International Journal of Engineering &Technology, 7 (4) (2018) 6565-6567

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET
doi: 10.14419/ijet.v7i4.28919
Research paper

Al-Tememe transform in solving developed missile automatic control problem

Emad Kuffi [1] *, Elaf Sabah Abbas[1], Salman Obeid Mebayet [1]

[1] *Communication Engineering department, Al-Mansour University Collage, Baghdad, Iraq*
**Corresponding author E-mail: elaf.abbas@muc.edu.iq*

Abstract

Al-Tememe transform is a new transform that is recently emerged, as a result to its modernity, it has not exploited properly in many applications. Missile automatic control problem is an application of the second order ordinary differential equation with constant coefficients, for that reason many methods in solving second order ordinary differential equations had been used in solving that problem, however, Al-Tememe transform had never been used in solving the problem of missile automatic control. In this paper Al-Tememe transform has been used to solve the developed problem of missile automatic control, which is an application of the second order ordinary differential equations with variable coefficients.

Keywords: *Al-Tememe Transform; Developed Missile Automatic Control Problem; Second Order Ordinary Differential Equations with Variable Coefficients, Torque; Angle of Error; Moment of Inertia; Angular Velocity.*

1. Introduction

A differential equation is the solving of a derivative containing equation, the derivatives could be for the function itself or other unknown functions to make the differential equation either ordinary or partial differential equation. Differential equations represent the mathematical description of the rate changing in real world events, and it is used in many science fields such as medical, engineering, chemistry, economics and physics [1] [3].

The work of this paper is on the solving of the second order ordinary differential equations, which is essential in the classical areas of mathematical physics. There are many methods to solve second order ordinary differential equations [3], Laplace and Al-Tememe transforms are methods that used to solve the ordinary differential equation [4 - 9]. Laplace transform has been used extensively in many applications related to differential equations, However Al-Tememe transform has not been sufficiently used in such applications due to its recent appearance, therefore, the benefits that such transform could provide in dealing with the applications that are related to differential equations are not exploited properly.

2. Basic concepts

It is necessary to provide some related definitions and theorems to make the later calculations clearer.

2.1. Definition of Al-Tememe transform [10]

Al-Tememe transform for the function $\mathbf{f(x)}$; $\mathbf{x > 1}$ is defined by the following integral $\mathbf{T[f(x)] = \int_1^\infty x^{-p} f(x) dx = F(p)}$. Such that this integral is convergent in some region, $\mathbf{p}$ is a positive constant, and $\mathbf{x^{-p}}$ the kernel of Al-Tememe transform.

2.2. Definition of inverse Al-Tememe transform [10]

Let $f(x)$ be a function where $x > 1$ and $T[f(x)] = F(p)$, $f(x)$ is said to be an inverse for Al-Tememe transform and written as:$T^{-1}[F(p)] = f(x)$, where T^{-1} returns the transform to the original function.

2.3. Propriety of Al-Tememe transform [10]

The transformation is characterized by the linear property, that is: $T[Af(x) \pm Bg(x)] = AT[f(x)] \pm BT[g(x)]$ where A and B are constants, the functions $f(x)$ and $g(x)$ are defined when $x > 1$.

2.4. Al-Tememe transformation for some functions, [11]

Functions	$T[f(x)] = \int_1^\infty x^{-p} f(x) dx$	Region of convergence
k, k is a constant	$\frac{k}{p-1}$	$p > 1$
x^n , $n \in R$	$\frac{k}{p-(n+1)}$	$p > n+1$
$\ln x$	$\frac{1}{(p-1)^2}$	$p > 1$
$\sin(a \ln x)$	$\frac{a}{(p-1)^2+a^2}$	$p > 1$
$\cos(a \ln x)$	$\frac{s-1}{(p-1)^2+a^2}$	$p > 1$
$x^n \ln x$, $n \in R$	$\frac{1}{(p-(n+1))^2}$	$p > n+1$
$\sinh(a \ln x)$	$\frac{a}{(p-1)^2-a^2}$	$\|p-1\| > a$
$\cosh(a \ln x)$	$\frac{p-1}{(p-1)^2+a^2}$	$\|p-1\| > a$
$(\ln x)^n$, $n \in Z^+$	$\frac{n!}{(p-1)^{n+1}}$	$p > 1$

2.5. Theorem [2]

Let $y(x)$ be defined function for $x > 1$, and its derivatives $y'(x), y''(x), \cdots, y^n(x)$ exist, then: $T^c[x^n y^n(x)] = -y^{(n-1)}(1) - (ip-n)y^{(n-2)}(1) - \cdots - (ip-n)(ip-(n-1)). \cdots .(ip-2)y(1) + (ip-n)(ip-(n-1)) \cdots (ip-1)F(ip)$ $n \in Z^+$.

3. Automatic control of missiles [12] [4]

One of the ordinary differential equations applications is the automatic control over a moving missile that followed a moving object such as an aircraft. If the moving missile is indicated by $\boldsymbol{M}$ and the turning angle of the of the aircraft is $\boldsymbol{\theta(t)}$at time $\boldsymbol{t}$, assuming the desired turning angle of the missile is $\boldsymbol{\alpha t}$, where $\boldsymbol{\alpha}$ is the angular velocity of the missile. It is an inevitable occurrence to have an error between the two angles: $\boldsymbol{error = \theta(t) - \alpha t}$.

Due to the error that happened between the two angles, it is necessary to produce a torque to reduce or eliminate the error. The relationship between the produced torque and the error is positive and in the opposite direction.

$\boldsymbol{torque = I\frac{d^2q}{dt^2}}$. Where $\boldsymbol{I}$ is the moment of inertia?

Since torque is directly proportional to the error

$$\boldsymbol{I\frac{d^2q}{dt^2}} \propto [\boldsymbol{\theta(t) - \alpha t}] .$$

$$\therefore \ \boldsymbol{I\frac{d^2q}{dt^2} = -k[\theta(t) - \alpha t]} \ \dots \boldsymbol{(3.1)} \ \boldsymbol{where\ k\ is\ a\ constant\ and\ > 0} .$$

The minus refer to the opposite direction of the torque in respect to the error.

Assuming that the initial angle and the angular velocity are zero $\boldsymbol{\theta = 0\ and\ \theta'^{(0)} = 0}$ then equation (3.1) will be: $\boldsymbol{I\frac{d^2q}{dt^2} = -k\theta(t) + k\alpha t}$

$$\Rightarrow \boldsymbol{\frac{d^2q}{dt^2} = \frac{-k}{I}\ \theta(t) + \frac{\alpha t}{I}}$$

$$\therefore\ \boldsymbol{\frac{d^2q}{dt^2} + \frac{k}{I}\ \theta(t) = \frac{k}{I}\alpha t} .$$

a) Solving developed missile automatic control problem using Al-Tememe transform

As an application to the second order ordinary differential equation with variable coefficients, the developed missile automatic control problem could be solved using Al-Tememe transform. It is necessary to multiply $\theta''$by t^2 in equation (3.1) in order to solve the developed missile automatic control problem using Al-Tememe transform.

Let's assume $\boldsymbol{\theta = y\ and\ t = x}$ then:

$$x^2y'' + \frac{k}{I}y = \frac{k}{I}\alpha x$$

$$y(1) = 0; y'(1) = 0.$$

By using Al-Tememe transform

$$-y'(1) - (p-2)y(1) + (p-2)(p-1)T(y) + \frac{k}{I}T(y) = \frac{k}{I}\alpha\frac{1}{p-2}$$

$$T(y)\left(p^2 - 3p + 2 + \frac{k}{I}\right) = \frac{\frac{k\alpha}{I}}{p-2} .$$

$$T(y) = \frac{k\alpha}{I}\left(\frac{1}{(p-2)\left(p^2-3p+2+\frac{k}{I}\right)}\right) .$$

$$y = T^{-1}\left(\frac{\frac{k\alpha}{I}}{(p-2)\left(p^2-3p+2+\frac{k}{I}\right)}\right) .$$

Now let: $\frac{A}{p-2} + \frac{Bp+C}{p^2-3p+2+\frac{k}{I}} = \frac{\frac{k\alpha}{I}}{(p-2)\left(p^2-3p+2+\frac{k}{I}\right)}$.

$$= \frac{Ap^2-3Ap+2p+\frac{kA}{I}+Bp^2-2Bp+Cp-2C}{(p-2)\left(p^2-3p+2+\frac{k}{I}\right)} .$$

$$A + B = 0 \quad (3.2)$$

$$-3A - 2B + C = 0 \quad (3.3)$$

$$2A + \frac{k}{I}A - 2C = \frac{k\alpha}{I} . \quad (3.4)$$

From (3.2) $B = -A$, substitute in (3.3) to find C .

$$-3A - 2(-A) + C = 0 .$$

$$-3A + 2A + C = 0 \ \therefore C = A .$$

$$2A + \frac{k}{I}A - 2A = \frac{k\alpha}{I} .$$

$$A = \frac{k\alpha}{I}.\frac{I}{k} = \alpha \ \therefore A = \alpha .$$

Then:

$$\frac{\alpha}{p-2} + \frac{-\alpha p+\alpha}{p^2-3p+2+\frac{k}{I}} = \frac{\frac{k\alpha}{I}}{(p-2)\left(p^2-3p+2+\frac{k}{I}\right)} .$$

Now:

$$y = \alpha T^{-1}\left(\frac{1}{p-2}\right) - \alpha T^{-1}\left(\frac{p-1}{p^2-3p+2+\frac{k}{I}}\right) .$$

$$y = \alpha T^{-1}\left(\frac{1}{p-2}\right) - \alpha T^{-1}\left(\frac{p-1}{\left(p-\frac{3}{2}\right)^2+\left(\frac{-1}{4}+\frac{k}{I}\right)}\right) .$$

$$y = \alpha T^{-1}\left(\frac{1}{p-2}\right) - \alpha T^{-1}\left(\frac{p-\frac{3}{2}}{\left(p-\frac{3}{2}\right)^2+\left(\frac{-1}{4}+\frac{k}{I}\right)} + \frac{\frac{1}{2}}{\left(p-\frac{3}{2}\right)+\left(\frac{-1}{4}+\frac{k}{I}\right)}\right) .$$

From the above the solution for the missile automatic control system is:

$$y = \alpha x - \alpha x^{\frac{1}{2}} cos\left(\sqrt{\frac{-1}{4}+\frac{k}{I}}\, ln\, x\right) - \frac{1}{2}\alpha x^{\frac{1}{2}} sin\left(\sqrt{\frac{-1}{4}+\frac{k}{I}}\, ln\, x\right) .$$

By using the original symbols

$\boldsymbol{\theta = \alpha t - \alpha t^{\frac{1}{2}} cos\left(\sqrt{\frac{-1}{4}+\frac{k}{I}}\, ln\, t\right) - \frac{1}{2}\alpha t^{\frac{1}{2}} sin\left(\sqrt{\frac{-1}{4}+\frac{k}{I}}\, ln\, t\right)}$. Is the required turn angle of the missile.

It is possible to proof that Al-Tememe transform could be used to solve the developed problem of missile automatic control

$$y' = \alpha - \alpha x^{\frac{1}{2}}\sqrt{-\frac{1}{4}+\frac{k}{I}}\,(-sin((\sqrt{-\frac{1}{4}+\frac{k}{I}}\, ln\, x)))x^{-1} - \frac{1}{2}\alpha\, x^{-\frac{1}{2}} cos\left(\sqrt{-\frac{1}{4}+\frac{k}{I}}\, ln\, x\right) - \frac{1}{2}\alpha x^{\frac{1}{2}}\left(\sqrt{-\frac{1}{4}+\frac{k}{I}}\right)x^{-1} cos(\sqrt{-\frac{1}{4}+\frac{k}{I}}\, ln\, x) - \frac{1}{4}\alpha x^{-\frac{1}{2}} sin(\sqrt{-\frac{1}{4}+\frac{k}{I}}\, ln\, x) .$$

$$y' = \alpha + \sqrt{-\frac{1}{4}+\frac{k}{I}}\,\alpha x^{-\frac{1}{2}} sin(\sqrt{-\frac{1}{4}+\frac{k}{I}}\, ln\, x) - \frac{1}{2}\alpha\, x^{-\frac{1}{2}} cos\left(\sqrt{-\frac{1}{4}+\frac{k}{I}}\, ln\, x\right) -$$

$\frac{1}{2}\left(\sqrt{-\frac{1}{4}+\frac{k}{I}}\right)\alpha x^{-\frac{1}{2}}cos(\sqrt{-\frac{1}{4}+\frac{k}{I}}\ln x) -$
$\frac{1}{4}\alpha x^{-\frac{1}{2}}sin(\sqrt{-\frac{1}{4}+\frac{k}{I}}\ln x)$.

$y'' = \left(-\frac{1}{4}+\frac{k}{I}\right)\alpha x^{-\frac{1}{2}}x^{-1}\cos\left(\sqrt{-\frac{1}{4}+\frac{k}{I}}\ln x\right) -$
$\frac{1}{2}\left(\sqrt{-\frac{1}{4}+\frac{k}{I}}\right)\alpha x^{\frac{-3}{2}} -$
$\frac{1}{2}\left(\sqrt{-\frac{1}{4}+\frac{k}{I}}\right)\alpha x^{\frac{-1}{2}}x^{-1}(-)\sin\left(\sqrt{-\frac{1}{4}+\frac{k}{I}}\ln x\right) +$
$\frac{1}{4}\alpha x^{\frac{-3}{2}}\cos\left(\sqrt{-\frac{1}{4}+\frac{k}{I}}\ln x\right) - \frac{1}{2}\Big(-\frac{1}{4}+$
$\frac{k}{I}\Big)\alpha x^{\frac{-1}{2}}x^{-1}(-)\sin\left(\sqrt{-\frac{1}{4}+\frac{k}{I}}\ln x\right) +$
$\frac{1}{4}\left(\sqrt{-\frac{1}{4}+\frac{k}{I}}\right)x^{\frac{-3}{2}}\cos\left(\sqrt{-\frac{1}{4}+\frac{k}{I}}\ln x\right) -$
$\frac{1}{4}\alpha\left(\sqrt{-\frac{1}{4}+\frac{k}{I}}\right)x^{\frac{-1}{2}}x^{-1}\cos\left(\sqrt{-\frac{1}{4}+\frac{k}{I}}\ln x\right) +$
$\frac{1}{8}\alpha x^{\frac{-3}{2}}\sin\left(\sqrt{-\frac{1}{4}+\frac{k}{I}}\ln x\right)$.

$y'' = \left(-\frac{1}{4}+\frac{k}{I}\right)\alpha x^{\frac{-3}{2}}\cos\left(\sqrt{-\frac{1}{4}+\frac{k}{I}}\ln x\right) -$
$\frac{1}{2}\left(\sqrt{-\frac{1}{4}+\frac{k}{I}}\right)\alpha x^{\frac{-3}{2}}\sin\left(\sqrt{-\frac{1}{4}+\frac{k}{I}}\ln x\right) +$
$\frac{1}{2}\left(\sqrt{-\frac{1}{4}+\frac{k}{I}}\right)\alpha x^{\frac{-3}{2}}\sin\left(\sqrt{-\frac{1}{4}+\frac{k}{I}}\ln x\right) +$
$\frac{1}{4}\alpha x^{\frac{-3}{2}}\cos\left(\sqrt{-\frac{1}{4}+\frac{k}{I}}\ln x\right) + \frac{1}{2}\Big(-\frac{1}{4}+$
$\frac{k}{I}\Big)\alpha x^{\frac{-3}{2}}\sin\left(\sqrt{-\frac{1}{4}+\frac{k}{I}}\ln x\right) + \frac{1}{4}\Big(-\frac{1}{4}+$
$\frac{k}{I}\Big)\alpha x^{\frac{-3}{2}}\cos\left(\sqrt{-\frac{1}{4}+\frac{k}{I}}\ln x\right) -$
$\frac{1}{4}\left(\sqrt{-\frac{1}{4}+\frac{k}{I}}\right)\alpha x^{\frac{-3}{2}}\cos\left(\sqrt{-\frac{1}{4}+\frac{k}{I}}\ln x\right) +$
$\frac{1}{8}\alpha x^{\frac{-3}{2}}\sin\left(\sqrt{-\frac{1}{4}+\frac{k}{I}}\ln x\right)$.

After performing the necessary mathematical simplifications the equation $x^2y'' + \frac{k}{I}y = \frac{k}{I}\alpha x$ is appeared and by using the original symbols, where $\theta = y$ and $t = x$ then the equation $t^2\theta'' + \frac{k}{I}\theta = \frac{k}{I}\alpha t$ will appear.

4. Conclusions

The developed missile automatic control problem is an application of the second order ordinary differential equations with variable coefficients, that can bs solved using Al-Tememe transform. The above calculations that are performed over the developed missile automatic control problem using Al-Tememe transform shows that it is possible to use Al-Tememe transform as a solving method to that problem, and a proof to the ability of Al-Tememe transform in solving the developed missile automatic control problem has also been provided.

References

[1] Dennis G. Zill, "A First Cource in Diffrential Equations with Modeling Applications",10th edition, Brooks/Cole: Boston, USA, 2011.

[2] Belkacem Said-Houari, "Differential Equations: Methods and Applications", Springer International Publishing: Switzerland, 2015.

[3] William E. Boyce, Richard C. DiPrima , "Elementary Differential Equations and Boundary Value Problems", 10th edition, Wiley: USA, 2012.

[4] L.S. Sawant, "Applications of Laplace Transform in Engineering Fields", International Research Journal of Engineering and Technology, Volume 5 Issue 5, 2018, Pages 3100-3105.

[5] Steven J. Desjardins, R´emi Vaillancourt, "Ordinary Differential Equations Laplace Transforms and Numerical Methods for Engineers", S. J. Desjardins and R. Vaillancourt: Ottawa, 2011.

[6] Elaf Sabah Abbas, Emad Kuffi, Sarah Faleh Maktoof Al Khozai, Solving an improved heat transmission measuring equation using partial differential equations with variable coefficients, International Journal of Engineering & Technology, Volume 7, Issue 4, 2018, pages 5258-5260

[7] Ali Hassan Mohammed, Mohammed Monther Neamah Kudaer, "Finding the complete solution of special types of the second order partial differential equations with variable coefficients", Journal of Kerbala University, Volume 12, Issue 4, 2014, Pages 61-69.

[8] Ali Hassan Mohammed, Ayman Mohammed Hassan, "Using AL-Tememe Transform to Solve System of Linear Second Order Ordinary Differential Equations with Variable Coefficients", Journal of Kerbala University, Volume 15, Issue 2, 2014, Pages 30-35.

[9] Ali Hassan Mohammed, Alaa Saleh Hadi, Hassan Nadem Rasoul, "Integration of the Al-Tememe Transformation To find the Inverse of Transformation and Solving Some LODEs With (I.C)". Journal of AL-Qadisiyah for computer science and mathematics, Volume 9, Issue 2, 2017; Pages 88-93.

[10] A. S., Hadi, M A. H. Mohammed, Z. M. Hussain. On Al-Temem Transform and Solving Some Kind of Ordinary Differential Equations with Initial Conditions and Without it and Some Applications in Another Sciences. A thesis of MSc. submitted to council of University of Kufa, Faculty of Education for girls. 2015.

[11] Zahir Muhsin Hussain, Ali Hassan Mohammed, "Using Al-Tememe Transform to Solve Linear Partial Differential Equations with Applications", MSc thesis submitted to the faculty of education for girls-University of Kufa, (2016).

[12] John H. Blakelock, "Automatic Control of Aircraft and Missiles", 2nd edition, Wiley: Canada, 1991.