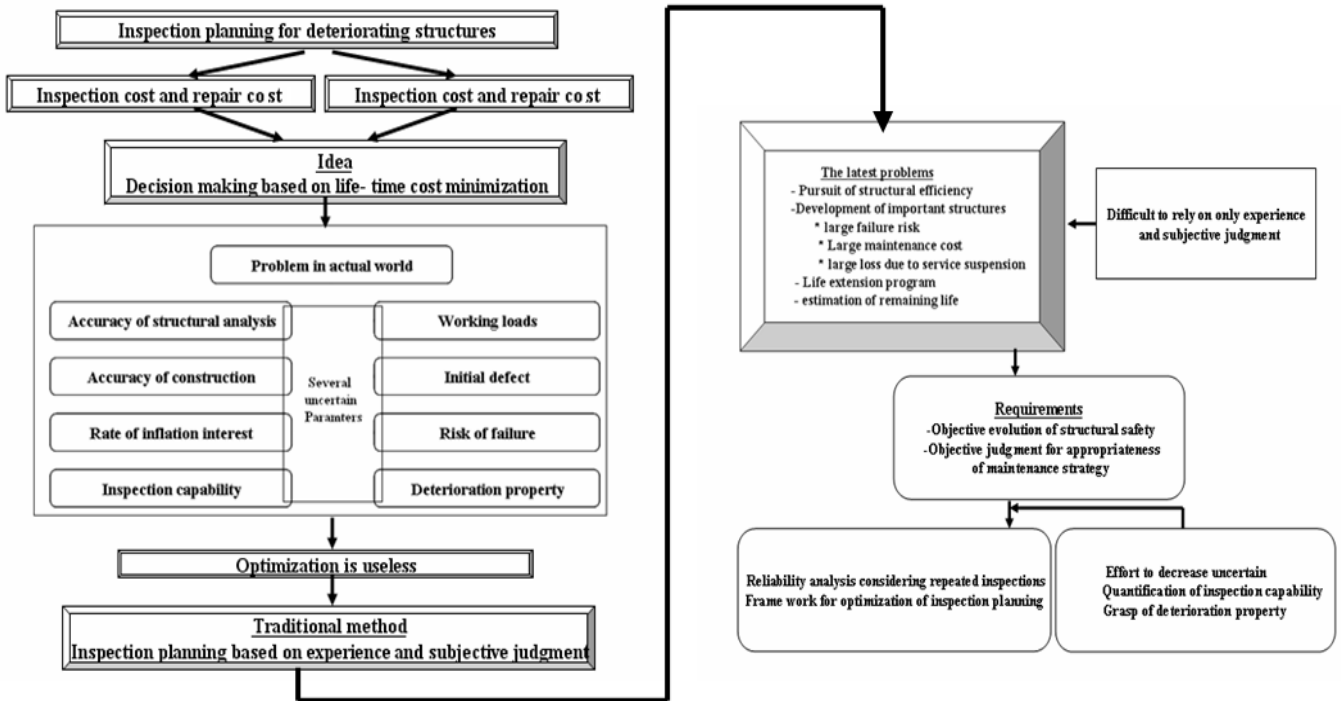


Fig. 1: Flow Chart for Inspection Planning of Deteriorating Structures

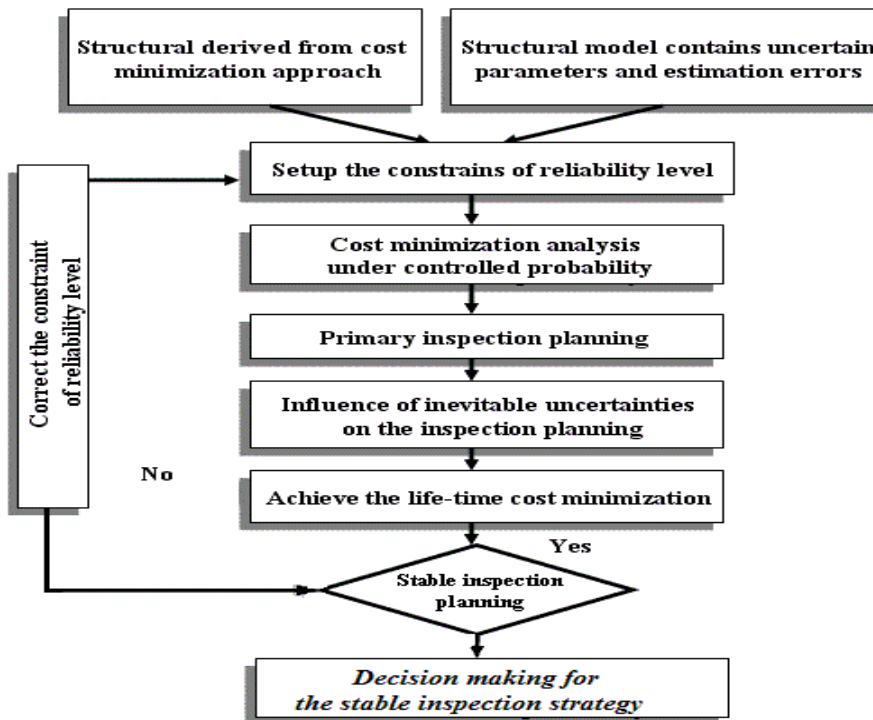


Fig. 2: Method to Determine an Appropriate Failure Probability

In the next section, the authors briefly review the content of the sequential cost minimization method.

### 3. Sequential cost minimization method

Optimization of the inspection strategy can be achieved by the appropriate selection of inspection intervals, inspection methods, repair qualities, and so on. However, the interval of periodic inspections is usually determined for structures, for example, once a year or once every two years. On the other hand, the pre-fixed inspection schedule is determined based on the synthetic consideration of a service plan, economic trend and loss due to maintenance downtime. Also, the repair quality is generally assumed to be perfect such that similar damage will never take place again in the member

after repair. Under such circumstances, the optimization is mainly achieved by the appropriate selection of inspection method and inspection interval.

### 3.1. Estimation of the total expected cost for the structure

The structure includes a variety of member sets which consist of different numbers of members. The following assumptions are made for the formulation of the problem. All the structural members in each set have the same strength property and are subjected to the same loading condition. Each member has a possibility of failure due to the deterioration damage. When any member fails, the service of the structure is always urgently suspended until the failed member is repaired. Before the structure returns to service, the member failure might develop into a catastrophic failure with a certain probability. Inspections are repeatedly carried out during the service life to find the damage at an early stage. The detected damage is perfectly repaired.

At a particular inspection during service, the total expected cost for the structure in the succeeding inspection interval is classified into two main groups: Costs necessary from the present inspection, and Risks (expected costs) during the service period until the next inspection. The former consists of the following cost items:

- $C_{\text{inspection}}$  ( $C_{\text{INS}}$ ): Inspection cost for each member set.
- $C_{\text{repair}}$  ( $C_{\text{REP}}$ ): Expected repair cost of the detected damages during the inspection.
- $C_{\text{scheduled system down}}$  ( $C_{\text{SSD}}$ ): Loss due to the service suspension caused by the present-time inspection and repair maintenance. This includes the losses for scheduled system down, system warm up and system downtime during the inspection and repair maintenance actions.

The latter consists of the following cost items:

- $C_{\text{member failure}}$  ( $C_{\text{MBF}}$ ): Expected loss due to a member failure. This includes repair cost of failed members, loss due to the service suspension caused by member failure, and other economic losses accompanied by the accidental system down.
- $C_{\text{catastrophic failure}}$  ( $C_{\text{CTF}}$ ): Risk against a catastrophic failure which may occur starting from member failure with a certain probability.
- Among the above cost items, only  $C_{\text{SSD}}$  is defined on the whole structure, and all the other costs which are different depending on the member set, are imposed on each member set.
- The total expected cost for the whole structure in an inspection interval, from time  $t$  to time  $t + 1$ , denoted by  $CT(t, t+1)$ , can be written as:

$$CT(t, t+1) = \sum_{j=Set1}^{SetL} C_j(t, t+1) + C_{SSD} \quad (1)$$

Where,

$$C_j(t, t+1) = C_{\text{INS}} + C_{\text{REP}} + C_{\text{MBF}} + C_{\text{CTF}} \quad (2)$$

Where 'j' means the j-th member set and 'L' means that the structure consists of L member sets.  $C_j(t, t+1)$  is the expected operating cost for the member set in the inspection interval (t, t+1). Of course, this value changes depending on the applied inspection method and the inspection interval.

### 3.2. The selection of the optimal inspection method for a member set

If the next inspection interval for the structure is predetermined, the optimal inspection method for each member set is selected when the total expected cost  $CT(t, t+1)$  in Eq. (1) is minimum. In Eq. (2), the values of  $C_j(t, t+1)$  are different depending on the member set and mutually independent. If it is assumed that  $C_{\text{SSD}}$  in Eq. (1) is independent on the applied inspection methods and repair qualities. Then, the minimization of  $CT(t, t+1)$  is achieved when the inspection method for each member set is selected so as to minimize  $C_j(t, t+1)$ . The selection is repeatedly carried out at every inspection from the following three proposal inspection methods:

- No inspection (NO).
- Visual inspection (VI) method.
- Mechanical (Precise) inspection (MI) method.

### 3.3. Selection of the appropriate inspection interval for a structure

The proposed method is basically applicable for selecting the appropriate inspection interval when there are two or more choices of inspection interval allowed for the structure. For simplicity, it is assumed that two inspection intervals are allowed for the structure, once a year and once every two years. The following procedures could be used to select the appropriate inspection interval:

- 1) Set the inspection interval for one year (t, t+1) and select the optimal inspection method for each member set of the structure.

- 2) Evaluate the total expected cost for the structure following Eq. (1) in the period between the time  $t$  and time  $t+1$ :  $CT(t, t+1)$ .
- 3) Repeat the upper steps 1) and 2) for the inspection interval after the next,  $(t+1, t+2)$ , to evaluate  $CT(t+1, t+2)$ .
- 4) Sum up these two costs for the structure and obtain the total expected cost in the period from the inspection time  $t$  to time  $t+2$ :

$$CT(t, t+2) = \sum_{j=Set1}^{Set.L} C_j(t, t+1) + \sum_{j=Set1}^{Set.L} C_j(t+1, t+2) + 2C_{SSD} \quad (3)$$

- 5) Change the inspection interval to two years  $(t, t+2)$ , and carry out the selection of the optimal inspection method for each member set.
- 6) Then, evaluate the total expected cost over two years adding a system down cost for the present inspection at time  $t$ . The total operating cost is given by the following equation:

$$CT(t, t+2) = \sum_{j=Set1}^{Set.L} C_j(t, t+2) + C_{SSD} \quad (4)$$

- 7) Lastly, the choice of the optimal inspection interval for the structure is to be done by comparing these two costs evaluated by the above two equations.

In the above procedure, if one year is chosen as the cost minimum, then the inspection methods selected in the 1<sup>st</sup> procedure are applied for each member set. And, the same procedures as those described from (1) to (7) are repeated at the inspection time of the next year. Or, if two years are selected, then the inspection methods chosen in the 5<sup>th</sup> procedure are applied. And the same procedures from (1) to (7) are repeated at the inspection time of two years later.

### 3.4. Cost evaluation equations

To estimate  $C_j(t, t+1)$  corresponding to each member set for the above mentioned three inspection methods, the cost evaluation equations are developed. In the formulations, the following assumptions are made.

- 1) The detection of defects in visual or mechanical inspection is probabilistic.
- 2) Costs due to the service suspension caused by accidental system down are to be taken into consideration. Member failure does not necessarily mean a collapse of the structure. However, the service of the structure is suspended urgently and the failed member has to be repaired. This accidental system down requires considerably larger cost factor than that of scheduled (predetermined) system down.
- 3) A member failure may result in a catastrophic failure with a certain transition probability. When the catastrophic failure occurs, the cost is due not only to the loss of the structure but also to the losses received from different portions of society, such as owners, clients, insurance, related industries, and so on.

The expected operating cost for a member set in an inspection interval  $(t, t+1)$  is evaluated by the following equations for the three inspection methods:

No inspection (NO)

$$C_j(t, t+1/NO) = G \times P_{F1} \times C_F \quad (5)$$

Visual inspection (VI) method

$$C_j(t, t+1/VI) = G \times \{C_{VI} + P_{DV} \times (C_{ZMI} + C_{RD}) + (1-P_{DV}) \times P_{F2} \times C_F\} \quad (6)$$

Mechanical inspection (MI) method

$$C_j(t, t+1/MI) = G \times \{C_{MI} + P_{DM} \times C_{RD} + (1-P_{DM}) \times P_{F3} \times C_F\} \quad (7)$$

In the above equations:

$$C_F = (1-P_{FC}) \times (C_{ASD} + C_{RF}) + P_{FC} \times C_{CF} \quad (8)$$

$$G = m \times P_{SV} \quad (9)$$

The meaning of the notations used in the above equations is as follows:

- $m$ : Number of members in the member set.
- $P_{F1}$ ,  $P_{F2}$ ,  $P_{F3}$ : Probabilities of occurrence of member failure in the succeeding inspection interval under the conditions that NO, VI and MI methods are applied at the present inspection, respectively.
- $P_{DV}$ ,  $P_{DM}$ : Probabilities of detecting a defect by VI and MI methods, respectively.
- $P_{FC}$ : Probability that a member failure develops into a catastrophic failure of the structure.
- $P_{SV}$ : Probability that a member has not experienced repair and failure until the present inspection.
- $C_{VI}$ ,  $C_{MI}$ : Visual and mechanical inspection costs for a member, respectively.
- $C_{ZMI}$ : When damage is detected by visual inspection, if that defect is again inspected mechanically for sizing or examining the repair method, the value of  $C_{ZMI}$  equals  $C_{MI}$ . Otherwise  $C_{ZMI}$  is zero.
- $C_{RD}$ : Repair cost of a damaged member detected by visual or mechanical inspection. This value is a function of degree of damage and elapsed service time.
- $C_{RF}$ : Repair cost of a failed member.
- $C_{CF}$ : Risk of a catastrophic failure.
- $C_{ASD}$ : Loss due to the service suspension caused by accidental system down.

The probabilities appearing in the cost evaluation equation are calculated by the Markov Chain Model (MCM) [11]. The merits of applying MCM to the fatigue reliability analysis are due to the procedure of periodic or non-periodic in-service inspection as well as pro-service inspection, and can be easily incorporated in the calculations. Furthermore, the probability of the existence of initial defects and the distribution of defect size can be considered in the initial state vector of MCM easily.

The event tree of the cost evaluation in case that visual inspection is performed is shown in Fig. 3. The event tree after member failure under any applied inspection method is shown in Fig. 4.

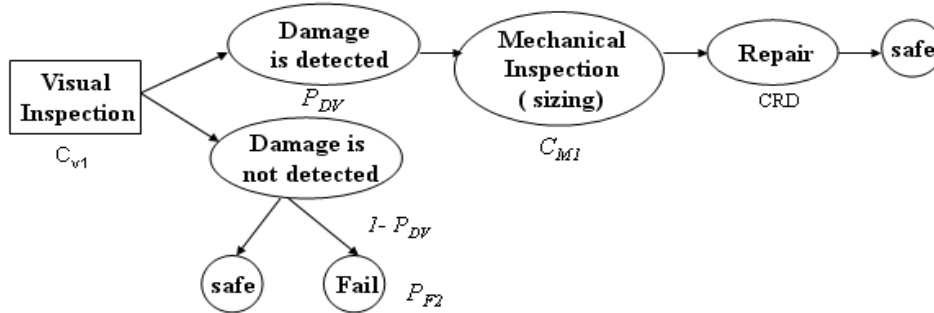


Fig. 3: Event Tree for Cost Evaluation

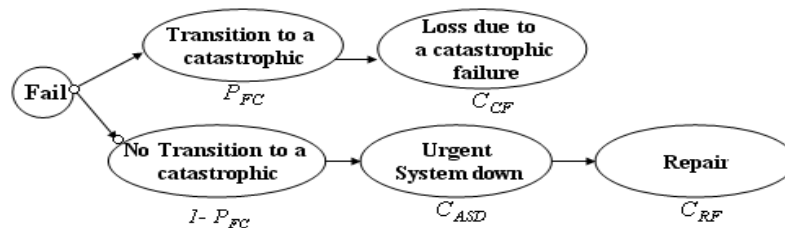


Fig. 4: Event Tree after Member Failure

### 3.5. Method of analysis

In this study, the estimation of the inspection planning for fatigue deteriorating structure is carried out based on three different approaches. Each approach has its own consistent formulations and assumptions.

For example, the sequential cost minimization method described in the previous section is used to estimate the inspection planning in case that the analysis is carried out on cost basis. The method aim is to find an optimal inspection planning so that the total expected cost in the period between the present inspection and the next is to be minimized. The optimization parameters are the inspection qualities for each member set and the inspection interval to the next inspection. The optimization is repeatedly carried out at every inspection.

However, if the estimation of the inspection planning is carried out based on reliability basis, a target failure probability was assumed for the members of the structure. The inspection timing is settled when the probability of failure of any member set reaches the target probability of failure. The inspection quality is predetermined to be either a visual inspection or mechanical inspection during the service of the structure. Fig. 5 shows the idea of the decision making for the inspection timing strategy if the reliability basis is considered in the analysis.

Finally, if the inspection planning is estimated based on cost minimization basis with constraint of reliability, the minimization process is performed under the constraint that the failure probabilities of the members are controlled below the respective target values for the members.

$$P_{F1}, P_{F2}, P_{F3} \leq \text{target failure probability} \tag{10}$$

Where,  $P_{F1}$ ,  $P_{F2}$ , and  $P_{F3}$  are the probabilities of occurrence of member failure in the succeeding inspection interval under the conditions that no inspection, visual inspection, and mechanical inspection methods are carried out at the present inspection, respectively. The initial values of the target failure probabilities are altered until an acceptable result is obtained. Life-time cost minimization method is used to investigate the inspection planning of deteriorating structures which contain uncertain parameters after adjusting the target probability of failure as well as the other parameters within the range of uncertainties.

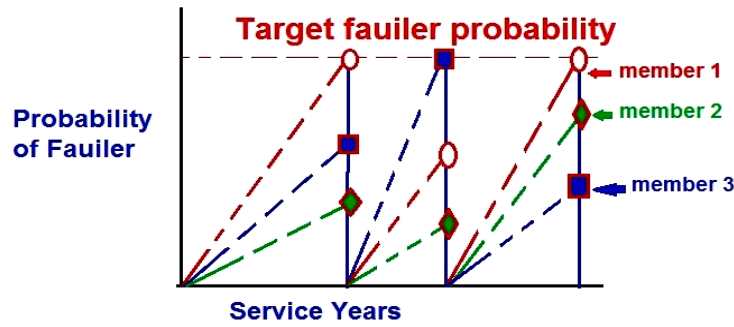


Fig. 5: Decision of Inspection Timing in the Strategy on Reliability Basis

## 4. Numerical example

### 4.1. Analysis of a structural member set

In this analysis, the estimation of the inspection planning is carried out based on cost basis. The sequential cost minimization method was applied to a structural member set consisting of 200 structural elements with a round fillet weld. The following assumptions were made for the formulation of the problem. All the elements have statistically identical strength properties and are subjected to the same loading condition. The surface fatigue crack initiated from the weld toe is treated as the deterioration damage. Moreover, the perfect repair model was employed to the repair of the detected cracks. The mean crack growth curve obtained by the model fatigue test was employed in the analysis. The fatigue crack initiation and propagation lives follow two parameter Weibull distributions with shape parameter 3.0 and 5.0, respectively.

$$f_N(N) = \frac{\gamma}{\beta} \times \left(\frac{N}{\beta}\right)^{\gamma-1} \times \exp\left\{-\left(\frac{N}{\beta}\right)^\gamma\right\} \tag{11}$$

The member failure was defined when the surface crack length reaches the plate width of 80mm. Fig. 6 shows the initial crack conditions for three cases analyzed. The capability of visual and mechanical inspections was assumed to be a function of surface crack length as follows:

$$\begin{aligned} \text{POD (VI)} &= 1.0 - \exp\{-0.025 \times (2a - 10.0)\} \\ \text{POD (MI)} &= 1.0 - \exp\{-0.10 \times (2a - 10.0)\} \end{aligned} \tag{12}$$

The following values were given for the cost items:

$C_{VI} = 10\$$ ,  $C_{MI} = 100\$$ ,  $C_{SSD} = 10^4\$$ ,  $C_{ASD} = 10^6\$$ .  $C_{RD} = 10^3\$$ ,  $C_{RF} = 5 \times 10^5\$$ ,  $C_{CF} = 2 \times 10^8\$$ . And  $P_{FC} = 0.01$  was assumed as the transition probability to a catastrophic failure.

Probability of Crack Existence	10 mm	20 mm	21 mm	24 mm
Case A	0.10	0.0	0.0	0.0
Case B	0.10	0.05	0.05	0.0
Case C	0.10	0.05	0.03	0.02

Fig. 6: The Probability of Existence Initial Defect

Table 1 shows the inspection timing and inspection qualities predicted by the sequential cost minimization method. The values of  $C_{OP}$  and  $P_F$  in the table are the accumulated costs and the failure probabilities during 32 years' service.

$$C_{OP} = \sum_{t=0}^{Life\ time} CT(t, t+i), \quad i=1 \text{ or } 2 \text{ or } 4 \tag{13}$$

Table 1: Results of Inspection Planning

Case	Selected inspection years and inspection qualities								$C_{OP}$ US \$ x 10 <sup>5</sup>	$P_f$ x 10 <sup>-4</sup>
A	4	8	12	16	20	24	28	32	3.2	2.06
	-	-	-	-	V	V	M	M		
B	4	8	12	16	20	24	28	32	4.0	2.27
	-	V	M	M	M	M	M	M		
C	4	8	12	16	20	24	28	32	4.6	2.57
	M	M	V	V	M	M	M	M		

V: Visual inspection

M: Mechanical inspection

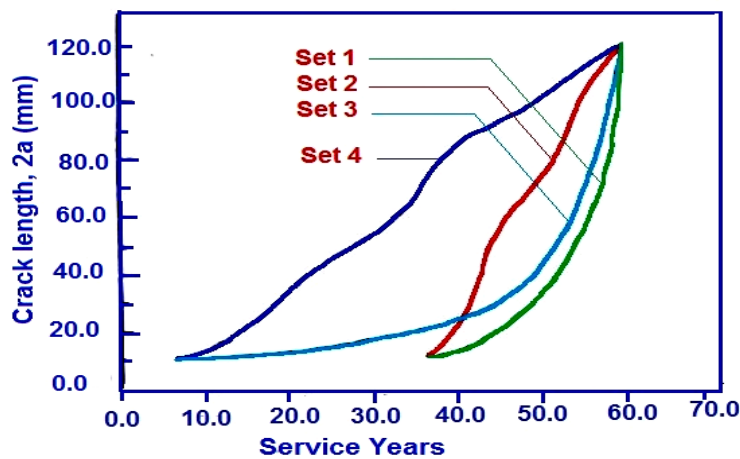
An interval of four years is selected for all the cases. It is seen that the first inspection timing becomes earlier when the initial crack condition becomes worse. If the inspection interval is pre-fixed for the member set, the inspection content will be changed. Table 2 shows the result of inspection planning for the member set for which the inspection interval is fixed at two years. Visual inspections are often selected in the fixed interval problems, and the values of  $C_{Op}$ 's are increased compared with those in Table 1.

**Table 2:** Results of Inspection Planning (Fixed Interval Inspection)

Case	Selected inspection years and inspection qualities																$C_{Op}$ (US \$ x 10 <sup>5</sup> )	$P_f$ x 10 <sup>-4</sup>
	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32		
A	-	-	-	-	-	-	-	-	-	-	V	V	V	M	M	M	4.2	3.04
B	-	-	-	-	V	V	V	M	V	V	V	M	V	V	M	V	5.7	4.28
C	-	V	M	V	V	V	V	V	V	V	V	V	V	V	M	V	5.3	2.83

**4.2. Analysis of a structure**

In this analysis, the estimation of the inspection planning is carried out based on the following: cost basis, reliability basis and cost basis with constraint of reliability. The proposed method was applied to an assumed structure which consists of four member sets with different mean fatigue property, as shown in Fig.7. The crack length in the figure expresses the observable length at the inspection. From Fig.7, it is seen that the mean crack propagation lives for member sets 1 and 2 are shorter than that for member sets 3 and 4. For all member sets, 60 years was assumed as the mean fatigue failure life  $N_f$ . The distribution of crack initiation and propagation lives follows two parameters, Weibull distribution with shape parameter,  $\gamma(N_c) = 4.0$  (for set 1 and set 2),  $3.0$  (for set 3 and set 4),  $\gamma(N_p) = 5.0$ , respectively.



**Fig. 7:** Mean Properties of Crack Growth Curves For the Members with Different Fatigue Properties

The contents of the fatigue properties for each member set are given in Table 3. The assumed values of the initial and critical crack length are 10 mm and 120 mm, respectively. Fig. 8 shows the fatigue sample functions generated for member set 3 by the Markov Chain model. The contents of cost, items, number of members, the target failure probability, and the POD curves for each member set are summarized in Table 4. The crack conditions at the initial state and after the repair are given in Table 5. Replacement model was employed as the repair method of the detected cracks. Three interval inspections are allowed for the structure, one year, two years and four years.

Table 3: Fatigue property of each member set.

Member Set	$N_p$ years	$N_c$ years	Weibull shape Parameter	
			$N_p$	$N_c$
Set 1	36	24	4	5
Set 2	36	24	4	5
Set 3	6	54	3	5
Set 4	6	54	3	5

Using the cost basis and applying the proposed method, the selected inspection timing and qualities for the structure are shown in Table 6 in which 7 inspection times are required. Four years' interval is always selected for all the members' sets. It is noticed that the first inspection timing becomes late for member set 4. This is because the crack detection is easy for the crack growth property of member set 4 (see Fig. 7). Mechanical inspections with long interval inspections are always selected. Fig. 8 shows the change of the failure probabilities of members in respective inspection intervals.

The failure probabilities are maintained in the range between  $10^6$  and  $2 \times 10^4$  which are acceptable from the reliability viewpoint.

If the inspection planning is estimated on cost basis with constraint of reliability for the same structure, then the selected inspection timing and qualities for the structure will be changed, and as shown in Table 7, twelve inspection times are necessary. Visual inspection with short interval inspections is always selected. The inspection timing for member set 4 based on cost minimization with constraint of reliability is started earlier than those when the analysis is carried out on cost basis only. If the allowable inspection methods are visual inspection or mechanical inspection, then the selected inspection qualities and timing will be changed, as shown in Table 8. The estimated operating cost during 30 years' service will become  $9.5 \times 10^5$  US\$ compared to  $9.4 \times 10^5$  US\$ in Table 7. The change of the failure probabilities of members in respective inspection intervals is shown in Fig. 8. From the figure, it is seen that the failure probabilities for member sets 1 and 2 as well as member set 3 are maintained less than  $10^{-5}$ . For member set 4, the failure probability is maintained under  $10^{-6}$ .

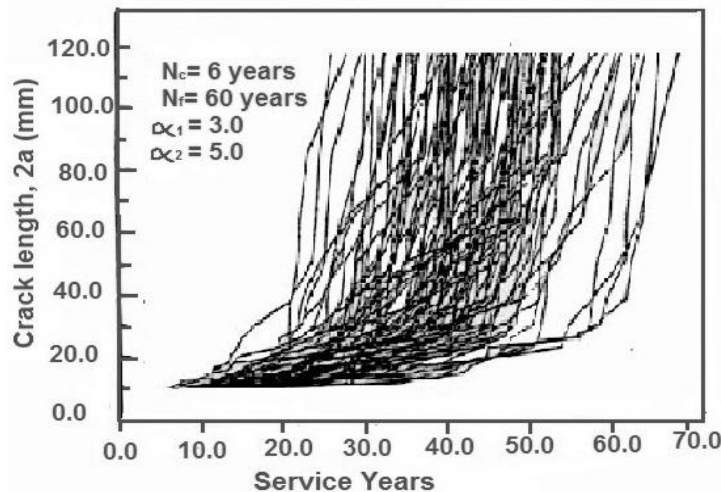


Fig. 8: Fatigue Sample Function Generated by Markov Chain Model (Member Set 3)

Table 4: Condition Analysis

Condition of analysis	Member set			
	Set 1	Set 2	Set 3	Set 4
No. of Members	100	100	100	100
$C_{VI}$ (US \$ / member)	50	50	50	50
$C_{MI}$ (US \$ / member)	200	200	200	200
$C_{RD}$ (US \$ / damage)	1000	1000	1000	1000
$C_{RF}$ (US \$ / failure)	$10^5$	$10^5$	$10^5$	$10^5$
$C_{CF}$ (US \$)	$10^8$	$10^8$	$10^8$	$10^8$
$P_{FC}$	0.001	0.01	0.01	0.1
$C_{SSD}$ (US \$)		$5 \times 10^4$		
$C_{ASD}$ (US \$)		$5 \times 10^8$		
POD curve				
VI: $POD = 1.0 - \exp \{-0.025 \times (2a - 10.0)\}$				
MI: $POD = 1.0 - \exp \{-0.10 \times (2a - 10.0)\}$				

Table 5: Crack Condition before the Start of Service and after Repair

Member Set	Crack condition	0.00 (mm)	10.0 (mm)	20.0 (mm)
Set 1	Initial state	80 %	10 %	10 %
Set 2				
Set 3				
Set 4				
	After repair (Replacement Model)	80 %	10 %	10 %

Table 6: Inspection Planning on Cost Basis

Member Set	Inspection years and qualities						
	4	8	12	16	20	24	28
Set 1	M	M	M	M	V	M	V
Set 2	-	M	V	V	V	V	V
Set 3	V	M	M	V	V	M	V
Set 4	-	-	-	V	V	V	V

**Table 7:** Inspection Planning on Cost Basis with Constraint of Reliability

Member set	Inspection years and qualities											
	4	6	8	9	10	12	14	16	20	24	25	29
Set 1	M	M	-	V	M	V	M	M	M	-	M	V
Set 2	V	V	V	-	V	V	V	V	M	-	V	-
Set 3	M	V	V	V	V	M	-	M	M	V	M	V
Set 4	V	V	-	-	-	V	V	V	V	-	M	-

**Table 8:** Inspection Planning on Cost Basis with Constraint of Reliability (VI and MI Methods Only)

Member set	Inspection years and qualities										
	4	6	8	10	12	16	18	22	26	28	
Set 1	M	M	V	M	M	M	M	M	V	M	
Set 2	V	V	V	V	M	V	V	V	V	V	
Set 3	M	V	M	V	M	V	M	M	M	V	
Set 4	V	V	V	V	V	V	V	V	V	V	

If the reliability is performed for the same structure to estimate the inspection planning, then the selected inspection timing is shown (in Table 9) in which visual inspection and mechanical inspection are performed for the structure, respectively. It is noticed that 20 inspection times are necessary during 30 years' service if VI method is used; however, 8 inspections are necessary if MI method is performed. It makes clear that if the analysis is performed on reliability basis, either mechanical inspection with long interval inspections or visual inspection with short interval inspections are selected, respectively. The change of the failure probabilities of members in respective inspection intervals is shown in Fig. 10, for member sets 1 and 3. From the figure, it is noticed that the failure probabilities are maintained within the range of  $10^{-5}$ .

Fig. 11 shows the cumulative operating costs expected for the structure during 32 years' service and calculated by cost basis, reliability basis and cost basis with constraint of reliability, respectively. From the figure, the expected cumulative operating cost on cost basis is usually the minimum. However, not only cost aspect, but also reliability aspect should be taken in consideration. Therefore, the decision making for the stable inspection planning can be obtained if the analysis of cost basis with constraint of reliability is performed.

## 5. Reflection of uncertainties

Most of the probabilities and the cost items required in the analysis contain several uncertainties in the actual world. In this study, the influence of inevitable uncertain parameters on inspection planning is discussed.

**Table 9:** Inspection Planning Reliability Basis

No. of Inspection	Inspection years	
	VI For all the member sets	MI For all the member sets
1	4.2	4.2
2	5	5.5
3	5.8	9.5
4	6.7	13.0
5	7.7	165.7
6	8.7	20.5
7	9.8	24.8
8	11.0	28.7
9	12.3	-
10	13.7	-
11	15.0	-
12	16.3	-
13	17.7	-
14	19.2	-
15	20.7	-
16	22.2	-
17	23.8	-
18	25.5	-
19	27.3	-
20	20.0	-

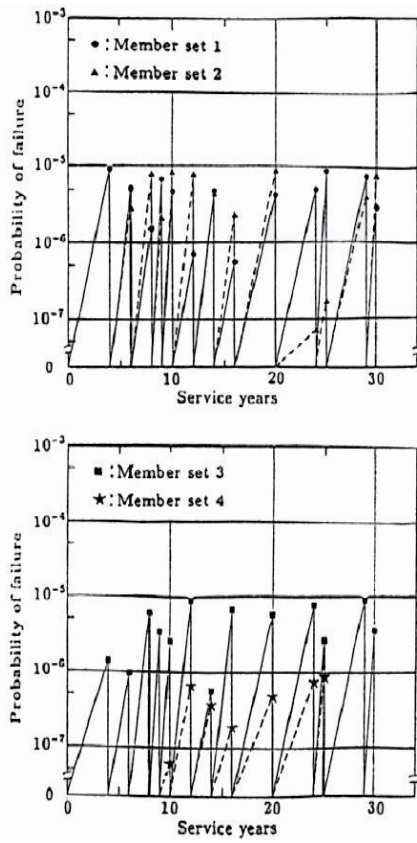


Fig. 9a: Change of Failure Probability in Respective Inspection Interval (Cost Basis)

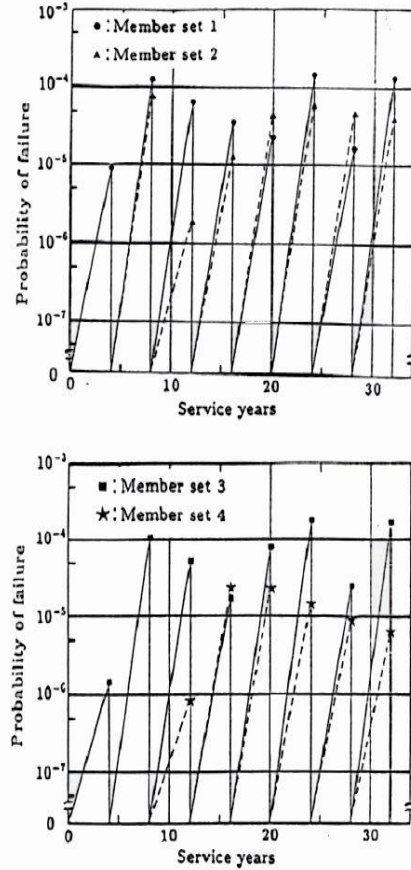


Fig. 9b: Change of Failure Probability in Respective Inspection Interval (Cost Basis with Constraint Reliability)

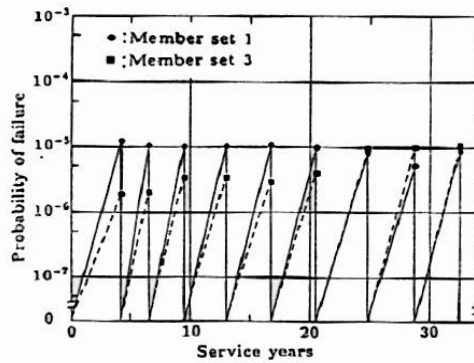


Fig. 10: Change of Failure Probability in Respective Inspection Interval (Reliability Basis)

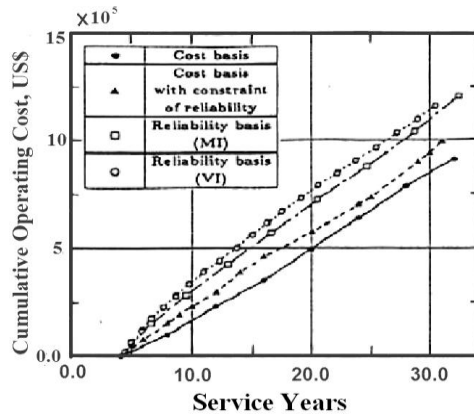


Fig. 11: Comparison of Cumulative Operating Cost among Three Inspection Plans

### 5.1. Influence of parameters on the inspection planning

The analysis is carried out by giving a small change for individual parameters; the degrees of influence of each parameter on the inspection content, the cumulative operating cost and failure probability are investigated. A single member set consisting of 200 numbers of welded joints, shown in Fig. 12, was chosen for the analysis. Table 10 shows the fatigue property, initial defect condition, inspection capability, and cost contents of the welded joint. These values were chosen as the basic condition of the analysis. Table 11 summarizes the result of inspection planning for the basic condition, in which the cumulative operation cost during 22 years' service and the cumulative failure probability of the member are shown as well as the inspection timing and qualities.

Table 10: Results of Inspection Planning

Number of members	200
Fatigue property of member $a_0 = 1.0$ mm $a_{max} = 20.0$ mm	$N_c = 50$ years Weibull shape parameter: 3 $N_p = 20$ years Weibull shape parameter: 4
Initial defect condition	$P_{ID} = 0.01$
POD Curve for VI and MI Methods	VI: $POD = 1.0 - \exp \{-0.2 \times (a - 3.0)\}$ MI: $POD = 1.0 - \exp \{-0.4 \times (a - 1.0)\}$
Cost item	
$C_{VI} = 10.0$ US \$ $C_{SSD} = 2.0 \times 10^4$ US \$ $C_{CF} = 2.0 \times 10^8$ US \$	$C_{MI} = 100.0$ US \$ $C_{ASD} = 10^6$ US \$ $P_{FC} = 0.05$

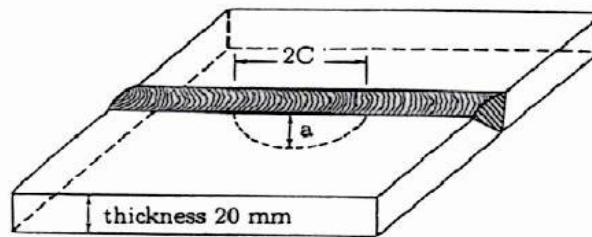


Fig. 12: Surface Crack Initiated From Butt Welding Joint

Table 11: Results of Inspection Planning

Parameter		Inspection years and qualities									$C_{op} \text{ \$ } \times 10^5$	$P_f \times 10^{-5}$
Basic cond.	$P_{FC}$ 0.05	4	8	10	12	14	16	18	20	22	3.42	9.7
		M	M	M	V	V	V	V	M	V		

The analysis was carried out by changing the individual parameters to half or to twice. Also, the sensitivities of each parameter were investigated from the viewpoints of the degree of influence on the inspection planning, the cumulative operating cost, COP, and the cumulative failure probability  $P_f$ .

Table 12 summarizes the result of the analysis. From the table, it is seen that the fatigue life ( $N_c, N_p$ ), inspection cost ( $C_{VI}, C_{MI}$ ), schedule system down cost ( $C_{SSD}$ ) and inspection capability (POD) are sensitive on the inspection planning, the cumulative operating cost,  $C_{OP}$ , and the cumulative failure probability  $P_f$ . Among them, the most sensitive parameter is the fatigue life.

When the fatigue life ( $N_c, N_p$ ) becomes half, mechanical inspections are frequently applied with a short interval. When ( $N_c, N_p$ ) becomes twice, visual inspections with a long interval are preferred. At the same time, both of the cumulative operating cost,  $C_{OP}$ , and the cumulative failure probability,  $P_f$ , are highly affected by the fatigue life. When the inspection cost ( $C_{VI}, C_{MI}$ ) becomes twice, the rate in use of visual inspection is increased. As a result, the failure probability of the member increases.

When schedule system down ( $C_{SSD}$ ) becomes half, visual inspections are frequently applied with short interval. However, when the  $C_{SSD}$  becomes twice, mechanical inspections with long interval are suggested.

When the inspection capability (POD curves) is good in quality, mechanical inspections with a long interval are selected. On the other hand, visual inspections with a short interval are preferred for the bad inspection capability. With respect to the insensitive parameters, repair cost ( $C_{RD}, C_{RF}$ ), accidental system down cost ( $C_{ASD}$ ), and transition probability of a catastrophic failure ( $P_{FC}$ ) have a slight effect on the inspection planning, the cumulative operating cost and the cumulative failure probability. Further, the initial defect condition,  $P_{ID}$ , and the risk against a catastrophic failure,  $C_{CF}$ , are insensitive on the cumulative operating cost, but sensitive on the inspection content and the cumulative failure probability. Large cumulative failure probability is allowed when the risk against a catastrophic failure becomes half, because the anxiety of catastrophic failure is decreased.

The quantitative property of the sensitivities obtained in the above analysis agrees well with reasoning of engineers who are familiar with the inspection planning of actual structures.

### 5.2. Influence of a large degree of uncertainty on the inspection strategy

In the previous section, the analysis was carried out by giving a small change for the individual parameter. In this section, the analysis is carried out giving a large change for two uncertain parameters, the fatigue failure life  $N_f$  and the probability of catastrophic failure  $P_{FC}$ .

The mean fatigue failure life  $N_f$  consists of the mean fatigue crack initiation  $N_c$  and propagation  $N_p$  lives.  $N_f$  and  $P_{FC}$  have a large influence on inspection contents, such as inspection qualities, inspection intervals, cumulative operating cost and cumulative probability of failure. The initial uncertainty of  $P_{FC}$  is usually maintained during the whole service life, because a catastrophic failure is a rare event. Therefore, a wide range of uncertainty of  $P_{FC}$  at the inspection planning must be prepared.

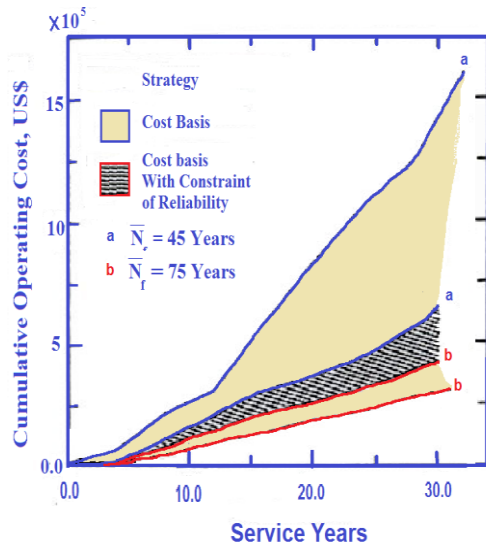
**Table 12:** Results of Changing the Parameters

Parameter		Number of inspections during 22 years' service			$C_{op} \times 10^5$ \$	$P_f \times 10^{-5}$
		VI	MI	Total		
$C_{VI}$	1/2	3	5	8	2.79	4.2
	1	4	4	8	3.42	4.9
	2	7	1	8	4.96	12.0
$C_{RD}$	1/2	4	4	8	3.40	4.9
	1	4	4	8	3.42	4.9
	2	4	4	8	3.55	4.9
$C_{RF}$	1/2	4	4	8	3.42	4.9
	1	4	4	8	3.42	4.9
	2	4	4	8	3.44	4.9
$C_{ASD}$	1/2	4	4	8	3.42	5.0
	1	4	4	8	3.42	4.9
	2	4	4	8	3.50	4.9
$C_{SSD}$	1/2	18	0	18	2.59	1.2
	1	4	4	8	3.42	4.9
	2	0	5	5	4.93	4.2
$C_{CF}$	1/2	2	3	5	3.25	5.0
	1	4	4	8	3.42	4.9
	2	3	5	8	3.91	4.3
$P_{FC}$	1/2	3	4	7	3.22	5.0
	1	4	4	8	3.42	4.9
	2	3	5	8	3.91	4.3
$P_{ID}$	1/2	1	5	6	3.48	5.2
	1	4	4	8	3.42	4.9
	2	7	4	11	3.93	5.0
POD	1/2	13	2	15	4.50	3.5
	1	4	4	8	3.42	4.6
	2	0	5	5	2.72	1.5
$N_c$	1/2	4	14	18	8.70	10.5
	1	4	4	8	3.42	4.9
	2	3	1	4	1.63	0.8

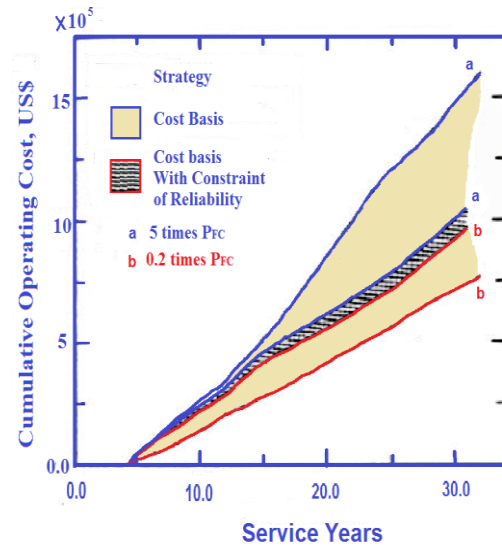
For the fatigue failure life, the analysis was carried out changing the  $N_f$  used in section 4 from 60 years to 45 and 75 years, respectively. It means that the change in  $N_f$  is  $\pm 15$ . Fig. 13(a) shows the influence of estimation errors of fatigue life on the cumulative operating cost calculated by the proposed method with different basis during 30 years' service. From the figure, it is clear that the uncertainty of the cumulative operating cost is large if the calculation is performed on cost basis only. However, the uncertainty in the operating cost can be reduced if the cost basis with constraint reliability is used. Fig. 13(b) shows the cumulative operating cost during 30 years' service for three inspection plans with different values of  $N_f$ . In the figure, the values of  $C_{OP}$ 's are influenced by the change of  $N_f$ , especially if the cost basis is used in the analysis. Table 13 gives the condition of analysis for all member sets.

**Table 13:** Condition of Analysis

Curves 13(a)	$(N_c, N_p)$ Years			
	Set 1	Set 2	Set 3	Set 4
a	(27.0, 18.0)	(27.0, 18.0)	(4.5, 40.5)	(4.5, 40.5)
b	(45.0, 30.0)	(45.0, 30.0)	(7.5, 67.5)	(7.5, 67.5)
Curves 13(b)	$P_{CF}$ - Probability of Catastrophic Failure			
a	0.005	0.05	0.05	0.5
b	0.0002	0.002	0.002	0.02

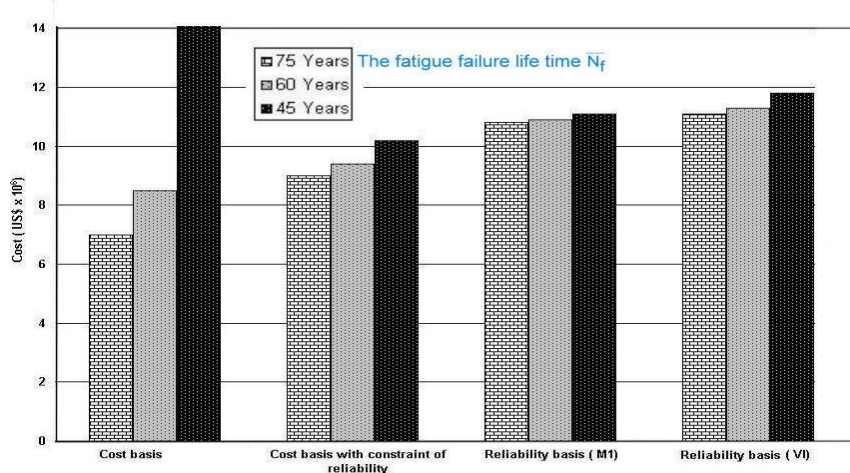


**Fig. 13a:** Influence Of Estimation Error Of Fatigue Life On Cumulative Operating Cost



**Fig. 13b:** Influence Of Estimation Error Of  $P_{FC}$  On Cumulative Operating Cost

For the probability of catastrophic failure  $P_{FC}$ , the analysis was carried out changing the  $P_{FC}$  used in the previous section to one fifth and five times, respectively. Fig. 14(a) shows the influence of  $P_{FC}$  on the cumulative operating cost calculated by the proposed methods with a different basis during 30 years' service. Fig. 14(b) shows cumulative operating cost during 30 years' service for three inspection plans with different values of  $P_{FC}$ . In the figure, the values of  $C_{OP}$ 's are influenced dramatically by the change of  $P_{FC}$  in the case of cost basis. The more  $P_{FC}$  increase, the more frequently and precisely the inspections are carried out. However, if inspection planning is carried out following the reliability basis or cost basis with constraint of reliability, the change of  $C_{OP}$  is relatively very small.



**Fig. 14a:** Comparison of  $C_{OP}$  Among Three Inspection Plans (Influence of Estimation Error of  $N_c, N_p$ )

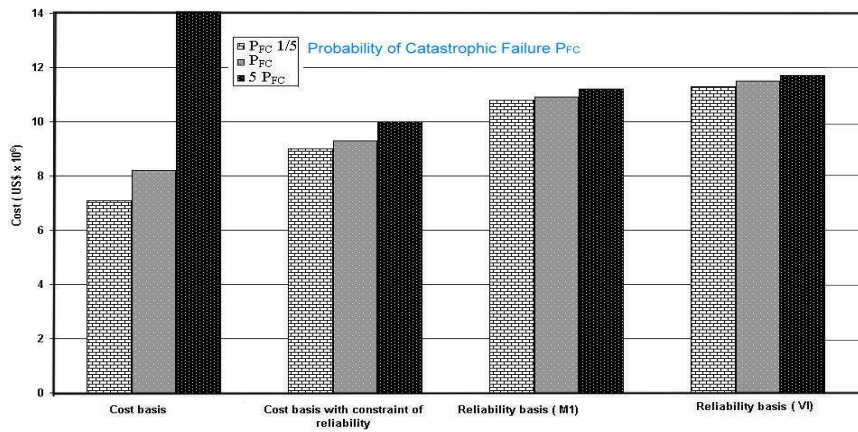


Fig. 14b: Comparison of C<sub>OP</sub> among Three Inspection Plans (Influence of Estimation Error of P<sub>FC</sub>)

From the above discussion, it can be concluded that the optimal decision making for the estimation of the inspection and repair maintenance strategy can be achieved by performing the calculation on cost basis with constraint of reliability.

## 6. Conclusions

In this paper, first, a sequential cost minimization method is used to estimate the inspection planning of fatigue deteriorating structures (cost basis). Second, reliability analysis is performed for the same purpose. Then, a cost minimization method with constraint of reliability is developed to estimate a stable and optimal inspection planning against the estimation errors of the parameters. The applicability of the method is examined for an actual structure consisting of four member sets of structural elements with a round fillet weld. Also, the influence of inevitable uncertain parameters on inspection planning is discussed. From the above discussion, the following conclusions can be drawn:

- 1) For the inspection planning problem, not only the cost aspect, but also the reliability aspect, should be taken into consideration. From this viewpoint, the decision making for a stable inspection planning can be obtained by performing the analysis on cost minimization basis with constraint of reliability.
- 2) The cost minimization approach has an effect to reduce the uncertainty of the estimated operating cost when a large uncertainty exists in the failure risk of members as well as the fatigue failure life.

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## Appendix

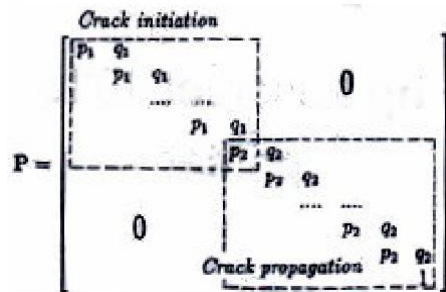
### Probability estimation using markov chain model

The probability appearing in the cost evaluation equations are calculated by the Markov Chain Model (MCM). Basically, in the simplest Bogdanoff and Kozin stationary Markov Chain Model, an initial state vector  $a(0)$  and a duty cycle independent basic transition matrix  $P$  are sufficient to describe the entire probabilistic structure. The probability distribution right after the elapsing of time  $t$ ,  $A_{BI}(t)$ , is obtained by:

$$A_{BI}(t) = A(0) \times P^n \tag{14}$$

Where,  $n$  is the number of duty cycles in the time  $t$  and ( $n=t/\Delta t$ , where,  $\Delta t$  is the time of a duty cycle).  $A_{BI}(t)$  and  $a(0)$  are  $M$ -dimensional vectors representing the probability distributions with respect to pre-defined  $M$  states.  $P$  is the basic  $M \times M$  probability transition matrix. The fatigue crack initiation and propagation processes are incorporated into the transition matrix of a single MCM. The probability of member failure is also considered in the absorbing term of the matrix:

$$A(0) = \left\{ \overbrace{a_1(0), a_2(0), \dots, a_{m_1}(0)}^{m_1}, \dots, \underbrace{a_{m_1+1}(0), \dots, a_{M-1}(0)}_{m_2}, 0 \right\} \tag{15}$$



$$\tag{16}$$

In the above equation, the first  $m_1$  rows are for crack initiation process, the last  $m_2$  rows for crack propagation process. The total size of the matrix is ( $M= m_1+m_2-1$ ). The probabilities  $p_1, q_1, p_2,$  and  $q_2$  are determined using the mean and variance of the crack initiation and propagation property of the structural member, respectively.

The merits of applying MCM to the fatigue reliability analysis are also due to the procedure of periodic or non-periodic in-service inspections, as well as the pre-service inspection, and can be easily incorporated into the calculations. Furthermore, the probability of the existence of initial defects and the distribution of defect size can be considered in the initial state vector of MCM.

The probability that the member survives until the time of the present inspection without any experience of repair  $P_{SV}$  can be expressed as:

$$P_{SV} = \sum_{m=1}^{M-1} a_m(t) \tag{17}$$

Where,  $a_m(t)$  are the elements of the vector  $A_{BI}(t)$ .

The detection probabilities for a crack of any length by visual inspection  $P_{DV}$  is calculated by the following equation:

$$P_{DV} = A_{BI} \times D^T \tag{18}$$

Where,  $D^T$  represents the detectability transposed matrix. The detection probabilities for all crack conditions are represented by  $D$ , ( $1 \times M$ ) row vector.

$$D = \left\{ \overbrace{d_1, d_2, \dots, d_{m_1}}^{m_1}, \dots, \underbrace{d_{m_1+1}, \dots, d_{M-1}}_{m_2}, 0 \right\} \tag{19}$$

The state vector right after the inspection can be given by the following equation:

$$A_{AI}(t) = A_{BI}(t) \times (C_1 + C_2) \tag{20}$$

$$C_1 = I - \text{diag.}(d_m) \tag{21}$$

$$C_2 = D^T \times R \tag{22}$$

$$R = \left\{ \overbrace{r_1, r_2, \dots, r_{m_1}}^{m_1}, \dots, \underbrace{r_{m_1+1}, \dots, r_{M-1}}_{m_2}, 0 \right\} \tag{23}$$

In the above equations, subscripts BI and AI express just before and after the inspection, respectively.  $I$  is the unit matrix of ( $M \times M$ ), and  $\text{diag.}(d_m)$  are the elements of the detectability vector  $D$ . The repair vector is represented by  $R$ , ( $1 \times M$ ) row vector, in which:

$$\sum_{m=1}^{M-1} r_m = 1.0 \tag{24}$$

The failure probabilities  $P_{F1}$ ,  $P_{F2}$  and  $P_{F3}$  are calculated as follows: when no inspection is carried out in the present inspection, the state vector at the next inspection becomes:

$$A_{BI}(t+1) = A_{AI}(t) \times P^k \quad (25)$$

Where,  $k$  is the number of duty cycles in the next inspection interval ( $k = 1/\Delta t$ ). The failure probability in this period is obtained from the following equation:

$$P_{F1} = \{a_M(t+1) - a_M(t)\} \quad (26)$$

In which,  $a_M(t+1)$  and  $a_M(t)$  are the absorbing terms of the state vector at the time  $(t+1)$  and time  $t$ , respectively. When visual inspection is applied, first the state vector is calculated immediately after the inspection,  $A_{AI}(t)$ . Then, calculate the state vector at the next inspection:

$$A_{BI}(t+1) = A_{AI}(t) \times P^k \quad (27)$$

The failure probability  $P_{F2}$  is obtained by the following equation:

$$P_{F2} = \{a_M(t+1) - a_M(t)\} / \{P_{SV} \times (1 - P_{DV})\}, \quad (28)$$

Similarly, if mechanical inspection is applied,  $A_{AI}(t \setminus MI)$ , the failure probability,  $P_{F3}$ , can be obtained.