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Research paper

Application of Infinite Memory Structure and Finite Memory Structure Filters for Electric Motor System

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Abstract

In this paper, both IMS and FMS filters are applied for the estimation filtering of electric motor systems. Firstly, the electric motor system and its state-space model is described, Secondly, IMS and FMS filters are briefly introduced and compared. These two filters are represented by the summation form and use the rotational speed as the output measurement unlike the existing work. Thirdly, comprehensive simulation works are performed for both certain system and temporarily uncertain system. Simulations results show that the FMS filter can be better than the IMS filter for the temporarily uncertain system. It is also shown that there can be the trade-off between two filtering performance indices, estimation error and tracking speed in terms of the memory length.

Keywords: *Electric motor system, Finite memory structure, Infinite memory structure, Nominal system, State estimation filter, Temporary uncertain system.*

1. Introduction

The direct current electric motor is the most widely adopted in motor control systems because of their inherent properties such as efficient cost, high performance, etc. Thus, this motor has been a common actuator in control systems, which can directly provide rotary motion as well as transitional motion related to wheels[1][2].

Meanwhile, state-space modeling is widely and generally used in diverse engineering areas to model linear and nonlinear dynamical system. In the electric motor system, the state-space model has been also used for control and estimation problems[3]-[5]. For the state-space model for electric motor system, a couple of estimation filters has been applied to provide filtered estimate for the state variable of the controller in each step in order to improve the system performance.

Depending on the measurement processing method, state estimation filters can be classified by the infinite memory structure (IMS) filter and the finite memory structure (FMS) filter. The IMS filter such as the well-known Kalman filter has been a standard choice for the state estimation and thus a beautiful reference for diverse engineering problems[5]-[7]. The IMS filter has the recursive formulation for computational efficiency. However, since the IMS filter processes all past measurements accomplished by equal weighting, it tends to accumulate estimation errors during its implementation. Therefore, the IMS filter has been known to show performance degradation and even divergence phenomena for mismodeling and temporary uncertainties. On the other hand, the FMS filter using finite measurements on the most recent window has been developed as an alternative to the IMS filter[8]-[11] and also applied successfully for various applications such as mobile target tracking, computer network, RFID system, global positioning system, wireless sensor network, fault diagnosis, etc., as shown in [12]-[15]. Especially, as shown in [15], the FMS filter with iterative and matrix forms was applied for the electric motor system when the armature current is chosen as the output measurement. Unlike the IMS filter, the FMS filter has been known inherently to show BIBO stability and robustness against round-off errors, mismodeling and temporary uncertainties.

In this paper, both IMS and FMS filters are applied for the estimation filtering of electric motor systems. Firstly, the electric motor system and its state-space model is described, Secondly, IMS and FMS filters are briefly introduced and compared. These two filters are represented by the summation form and uses the rotational speed as the output measurement unlike the existing work of [15]. Thirdly, to apply these estimation filters to the electric motor system, comprehensive simulation works are performed are performed for both certain and temporarily uncertain systems. Simulations results such as estimation error, error convergence, and noise suppression show that the FMS filter is more robust than the IMS filter when applied to temporarily uncertain systems, although the FMS filter is designed with no consideration of robustness. Of course, the IMS filter can outperform or be comparable to FMS filters after the effect of temporary modeling uncertainty completely disappears. In addition, it is also shown that the noise suppression of the state estimation filter might be closely related to the memory length of past measurements. Therefore, there can be the trade-off between two filtering performance indices, estimation error and tracking speed in terms of the memory length.

The structure of this paper is as follows. In Section 2, an electric motor system and its state-space model are described. In Section 3, FMS and IMS estimation filters are briefly introduced and compared. In Section 4, extensive computer simulations are performed. Then, concluding remarks are given in Section 5.

2. State-Space Model Representation for Electric Motor System

For the electric motor system such as direct current motor, the armature's electric circuit and the rotor's free body diagram are shown in Fig. 1.

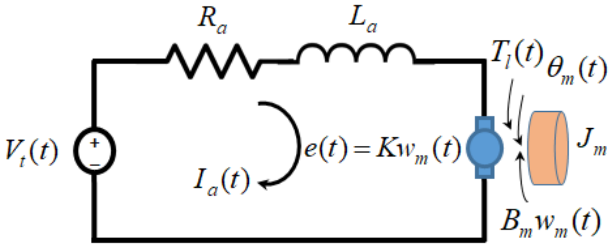

Figure 1: Direct current electric motor system model

The direct current electric motor system consists of many kinds of variable and parameters : the armature current $I_a(t)$, the rotational speed $w_m(t)$, the drive voltage $V_t(t)$, the emf $e(t)$, the load torque $T_l(t)$, the armature resistance R_a, the armature inductance L_a, the motor inertial coefficient J_m, the motor viscous friction coefficient B_m, motor torque and back emf constants K.

The state-space approach has been a general method for modeling, analyzing and designing a wide range of control systems in time-domain and is especially suitable for digital computation techniques. In this paper, the state-space realization is also required for the electric motor system to apply the state estimation filtering.

A continuous-time state-space model of the electric motor system can be represented by[2][15]

$$\dot{x}(t) = A_c x(t) + B_c u(t) \\ y(t) = Cx(t). \tag{1}$$

If the goal of control is to drive the electric motor to a desired rotational speed, variable and parameters are expressed by

$$x(t) = \begin{bmatrix} I_a(t) \\ w_m(t) \end{bmatrix},\ u(t) = V_t(t),$$

$$A_c = \begin{bmatrix} -\dfrac{R_a}{L_a} & -\dfrac{K}{L_a} \\ \dfrac{K}{J_m} & -\dfrac{B_m}{J_m} \end{bmatrix},\ B_c = \begin{bmatrix} \dfrac{1}{L_a} \\ 0 \end{bmatrix},\ C = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

State variables to be estimated consist of the armature current $I_a(t)$ and the rotational speed $w_m(t)$ for the electric motor system. The control input is taken by the armature voltage $V_t(t)$. The measure output is taken by the rotational speed.

Using the discretization with a sampling period T, the continuous-time state-space model (1) can be represented by

$$x_{i+1} = Ax_i + Bu_i, \\ y_i = Cx_i, \tag{2}$$

where

$$A = e^{A_c T},\ B = \left(\int_0^T e^{A_c \varepsilon} d\varepsilon\right) B_c = \left(e^{A_c T} - I\right) A_c^{-1} B_c.$$

Thus, the discretized model (2) can be used for the discrete-time state-space model with noise-free.

3. IMS and FMS Filters with Summation Form for State Estimation

In actual situations, there can be system and measurement noises in the electric motor system. Then, the discretized electric motor system (2) can be extended by the ultimate discrete-time state-space model using system and measurement noises w_i, v_i as follows:

$$x_{i+1} = Ax_i + Bu_i + Gw_i, \\ y_i = Cx_i + v_i, \tag{3}$$

where the initial state of the state variable $\hat{x}_i$ is notated by a random variable $\hat{x}_{i_0}$ with a mean $\bar{x}_{i_0}$ and a covariance Ω_{i_0}. The system noise w_i and the measurement noise v_i are zero-mean white Gaussian. Covariances for these noises are notated by Q and R, respectively, and assumed to be positive definite matrices.

The infinite memory structure (IMS) filter such as the well-known Kalman filter gives an optimal state estimate $\hat{x}_i^{ims}$ for the system state x_i. The IMS filtered estimate $\hat{x}_i^{ims}$ can be represented by the summation form with the initial condition $\hat{x}_{i_0}^{ims} = \bar{x}_{i_0}$ as follows:

$$\begin{aligned} \hat{x}_i^{ims} &= \Phi_i \hat{x}_{i_0} + \sum_{j=i_0}^{i-1} \Phi_{i-j} \Sigma_j C^T R^{-1} y_j + \sum_{j=i_0}^{i-1} \Phi_{i-j} Bu_j \\ &= \Phi_i \bar{x}_{i_0} + \sum_{j=i_0}^{i-1} \Phi_{i-j} \Sigma_j C^T R^{-1} y_j + \sum_{j=i_0}^{i-1} \Phi_{i-j} Bu_j \end{aligned} \tag{4}$$

where Φ_j is the transition matrix given by

$$\Phi_{j+1} = \Phi_j A\left[I + \Sigma_{i-j-1} C^T R^{-1} C\right]^{-1},\ \Phi_0 = I,\ i_0 \le j \le i-1$$

and Σ_i is the error covariance of the estimate $\hat{x}_i^{ims}$ given by

$$\Sigma_{i+1} = A(I + \Sigma_i C^T R^{-1} C)^{-1} \Sigma_i A^T + GQG^T$$

with the initial value Σ_{i_0}, which is the covariance of $\hat{x}_{i_0}^{ims}$.

The IMS filter has been adopted generally for the optimal state estimation and thus applied successfully for diverse engineering problems and applications. However, because the IMS filter processes all past measurements accomplished by equaling weighting and has a recursive formulation, it tends to accumulate the estimation error as time goes, which can show even divergence phenomenon for mismodeling and temporary uncertainties.

Hence, as an alternative to the IMS filter, the finite memory structure (FMS) filter has been developed by the optimization problem under a weighted least square criterion using only the most recent finite measurements on the window $[i-M, i]$. For simplicity, the window initial time $i-M$ will be notated by i_M hereafter. The FMS filter gives an optimal state estimate $\hat{x}_i^{fms}$ for the system state x_i. The FMS filtered estimate $\hat{x}_i^{fms}$ can be represented by the summation form with the window initial condition $\hat{x}_{i_M} = \left(\Gamma^T \Pi^{-1} \Gamma\right)^{-1} \Gamma^T Y_i$ as follows:

$$\begin{aligned}\hat{x}_i^{fms} &= \Phi_M \hat{x}_{i_M} + \sum_{j=0}^{M-1} \Phi_{M-j}\Sigma_{i_M+j} C^T R^{-1} z_{i_M+j} \\ &\quad + \sum_{j=0}^{M-1} \Phi_{M-j} B u_{i_M+j} \\ &= \Phi_M \left(\Gamma^T \Pi^{-1} \Gamma\right)^{-1} \Gamma^T Y_i \\ &\quad + \sum_{j=0}^{M-1} \Phi_{M-j}\Sigma_{i_M+j} C^T R^{-1} z_{i_M+j} \\ &\quad + \sum_{j=0}^{M-1} \Phi_{M-j} B u_{i_M+j}\end{aligned} \tag{5}$$

where Φ_j is the transition matrix given by

$$\Phi_{j+1} = \Phi_j A\left[I + \Sigma_{i_M+M-j-1} C^T R^{-1} C\right]^{-1},$$
$$\Phi_0 = I, 0 \le j \le M-1$$

and Σ_i is the error covariance of the estimate $\hat{x}_i^{fms}$ given by

$$\Sigma_{j+1} = A(I + \Sigma_j C^T R^{-1} C)^{-1} \Sigma_j A^T + GQG^T,$$
$$\Sigma_0 = \left(\Gamma^T \Pi^{-1} \Gamma\right)^{-1}.$$

The most recent finite measurements Y_i and matrices Γ, Λ, and Π are as follows.

$$Y_i \equiv \begin{bmatrix} y_{i_M} \\ y_{i_M+1} \\ \vdots \\ y_{i-1} \end{bmatrix}, \quad \Gamma \equiv \begin{bmatrix} CA^{-M} \\ CA^{-M+1} \\ \vdots \\ CA^{-1} \end{bmatrix}$$

$$\Lambda \equiv -\begin{bmatrix} CA^{-1}G & CA^{-2}G & \cdots & CA^{-M+1}G & CA^{-M}G \\ 0 & CA^{-1}G & \cdots & CA^{-M+2}G & CA^{-M+1}G \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & CA^{-1}G \end{bmatrix},$$

$$\Pi \equiv \Lambda\left[diag\left(\overbrace{Q \quad Q \quad \cdots \quad Q}^{M}\right)\right]\Lambda^T + \left[diag\left(\overbrace{R \quad R \quad \cdots \quad R}^{M}\right)\right].$$

The FMS filter has been known to have a couple of inherent good properties such as unbiasedness and deadbeat. These two properties cannot be obtained by the IMS filter. Moreover, in contrast to the IMS filter that tends to accumulate the estimation error with the progression of time, the FMS filter is more robust against round-off errors, mismodeling and temporary uncertainties. due to the finite memory structure as shown in [8]-[11]. Thus, the FMS filter has been applied successfully for various applications.

4. Extensive Computer Simulations

4.1. Simulation Specification

Because the most basic requirement of an electric motor is that it should rotate at the desired speed, the steady-state error of the rotational speed should be smaller than 1%. The other performance requirement is that the motor will accelerate to its steady-state speed quickly after it's turned on. In this case, a settling time of 2 seconds is also required. In addition, an overshoot smaller than 5% is also required because a speed faster than the requirement may damage the equipment. Therefore, when the reference input is simulated by a unit step input with 1 Volt, the output, that is, the rotational speed should have settling time less than 2 seconds, overshoot less than 5%, steady-state error less than 1%. With these specifications, physical parameters for the electric motor system is set as shown in Table 1[2][15].

Table 1: Values of physical parameters for simulations

Physical Parameters	Values
Motor inertial coefficient J_m [Nm/(rad/sec^2)]	0.01
Motor viscous friction coefficient B_m [Nm/(rad/sec)]	0.1
Motor torque constant K [Nm/A]	0.01
Armature resistance R_a [Ohm]	1
Armature inductance L_a [H]	0.5

System and measurement noise covariances are taken by $Q = 0.01^2 I_{2\times2}$ and $R = 0.02^2$, respectively. Four kinds of filters, the IMS filter, the FMS filter with $M = 30$, $M = 20$, and $M = 10$, are compared. Simulations of 20 runs are performed for clearer comparison of performances between IMS and FMS filters. Each single simulation uses different system and observation noises and runs lasts 500 samples.

4.2 Simulation for Nominal System

Through the discretization with physical parameters in TABLE 1, system matrices for discrete-time state-space model for the electric motor system can be obtained by

$$A = \begin{bmatrix} 0.8178 & -0.0011 \\ 0.0563 & 0.3678 \end{bmatrix}, \; B = \begin{bmatrix} 0.1813 \\ 0.0069 \end{bmatrix}$$
$$G = \begin{bmatrix} 0.0006 & 0 \\ 0 & 0.0057 \end{bmatrix}, \; C = \begin{bmatrix} 0 & 1 \end{bmatrix}, \tag{6}$$

where the rotational speed is chosen as the output measurement while the armature current is chosen in the existing work [15].

For the nominal system (6) where there is no temporary uncertainty, four filters are compared by estimation error for one of 20 simulations. As shown in Fig. 2, estimation errors of all filters can be comparable. However, the IMS filter and two FMS filters $M = 30$ and $M = 20$ are better than the FMS filter with $M = 10$ for the noise suppression, which shows that the estimation filter can have greater noise suppression as the measurement memory length increases.

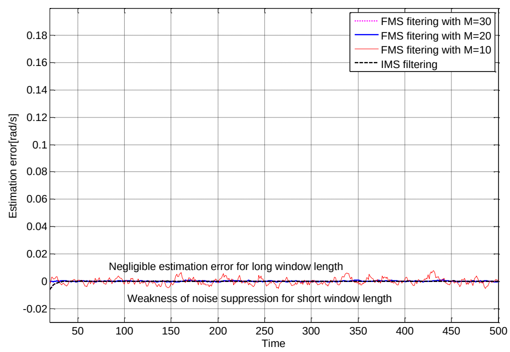

Figure 2: Estimation errors of 2nd state variable (rotational speed) for nominal system

4.3 Simulation for Nominal System

Even if the electric motor system is represented in state-space model accurately on a long time scale, there can be unpredictable changes, such as jumps in frequency, phase, and velocity. Since these effects typically occur over a short time horizon, they are called temporary uncertainties.
For the nominal system (6) of discretized electric motor system, the temporary uncertainty δ_i is considered as

$$A_{\text{ in (3)}} \Rightarrow \begin{bmatrix} 0.8178+\delta_i & -0.0011 \\ 0.0563 & 0.3678+\delta_i \end{bmatrix}$$
$$C_{\text{ in (3)}} \Rightarrow \begin{bmatrix} 0.2*\delta_i & 1+0.2*\delta_i \end{bmatrix} \quad (7)$$
$$\delta_i = 0.1, \quad 50 \le i \le 250.$$

That is, although the IMS filter and FMS filters with $M=30$, $M=20$, $M=10$ are computed by the nominal discrete-time state-space model (6) for the electric motor system, actual measurements for these four filters are obtained from the temporarily uncertain system (7). As shown in Fig. 3 and 4, the FMS filter with $M=20$ can be better than the IMS filter in terms of both error magnitude and error convergence. The estimation error of the FMS filter with $M=20$ is smaller than that of the IMS filter on the interval where modeling uncertainty exists. In addition, the convergence of estimation error is faster than that of the IMS filter after temporary modeling uncertainty disappears. Therefore, the FMS filter with $M=20$ can be more robust than the IMS filter when applied to temporarily uncertain systems, although the FMS filter with $M=20$ is designed with no consideration of robustness.
Although the FMS filter with $M=10$ has smaller estimation error and more fast error convergence than the IMS filter and two FMS filters with $M=20$ and $M=30$, there can be weakness for the noise suppression. On the other hand, although the FMS filter with $M=30$ has better noise suppression than two FMS filters with $M=20$ and $M=30$, there can be weakness for the estimation error and the error convergence. That is, the FMS filter with $M=30$ might be comparable to the IMS filter.
These observations show the noise suppression of the state estimation filter might be closely related to the memory length for past measurements. The estimation filter can have greater noise suppression as memory length increases. However, the convergence speed of estimation error worsens as memory length increases. Thus, there can be the trade-off between two filtering performance indices, estimation error and tracking speed in terms of the memory length.

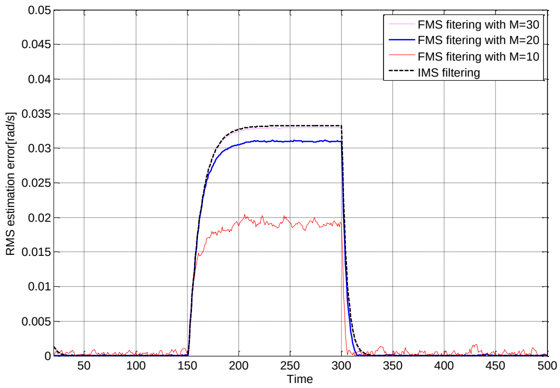

Figure 3: RMS estimation error of 2nd state variable (rotational speed) for temporarily uncertain system

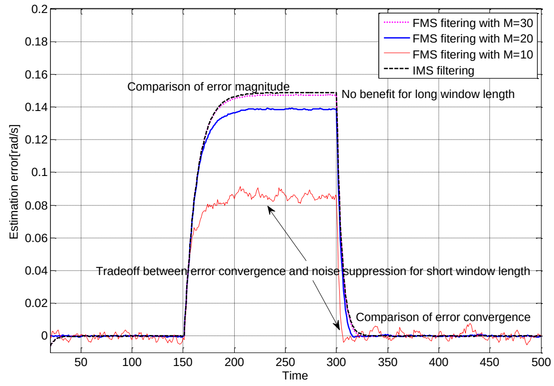

Figure 4: Estimation error of 2nd state variable (rotational speed) for temporarily uncertain system

5. Conclusion

This paper has applied both IMS and FMS filters for the estimation filtering of electric motor systems. The direct current motor system and its state-space model has been described, Then, IMS and FMS filters have been briefly introduced and compared, Finally, comprehensive simulation works have been performed are performed for both certain and temporarily uncertain systems. Simulations results such as estimation error, error convergence, and noise suppression has shown that the FMS filter can be better that the IMS filter for the temporarily uncertain system. It has been also shown that there can be the trade-off between two filtering performance indices, estimation error and tracking speed in terms of the memory length.

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