

# Modal Analysis for Assessing the Durability of an Automobile Coil Spring Under Random Load

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## Abstract

The aim of this paper is to analyse modal analysis for assessing the durability of an automobile coil spring under random load. Coil spring is subjected to unpredictable loads and random loads; therefore, fatigue life assessment was investigated as a problem in ground vehicle suspension systems. It is important to design coil springs as vital part of ground motor vehicles. In this study, steel SAE 5160 alloy selected as material properties. Finite element analysis was carried out to determine maximum stress/strain von Mises by using linear static approach. The original strain-time history signal captured from coil spring of a car during service load. A power spectral density function obtained to analyse modal participations using specific software based on strain history signal. The Coffin-Manson strain life model performed to analyse fatigue life and damage. The results show that maximum fatigue life take-place about  $3.8 \times 10^5$  cycles based on critical plane strain at critical point.

**Keywords:** coil spring, critical plane strain, durability, modal analysis, random load.

## 1. Introduction

Modal analysis is a major technology for assessing, improving and optimising the inherent dynamic characteristics of engineering structures. Reducing of production body weight is a major problem in automobile manufacturers in microscopic scale. For this purpose, design of complex mechanical, aerospace industry and civil structures require to become increasingly lighter, powerful and more flexible. In order to resist compression loads and recover to original shape, coil springs are considered as an elastic part in automotive, when the load is removed. Hence, susceptibility of structures to damage can be reduced in suspension structures [1].

An important aspect of engineering structures are assessment of dynamic stresses for predicting life and safe design based on random loading. Xie and Xue [2] offered a new approach in dynamic stresses, which it was successful in high computational performance. In order to predict fatigue life, dynamic stresses are important for better modal responses. The modal stress superposition and the equivalent behaviour able to transfer Power Spectral Density (PSD) into harmonic functions. Vibration fatigue strength is determined as a vital part in mechanical design; therefore, fatigue life prediction must be developed for test characterisation of components in primary design of product [3]. Wannenburg and Heyns [4] described a framework of numerical methodologies used to determine the durability of automotive structures that considered broad range of procedures as input loading, likewise carried out fatigue analysis and stress analysis. Cianetti [5] et al. introduced a durability acceleration tests as a technique on dynamics component, which considered a method of fatigue damage spectrum. The technique revealed efficacy of the multiple loading compound to an equivalent conditions. The purpose of this investigation is to determine fatigue life and fatigue damage using the modal analysis for

coil spring of suspension systems. The objective of this study characterise durability investigation of coil spring under PSD as a load displacement during service load, when subjected to unpredicted loads. Car manufactures consider coil springs as a safety parts in vehicle, because of sequence compression loads when the vehicle is moving on the road. In this research, the Coffin-Manson strain life model is a method to predict the fatigue life. The results of PSD amplitude test applied to structures as a durability test. Hence, modal displacement analysis is a method to present displacements for coil spring, which defined in terms of distributed coordinates, likewise related to specify sets of normalised displacement functions.

## 2. Theoretical background

### 2.1. Theory for modal based on random loading response

The modal based on random loading response and sensitivity case allow calculating the vibration response and response sensitivity with respect to the design variables. In real modes that are normalised to unity modal mass, the modal displacement response at node is the sum of participation factors at node  $j(U_j)$ , which multiplied by the mode displacement value in the form equation (1).

$$U_j(\omega) = \sum_i \phi_{1,j} q_i(\omega) \quad (1)$$

where,  $q_i$  is the participation factor for mode,  $i$  and  $\phi_{1,j}$  is the mode displacement value for mode  $i$  at node  $j$ . The participation factor for mode  $i$  is in the form equation (2).

$$q_i(\omega) = \sum_i \frac{\phi_{1,L}}{\omega_i^2 - \omega^2} F_L(\omega) \tag{2}$$

where  $\phi_{1,L}$  is the mode displacement for mode  $i$  at the excitation force location  $L$ ,  $\omega_i$  is the mode  $i$  pulsation and  $F_L(\omega)$  is the force at location  $L$ . The modal participation factor sensitivity with respect to the design variable  $\alpha$  has written in the equation (3).

$$\frac{\partial q_i(\omega)}{\partial \alpha} = \sum_i \frac{\frac{\partial \phi_{1,L}}{\partial \alpha} (\omega_i^2 - \omega^2) - 2 \frac{\partial \omega_i}{\partial \alpha} \omega_i \phi_{1,L}}{(\omega_i^2 - \omega^2)^2} F_L(\omega) \tag{3}$$

where,  $\frac{\partial \phi_{1,j}}{\partial \alpha}$  the mode is shape sensitivity,  $\frac{\partial \omega_i}{\partial \alpha}$  is the mode frequency sensitivity and the displacement response sensitivity in the form equation (4).

$$\frac{\partial U_j}{\partial \alpha}(\omega) = \sum_i \phi_{1,j} \frac{\partial q_i(\omega)}{\partial \alpha} + \frac{\partial \phi_{1,j}}{\partial \alpha} q_i(\omega) \tag{4}$$

### 2.1.1. Fatigue life analysis models

Fatigue life assessment can determine by one of the Smith-Watson-Topper (SWT), Morrow and Coffin-Manson strain fatigue life models. Strain life fatigue approach mostly deals with localised plasticity flow and withstand a specified grade of structural limitation. Total life divided to the crack initiation and crack propagation approach so-called linear elastic fracture mechanics (LEFM). The crack initiation describes elastic-plastic of local stresses or strains and the crack propagation method represents pre-existing crack growth rate. In addition, the crack propagation estimates the number of loading cycles to grow critical size of crack when the failure suddenly accrue [1]. During the last few decades, research studies showed for mean stress effect on fatigue life according to strain life models. The mean stress and mean strain effect are important to fatigue life prediction, since the engineering components mostly subjected to cyclic loads.

Equation (5) is known as the Coffin-Manson strain life model [6], which expressed the relationship between total strain amplitude ( $\epsilon_a$ ) and fatigue life ( $N_f$ ).

$$\epsilon_a = \frac{\Delta \epsilon_e}{2} + \frac{\Delta \epsilon_p}{2} = \frac{\sigma_f}{E} (2N_f)^b + \epsilon_f' (\sigma_f')^c \tag{5}$$

where,  $\sigma_f'$  is the fatigue strength coefficient,  $E$  is the modulus of

elasticity,  $\epsilon_f'$  is the fatigue ductility coefficient,  $\frac{\Delta \epsilon_e}{2}$ ,  $\frac{\Delta \epsilon_p}{2}$  are

elastic and plastic strain amplitude respectively,  $C$  and  $b$  are fatigue ductility exponent. By modifying mean stress effect parameter in the equation (5), the fatigue life can be assessed at zero mean stress. Mean stress effect ( $\sigma_m$ ) is modified for elastic part of strain life model [1]; therefore, the Morrow strain fatigue life model can be applied in the form equation (6).

$$\epsilon_a = \frac{\sigma_f' - \sigma_m}{E} (2N_f)^b + \epsilon_f' (\sigma_f')^c \tag{6}$$

The SWT strain life model considers another mean strain effect (SWT mean stress correction model), therefore, it is mathematically expressed in equation (7), where  $\sigma_{max}$  is the maximum stress [7].

$$\sigma_{max} \epsilon_a = \sigma_{max} \frac{\Delta \epsilon}{2} = \frac{(\sigma_f')^2}{E} (2N_f)^{2b} + \epsilon_f' \sigma_f' (2N_f)^{b+c} \tag{7}$$

The linear damage accumulation rule as known the Palmgren-Miner approach. In order to assess fatigue life, it computes the total damage. In order to calculate fatigue damage of structure this approach is very effective and useful in automobile industry [8]. The linear damage accumulation rule can be determined by using sum of total partial damage in the equation (8) as follows.

$$D = \sum_{i=1}^n \frac{n_i}{N_{f_i}} = 1 \tag{8}$$

Where,  $D$  is the accumulated damage,  $n_i$  is the number of applied cycles and  $N_{f_i}$  is the number of constant amplitude cycles to failure. The structure is failed when the total damage reached to value of 1.

## 3. Material and method

Strain time history signal computed from coil spring using strain gauge and all fatigue data captured by data acquisition system. Therefore, all fatigue data input to software interface to calculate strain values based on time domain. Figure 1, illustrates the flow chart to get the objective of current study in 3 steps. In this research, a coil spring selected from a vehicle, in order to investigate correlation between fatigue life and modal analysis. Strain time history collected from coil spring according to road testing. Strain time history considered as inputting data that known as the fatigue signal. In step 1, by using specific software the PSD data obtained from road data (strain time history).

Step 2, the durability analysis case carried out to analyse results of fatigue damage and fatigue life based on modal force by using the Coffin-Manson strain life model. Finally, fatigue life prediction can be specified when the results are satisfactory and accurate. Table 1, showed material properties and their definitions, which used to the automotive suspension component in wide number of different heavy spring applications, especially for helical coil and leaf springs. The SAE 5160 steel [1] was selected as the mechanical properties in this research. It has made in high carbon and chromium spring steel.

**Table 1:** Mechanical and cyclic properties of the SAE5160 steel

Properties	Values
Yield strength (MPa)	1070
Ultimate tensile strength (MPa)	1550
Modulus of elasticity (GPa)	207
Fatigue strength coefficient (MPa)	2063
Fatigue strength exponent	-0.08
Fatigue ductility exponent	-1.05
Fatigue ductility coefficient	9.56
Cyclic-strain hardening exponent	0.10
Cyclic strength coefficient	2000

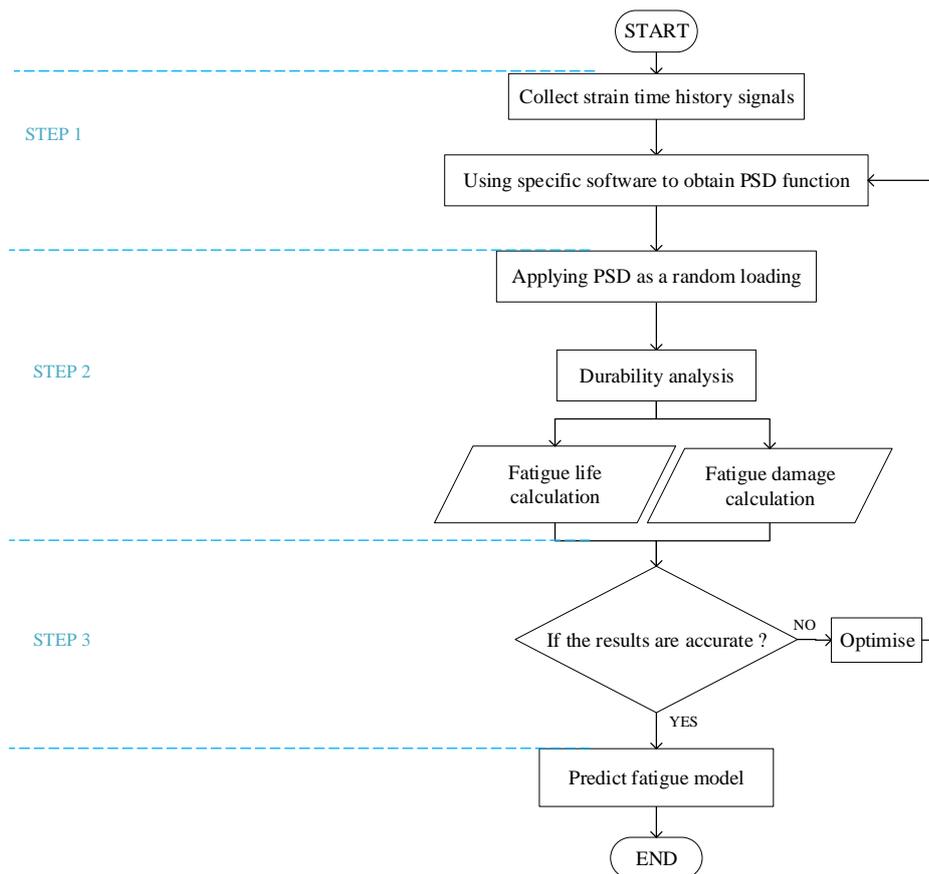
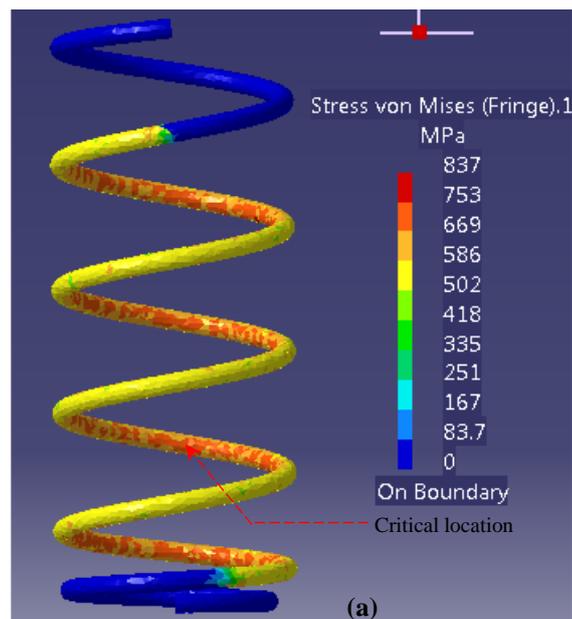


Fig. 1: The flow chart of the study.

#### 4. Results and discussions

Coil spring is the one of the important and vital component in ground vehicles. In this study, geometry model of coil spring is considered same as a real part of vehicle. Figure 2, illustrates the finite element model (FEM) of coil spring based on (a) stress von Mises

and (b) strain von Mises respectively, which applied 4 nodes tetrahedral element for the solid mesh. Coil spring is fixed from down and applied 1900 N distribution load in top. Maximum of equivalent von Mises stresses/strains obtained 837 MPa and  $4.04 \times 10^{-3}$  mm at critical location respectively as shown in Figure 2.



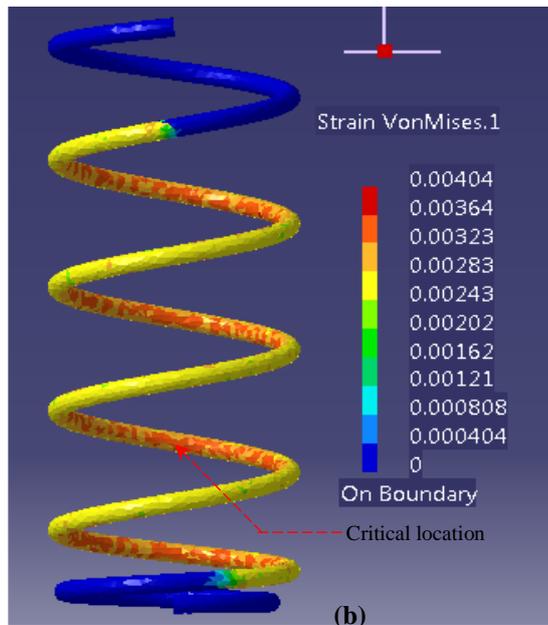


Fig. 2: FEM results based on (a) stress von Mises and (b) strain von Mises.

Figure 2 shows the damage and original strain signal at 80 seconds of time value in a plot together. As mentioned in the plot, part (a) the higher fatigue damage can be related to higher amplitude or higher energy. According to damage and original strain signal, the high strain amount can lead to fatigue damage value because of the large strain values throughout the signal. Therefore, the high amplitudes and low fatigue damage contributed to higher and lower energies from the potential fatigue damage strain signals respectively.

Coil spring is frequently under random loading by the wheels on the road. Signals of random loading (strain time history signal) and acceleration occur in a term. These signals are usually Gaussian and linear, which can be fundamental criteria of fatigue vibration. These signals assist to obtain the fast Fourier transform (FFT) and PSD functions [9]. Figure 3, shows the process of PSD production from original strain time history signal by using specific software.

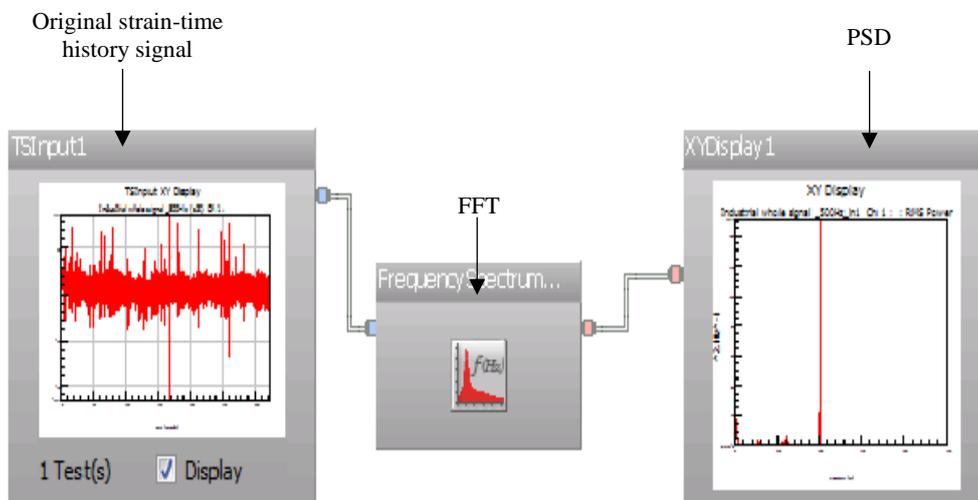


Fig. 3: The PSD process from original strain-time history signal.

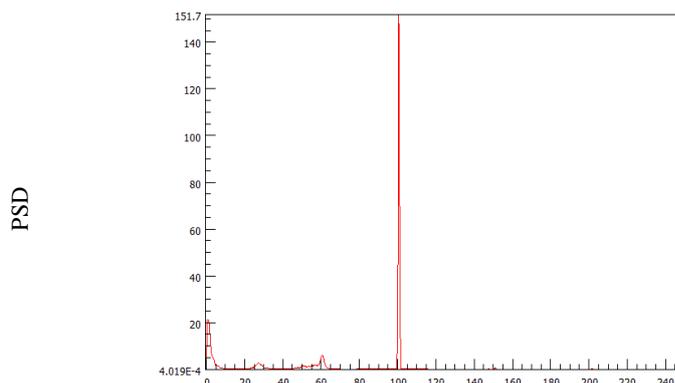


Fig. 4: Power spectral density distribution.

Figure 4 illustrates modal displacement load based on PSD function to analyse modal case, when coil spring is subjected to random loading in service road. In this plot, x-axis shows frequency in Hz against y-axis that determines PSD values. Total length of fre-

quency is around 250 Hz. This plot is very useful to identify oscillatory signals in frequency data and amplitude. Hence, PSD is determined the mean square amplitude by measuring the area under PSD over the desired frequency range.

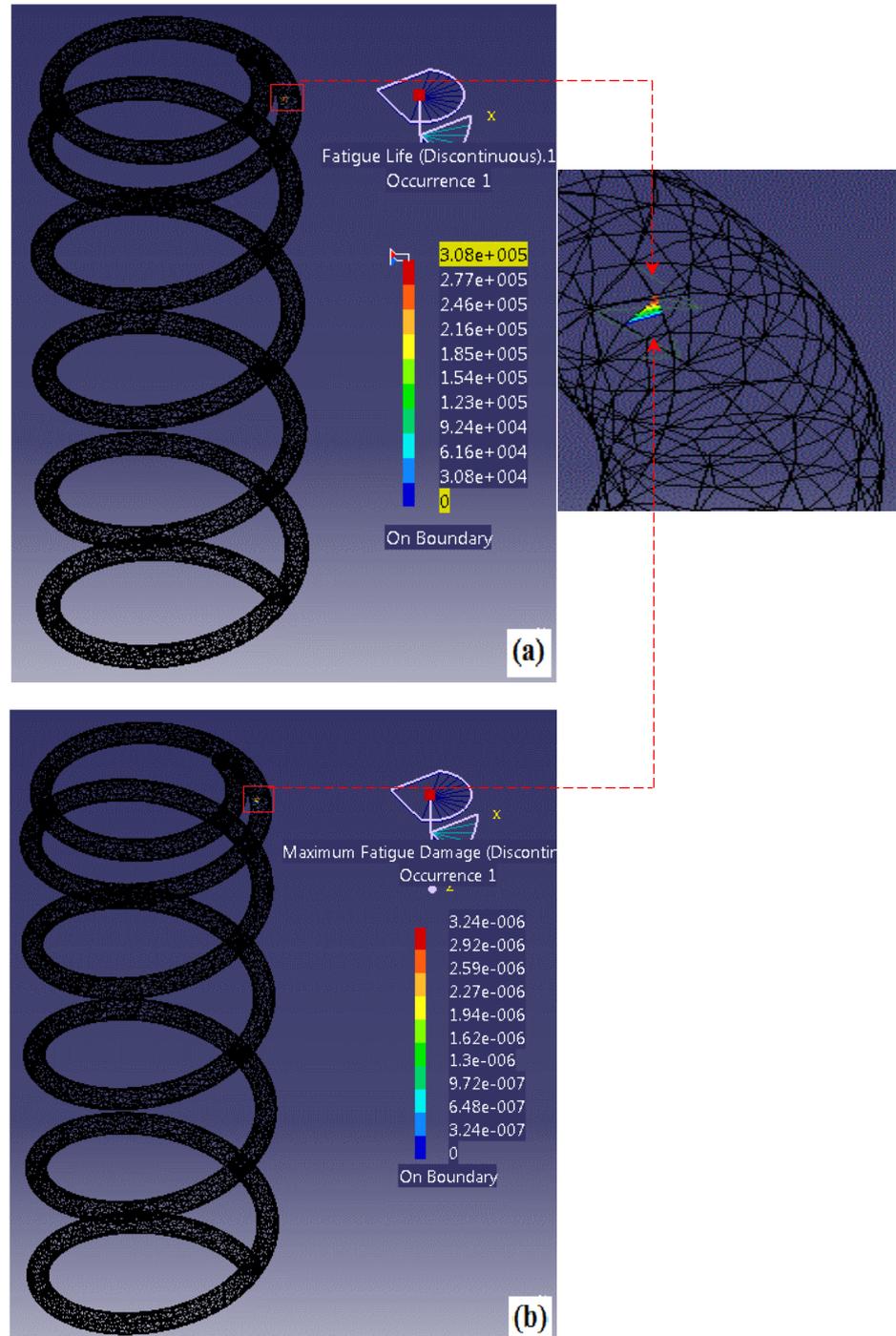


Fig. 5: Strain durability analysis (a) fatigue life and (b) fatigue damage.

As shown in Figure 5, fatigue life and fatigue damage focused on results using the specific software at critical location. Number of repeats of each time history applied as loading is calculated. Maximum fatigue life and fatigue damage computed based on the Coffin-Manson strain fatigue life around  $3.08 \times 10^5$  and  $2.24 \times 10^{-6}$  cycles at the critical location respectively. Coil spring tends to fail at fixed coil (inactive coil) far from free coils (active coils) because of fatigue failure [10].

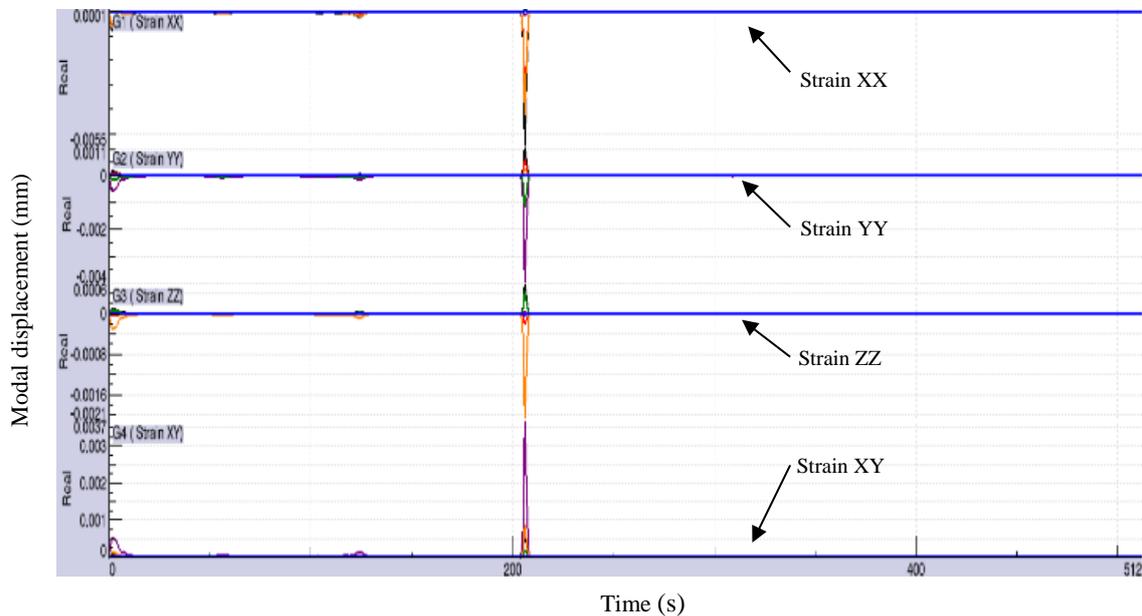
Critical plane search for a plan that is most favourable for crack initiation and propagation, where occurred fatigue. In the high cycle

fatigue domain plasticity is very limited; therefore, stress based models are typically used. Strain based models evaluate strains or combinations of strains and stresses when defining a critical plane. Once the critical plane identified, strain models predict the number of cycles to failure. Therefore, Figure 6 shows modal displacement against time for automobile coil spring suspension system under random loading at critical area during service load.

**Table 2:** Modal displacement in strain critical plane.

Critical plane	Maximum displacement (mm)	Minimum displacement (mm)
Strain XX	0.0001	-0.0055
Strain YY	0.0011	-0.004
Strain ZZ	0.0006	-0.0021
Strain XY	0.0037	0

Maximum and minimum modal displacement occurred in critical plane strain XY and XX around 0.0037 mm and -0.0055 mm respectively. Results of maximum and minimum modal displacement tabulated in Table 2. While, strain XY critical plane showed no minimum displacement.

**Fig. 6:** Modal analysis plot based on critical plane.

## 5. Conclusion

Coil spring works as a shock absorbers and avoid sudden loads from a vehicle suspension system in service load. SAE 5160 steel selected as material properties for coil spring in this research. Modal analysis expressed by applying a strain signal in service load test. It known as random loading, where the response is categorised using PSD function. FEA was evaluated maximum von Mises stress/strain distribution at critical locations. FEA illustrated inner surface of coil spring is located at critical area, when subjected to service load. According to strain-time history signal, the high amplitudes identified as the potential fatigue damage signals. Durability results illustrated maximum fatigue damage around  $2.63 \times 10^{-6}$  and fatigue life about  $3.8 \times 10^5$  cycles based on the Coffin-Manson strain fatigue life model at critical plane. In this study, modal analysis investigated to determine modal displacement for automotive coil spring suspension system during service loads at XX, YY, ZZ and XY strain critical plane. The planes that experience the highest normal stresses/strains are usually good candidates for a critical plane.

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