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Research paper

On generalizations of Soft Fuzzy G_δ Semi-Closed Sets

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Abstract

In this paper, soft fuzzy G_δ semi closed set is introduced. Using soft fuzzy G_δ semi closed set, generalized soft fuzzy G_δ semi-continuous function, generalized soft fuzzy G_δ semi-connected space, generalized soft fuzzy F_σ semi-compact space are introduced and studied. In this connection, properties and characterizations are established. Also soft fuzzy T-F_σ space is introduced and studied. Examples and counter examples are provided wherever necessary.

Keywords: *$gsfG_\delta s$-closed; $gsfG_\delta s$-continuous; $gsfG_\delta s$-connected space; $gsfG_\delta s$-extremally disconnected space; $gsfF_\sigma s$-compact space.*

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1. Introduction

The fundamental concepts of uncertainity fuzzy sets was introduced by Zadeh in his classical paper[12]. Fuzzy sets have applications in many fields such as information[6] and control [7]. In mathematics, topology provided the most natural framework for the concepts of fuzzy sets to flourish. Fuzzy topological spaces are introduced and many of its applications are discussed by Chang [3]. The concept of soft fuzzy topological space is introduced by [8]. Various properties of soft fuzzy topological space was discussed by the authors D. Vidhya [9], V.Visalakshi[10], T. Yogalakshmi [11]. V.Visalakshi [9] introduced the concept of soft fuzzy $G_\delta$set. Jin Han Park and Jin Keun Park [5] introduced and studied the concepts of generalized fuzzy semi-continuous, gfs-irresolute, strongly gfs-continuous and perfectly gfs-continuous functions. He also discussed gfs-connected space, gfs-extremally disconnected space and gfs- compact space. By using the concept of generalized fuzzy semi closed sets [5] and, the concept of generalized soft fuzzy $G_\delta$semi closed set is introduced. The concepts of generalized soft fuzzy G_δ semi-continuous functions, gsfG_δs-irresolute functions, strongly gsfG_δs-continuous functions, perfectly gsfG_δs-continuous functions are introduced and interrelations among these functions are discussed with suitable examples. The concepts of gsfG_δs-connected space, gsfG_δs-extremally disconnected space, gsfF_σs-compact space and soft fuzzy T-F_σ space are established.

2. Preliminaries

Definition: 2.1[9]
Let (X, T) be any soft fuzzy topological space and (λ,M) be a soft fuzzy set in (X, T). (λ,M) is said to be soft fuzzy G_δ set if $(\lambda,M) = \bigwedge_{i=1}^{\infty} (\lambda_i,M_i)$ where each $\lambda_i \in T$, $i \in I$, $M_i \in X$. (λ,M) is said to be soft fuzzy F_σ set if $(\lambda,M) = \bigvee_{i=1}^{\infty} ((1,X)-(\lambda_i,M_i))$where each $\lambda_i \in T$, $i \in I$.

3. Soft fuzzy semi closed sets

Definition: 3.1
Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. A mapping $k : (X, T) \to (Y, S)$ is said to be soft fuzzy continuous if the inverse image of every soft fuzzy closed set in (Y, S) is soft fuzzy closed in (X, T).

Definition: 3.2
A soft fuzzy topological space X is said to be soft fuzzy connected if it has no proper soft fuzzy closed and soft fuzzy open set.

Definition: 3.3
A soft fuzzy set (λ,M) in a soft fuzzy topological space (X, T) is said be soft fuzzy semi-closed (in short, sfs-closed) if $(\lambda_1,M) \sqsupseteq$ int (cl (λ_1,M))). Afuzzy set (λ_1,M) in a soft fuzzy topological space (X, T) is said be soft fuzzy semi-open (in short, sfs-open) $(\lambda_1,M) \sqsubseteq$ cl (int (λ_1,M)).

Definition :3.4
Let(λ_1M) be any soft fuzzy set in asoft fuzzy topological space(X, T). Soft fuzzy semi closure of λ is defined as
sfscl (λ_1,M) = $\sqcap\{(\mu_1,N) / (\mu_1,N) \sqsupseteq (\lambda_1,M)$ and (μ_1,N) is sfs-closed}.

Definition: 3.5
Let(λ_1,M) be any soft fuzzy set in a soft fuzzy topological space(X, T).Soft fuzzy semi interior of (λ_1,M) is defined as

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sfsint (λ_1,M) = $\bigsqcup$ {(μ_1,N) / (μ_1,N) $\sqsubseteq$ (λ_1,M) and (μ_1,N) is sfs-open}.

Definition : 3.6
A soft fuzzy set (λ_1,M) in a soft fuzzy topological space (X, T) is called generalized soft fuzzy semi closed (in short, gsfs-closed) if sfscl (λ_1M) $\sqsubseteq$(μ_1,N) whenever (λ_1,M) $\sqsubseteq$(μ_1,N) and (μ_1,N) is soft fuzzy open.A soft fuzzy set (λ_1,M) is called generalized soft fuzzy semi open (in short, gsfs-open) if its complement (1,X)–(λ_1,M) is gsfs-closed.

Definition : 3.7
Let (X, T) and (Y, S) be any two fuzzy topological spaces. A function k : (X, T) $\rightarrow$ (Y, S) is called generalized soft fuzzy semi-continuous (in short, gsfs-continuous) if the inverse image of every soft fuzzy closed set in (Y, S) is gsfs-closed in (X, T).

Definition :.3.8
Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. A function f : (X, T) $\rightarrow$ (Y, S) is said to be strongly gsfs-continuous if the inverse image of every gsfs-open set in (Y, S) is soft fuzzy open in (X, T).

Definition : 3.9
Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. A function f : (X, T) $\rightarrow$ (Y, S) is said to be perfectly gsfs-continuous if the inverse image of every gsfs-open set in (Y, S) is both soft fuzzy open and soft fuzzy closed in (X, T).

Definition : 3.10
A soft fuzzy topological space (X, T) is said to be generalized soft fuzzy semi connected (in short, gsfs-connected) if the only soft fuzzy sets which are both gsfs-closed and gsfs-open are (0,ϕ) and (1,X).

Definition: 3.11
Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. A mapping$k$: (X, T) $\rightarrow$ (Y, S) is called soft fuzzy G_δ-irresolute ifk^{-1} (μ_1,N) is soft fuzzy G_δ set in (X, T) for each soft fuzzy G_δ set (μ_1,N)in (Y, S).

4. Generalized soft fuzzy g$_\delta$semi closed sets

Definition :4.1
A soft fuzzy set (λ_1,M) in a soft fuzzy topological space (X, T) is called generalized soft fuzzy G_δ semi closed (in short, gsfG$_\delta$s-closed) if sfscl (λ_1) $\sqsubseteq\mu_1$ whenever $\lambda_1\sqsubseteq\mu_1$ and μ_1 is soft fuzzy G_δ.
A soft fuzzy set λ_1 is called generalized soft fuzzy F_σ semi open (in short, gsfF$_\sigma$s-open) if its complement (1,X) – (λ_1,M) is gsfG$_\delta$s-closed.

Example : 4.1
Let X = [0, 1] and T = {(0,ϕ), (1,X), (λ_n,M)} where λ_n(x) = $\frac{5n^2}{7(n^2-1)}$ (n = 2, 3, ...),$\forall$ x $\in$ X, M={0,1}. Then (λ_2,M) *is gsfG$_\delta$s-closed in (X, T).*

Definition : 4.2
Let (X, T) be any soft fuzzy topological space. Let (λ,M) be any soft fuzzy set, generalized soft fuzzy G_δ semi closure of λ is defined as gsfG$_\delta$s-cl (λ_1,M) = $\bigsqcap${(μ,N) / (μ,N) $\sqsupseteq$ (λ,M) and (μ,N) is gsfG$_\delta$s-closed}.

Definition : 4.3
Let (X, T) be any soft fuzzy topological space. Let (λ_1,M) be any soft fuzzy set, generalized soft fuzzy F_σ semi interior of (λ ,M)is defined as
gsfF$_\sigma$s-int (λ_1,M) = $\bigsqcup${(μ_1,N) / (μ_1,N) $\sqsubseteq$ (λ_1,N) and (μ_1,N) is gsfF$_\sigma$s-open}.

Property : 4.1
Every gsfG$_\delta$s-closed set is gsfs-closed.

Remark : 4.1
Every gsfs-closed set need not be gsfG$_\delta$s-closed as shown in the following example.

Example : 4.2
Let X = [0,1] and T = {(0,ϕ), (1,X), (λ_n,M)} where λ_n(x) = $\frac{2n+3}{5n+1}$(n = 1, 2, 3,.....), $\forall$ x$\in$ X, M={0,1}. Then (λ_1,M)*is gsfs-closed but notgsfG$_\delta$s-closed in (X,T).*

Remark : 4.2
The intersection of any two gsfG$_\delta$s-closed sets need not be gsfG$_\delta$s-closed as seen in the following example.

Example :4.3
Let X = {a, b, c} and T = {(0,ϕ), (1,X), (λ,M)}where λ(a) = 1, λ(b) = 0, λ(c) = 0, M={a}. Define fuzzy sets (λ_1,M$_1$) and (λ_2,M$_2$) in (X, T) as follows: λ_1(a) = 1, λ_1(b) = 1, λ_1(c) = 0 ; M$_1$={a,b} andλ_2(a) = 1, λ_2(b) = 0, λ_2(c) = 1; M$_2$={a,c}. Then (λ_1,M$_1$) and (λ_2,M$_2$) *are* g*sfG$_\delta$s-closed but* (λ_1,M$_1$) $\bigsqcap$(λ_2,M$_2$) *is not* g*sfG$_\delta$s-closed in (X, T).*

Remark : 4.3
The union of any two gsfF$_\sigma$s-open sets need not be gsfF$_\sigma$s-open as seen in the following example.

Example : 4.4
Let X = {a, b, c} and T = T = {(0,ϕ), (1,X), (λ,M)} whereλ(a) = 1, λ(b) = 0, λ(c) = 0; M={a}. Define fuzzy sets λ_1 and λ_2 as follows: λ_1(a) = 0, λ_1(b) = 0, λ_1(c) = 1 ; M$_1$={a,b} ,λ_2(a) = 0, λ_2(b) = 1, λ_2(c) = 0,M$_2$={b}. Then (λ_1,M$_1$) and (λ_2,M$_2$) *are* g*sfF$_\sigma$s-open but* (λ_1,M$_1$) $\bigsqcup$ (λ_2,M$_2$) *is not* g*sfF$_\sigma$s-open in (X, T).*

Property : 4.2
Let (X,T) be any soft fuzzy topological space. A soft fuzzy set(λ,M) in (X,T) is gsfF$_\sigma$s-open if and only if (δ,N) $\sqsubseteq$ sfsint (λ,M) whenever (δ,N) is soft fuzzy F_σ in (X, T) and(δ,N) $\sqsubseteq$(λ,M).

Proof :
Assume that (λ,M) is a gsfF$_\sigma$s-open set in (X, T). Let (δ,N) be soft fuzzy F_σ in (X, T) such that (δ,N) $\sqsubseteq$ (λ,M). Then, (1,X)–(δ,N) is soft fuzzy G_δ and (1,X)–(λ,M) $\sqsubseteq$ (1,X)–(δ,N). Since (1,X)–(λ,M) is gsfG$_\delta$s-closed, sfscl (1,X)–(λ,M) $\sqsubseteq$ (1,X)–(δ,N). That is, (1,X)–sfsint (λ,M) $\sqsubseteq$ (1,X)–(δ,N). Hence (δ,N) $\sqsubseteq$ sfsint (λ,M).Conversely, suppose that (λ,M) is a soft fuzzy set such that (δ,N) $\sqsubseteq$sfsint (λ,M) whenever (δ,N) is soft fuzzy F_σ and (δ,N) $\sqsubseteq$(λ,M). Let (1,X)–(λ,M) $\sqsubseteq$(μ,F), (μ,F) is a soft fuzzy G_δ set. Then, (1,X)–(μ,F) $\sqsubseteq$(λ,M). By assumption,(1,X)–(μ,M) $\sqsubseteq$sfsint (λ,M). That is, (1,X)–sfsint (λ,M) $\sqsubseteq$(μ,F). Which implies sfscl ((1,X)–(λ,M)) $\sqsubseteq$ (μ,F). Thus (1,X)–(λ,M) is a gsfG$_\delta$s-closed set in (X, T). Hence (λ,M) is a gsfF$_\sigma$s-open set in(X, T).

Property : 4.3

Let (X,T) be any soft fuzzy topological space. Let (λ_1,M_1), (λ_2,M_2)be soft fuzzy sets in a soft fuzzy topological space (X, T). Then the following hold :

(i) If $(\lambda_1,M_1)$is gsfG$_\delta$s-closed in (X, T) and $(\lambda_1,M_1) \sqsubseteq (\lambda_2,M_2) \sqsubseteq$sfscl (λ_1,M_1), then (λ_2,M_2) is gsfG$_\delta$s-closed.

(ii) If $(\lambda_1,M_1)$is gsfF$_\sigma$s-open in (X, T) and sfsint (λ_1,M_1) $\sqsubseteq (\lambda_2,M_2) \sqsubseteq (\lambda_1,M_1)$, then (λ_2,M_2) is gsfF$_\sigma$s-open.

Property : 4.4

Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. If (λ,M) is a gsfG$_\delta$s-closed set in (X, T) and if k : (X, T) $\rightarrow$ (Y, S) is soft fuzzy G$_\delta$ irresolute and strongly sfs-closed, then f (λ,M) is gsfG$_\delta$s-closed in (Y, S).

Proof :

Let (μ,N) be any soft fuzzy G$_\delta$ set in (Y, S) such that$k$ (λ,M) $\sqsubseteq (\mu,N)$. Then $(\lambda,M) \sqsubseteq k^{-1}$(μ,N). Since (λ,M) is gsfG$_\delta$s-closed andk^{-1}(μ,N) is soft fuzzy G$_\delta$, sfscl (λ,M) $\sqsubseteq k^{-1}$(μ,N) . That is,

k (sfscl (λ,M)) $\sqsubseteq (\mu,N)$. (3.1)

Since f is strongly sfs-closed, k (sfscl (λ,M)) is soft fuzzy closed in (Y, S). Now,k (λ,M) $\sqsubseteq k$ (sfscl (λ,M)). Thus, sfscl (k (λ,M)) $\sqsubseteq$sfscl (k(sfscl (λ,M))). Since k (sfscl (λ,M)) is soft fuzzyclosed in (Y, S), sfscl (k (λ,M)) $\leq k$ (sfscl (λ,M)). By (3.1), sfscl (k (λ,M)) $\sqsubseteq (\mu,N)$.Hence k (λ,M) is gsfG$_\delta$s-closed in (Y, S).

5. Generalizations of soft fuzzy g$_\delta$ semi-continuous functions

Definition : 5.1

Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. A function k : (X, T) $\rightarrow$ (Y, S) is called generalized soft fuzzy G$_\delta$ semi-continuous (in short, gsfG$_\delta$s-continuous) if the inverse image of every soft fuzzy closed set in (Y,S) is gsfG$_\delta$s-closed in (X, T).

Property : 5.1

Let (X, T) and (Y, S) be any two soft fuzzy topological spaces and f : (X, T) $\rightarrow$ (Y, S) be gsfG$_\delta$s-continuous then f is gsfs-continuous.

Remark : 5.1

Every gsfs-continuous function need not be gsfG$_\delta$s-continuous as shown in the following example.

Example : 5.1

Let X = [0, 1], T = {(0,ϕ), (1,X), (λ_n,M)} where λ_n(x) = $\frac{2n+3}{5n+1}$ (n = 1, 2, 3, ...), $\forall$ x$\in$ X, M={0,1} and Y = {a}, S={(0,ϕ),(1,Y), (μ,Y)} where μ(a) = 0.62. Let k : (X, T) $\rightarrow$ (Y, S) be the identity function. Then$k$ is gsfs-continuous but not gsfG$_\delta$s-*continuous.* Since k^{-1}((1,Y)–(μ,Y)) is not gsfG$_\delta$s-closed in (X, T) for the soft fuzzy closed set ((1,Y)–(μ,Y)) in (Y, S).

Property : 5.2

Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. For a function$k$: (X, T) $\rightarrow$ (Y, S), the following statements are equivalent :

(a) k is gsfG$_\delta$s-continuous.

(b) The inverse image of every soft fuzzy open set in (Y, S) is gsfF$_\sigma$s-open in (X, T).

Property : 5.3

Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. If k : (X,T) $\rightarrow$ (Y,S) is gsfG$_\delta$s-continuous, then k (gsfG$_\delta$s-cl (λ,M)) $\sqsubseteq$sfcl (f (λ,M)) for any fuzzy set(λ,M) in (X, T).

Proof :

Let (λ,M) be any fuzzy set in (X, T). Then, (λ,M) $\sqsubseteq k^{-1}$(k (λ,M)) $\sqsubseteq k^{1}$(sfcl (f (λ,M))). Thus, (λ,M) $\sqsubseteq k^{-1}$(sfcl (k(λ,M))). Since k is gsfG$_\delta$s-continuous,k^{-1}(sfcl (k (λ,M))) is gsfG$_\delta$s-closed in (X, T). Now, gsfG$_\delta$s-cl (λ,M) gsfG$_\delta$s-cl (k^{1}(sfcl (k (λ,M)))). Sincek^{1}(sfcl (k (λ,M))) is gsfG$_\delta$s-closed in (X, T), gsfG$_\delta$s-cl (λ,M) $\sqsubseteq k^{-1}$(sfcl (k (λ,M))).Hence,k (gsfG$_\delta$s-cl (λ,M)) $\sqsubseteq$sfcl (f (λ,M)).

Property : 5.4

Let (X, T), (Y, S) and (Z, R) be any three soft fuzzy topologicalspaces. If k : (X, T) $\rightarrow$ (Y, S) is gsfG$_\delta$s-continuous and h : (Y, S) $\rightarrow$ (Z, R) is soft fuzzy continuous, then the composition hok : (X, T) $\rightarrow$ (Z, R) is gsfG$_\delta$s-continuous.

Remark : 5.2

Composition of two gsfG$_\delta$s-continuous functions need not be gsfG$_\delta$s-continuous as shown in the following example.

Example : 5.2

Let X = [0, 1], T = {(0,ϕ), (1,X), (λ_n,N)} where λ_n(x) = $\frac{2n+3}{5n+1}$ (n = 1, 2, 3, ...), $\forall$ x $\in$ X, N={0.5} . Y = {p}, S = {(0,ϕ), (1,Y),(μ_1,Y), (μ_2,Y)} where μ_1(p) = 0.75, μ_2(p) = 0.52. Z = {p}, R = {(0,ϕ), (1,Z),(δ,Z)} where δ(p) = 0.62. Let k : (X, T) $\rightarrow$ (Y, S) and h : (Y, S) $\rightarrow$ (Z, R) be identity functions. Then k and h are gsfG$_\delta$s-continuous, but the composition is not gsfG$_\delta$s-continuous**.** Since $(hok)^{-1}$ ((1,Z)–(δ,Z)) is not gsfG$_\delta$s-closed in (X, T) for the soft fuzzy closed set ((1,Z)–(δ,Z)) in (Z, R).

Definition : 5.2

A soft fuzzy topological space (X, T) is called soft fuzzy T-F$_\sigma$ space if every gsfF$_\sigma$s-open set is soft fuzzy open. Equivalently every gsfG$_\delta$s-closed set is soft fuzzy closed.

Property : 5.5

Let (X, T), (Y, S) and (Z, R) be any three soft fuzzy topological spaces and k : (X, T) $\rightarrow$ (Y, S) be soft fuzzy continuous and h : (Y, S) $\rightarrow$ (Z, R) be gsfG$_\delta$s-continuous. If (Y, S) is soft fuzzy T-F$_\sigma$ space, then the composition gof : (X, T) $\rightarrow$ (Z, R) is soft fuzzy continuous.

Remark : 5.3

The above Property : 5.5 is not valid if (Y, S) is not soft fuzzy T-F$_\sigma$as shown in the following example.

Example : 5.3

Let X ={a} and T={(0,ϕ), (1,X), (λ_1,X), (λ_2,X), (λ_3,X)} where λ_1(a) = 0.67, λ_2(a) = 0.20, λ_3(a) = 0.50. Y ={p} and S = {(0,ϕ), (1,Y), (μ_1,Y), (μ_2,Y)} where μ_1(p) = 0.67, μ_2(p) = 0.20. Z = {p} and R = {(0,ϕ), (1,Z),(δ,Z)} where δ(p) = 0.40. Let k : (X, T) $\rightarrow$ (Y, S) and h : (Y, S) $\rightarrow$ (Z, R) be the identity functions. Then *kis soft fuzzy continuous and h is gsfG$_\delta$s-continuous, but the composition hok is not soft fuzzy continuous*. Since $(hok)^{-1}$((1,Z)–δ,Z)) is not soft fuzzy closed in (X, T), for the soft fuzzy closed set ((1,Z)–δ,Z)) in (Z, R).

Property : 5.6
Let (X, T), (Y, S) and (Z, R) be any three soft fuzzy topological spaces and k : (X, T) → (Y, S) be gsfs-continuous and h : (Y, S) → (Z, R) be gsfG$_\delta$s-continuous. If (Y, S) is soft fuzzy T-F$_\sigma$ space, then the composition *hok* : (X, T) → (Z, R) is gsfs-continuous.

Remark : 5.4
The above Property : 5.6 is not valid if (Y, S) is not soft fuzzy T-F$_\sigma$as shown in the following example.

Example : 5.4
Let X ={a, b, c} and T_1 ={(0,ϕ), (1,X), (λ_1,N)}, T_2 = {((0,ϕ), (1,X),(λ_2,M), (λ_3,P)}, T_3 ={(0,ϕ), (1,X),(λ_4,R)} where λ_1(a) =1,λ_1(b)=1, λ_1(c)= 0, N={a, b}; λ_2(a)= 0, λ_2(b) = 1, λ_2(c) = 1, M={b, c}; λ_3(a)= 1, λ_3(b) = 0, λ_3(c) = 0, P={a}; λ_4(a) = 1,λ_4(b) = 0, λ_4(c) = 1, R={a,c}. Let *f* : (X, T_1) → (X, T_2) be a function defined by *f* (a)= *f* (c)=c, *f* (b) = b and *g* : (X, T_2) → (X, T_3) be the identity function. Then *k is gsfs-continuous and h is gsfG$_\delta$s-continuous*. But the composition *hok is not gsfs-continuous***.** Since $(hok)^{-1}$ ((1,X)–(λ_4,R)) is not gsfs-closed in (X, T_1), for the soft fuzzy closed set ((1,X)–(λ_4,R)) in(X, T_3).

Definition : 5.3
Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. A function *k* : (X, T) → (Y, S) is called gsfG$_\delta$s-irresolute if the inverse image of every gsfG$_\delta$s-closed set in (Y, S) is gsfG$_\delta$s-closed in (X, T).

Property : 5.7
Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. If *k* : (X, T) → (Y, S) is gsfG$_\delta$s-irresolute then *k* is gsfG$_\delta$s-continuous.

Remark : 5.5
Every gsfG$_\delta$s-continuous function need not be gsfG$_\delta$s-irresolute as shown in the following example.

Example : 5.5
Let X = [0, 1], T = {(0,ϕ), (1,X), (λ_n,N)} where $\lambda_n(x) = \frac{2n+3}{5n+1}$ (n = 1, 2, 3, ...), ∀ x ∈ X, N={1}. Y = {p}, S = {(0,ϕ), (1,Y), (μ_1,Y), (μ_2,Y)} where μ_1(p) = 0.64and μ_2(p) = 0.52. Let k : (X, T) → (Y, S) be the identity function. Then *k is gsfG$_\delta$s-continuous but not gsfG$_\delta$s-irresolute*, for the soft fuzzy set (λ,Y) defined by λ(p) = 0.38 which is gsfG$_\delta$s-closed in (Y, S) butf^{-1} (λ,Y) is not gsfG$_\delta$s-closed in (X, T).

Property : 5.8
Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. For a function *k* : (X, T) → (Y, S), the following statements are equivalent :
(a) *k* is gsfG$_\delta$s-irresolute.
(b) The inverse image of every gsfF$_\sigma$s-open set in (Y, S) is gsfF$_\sigma$s-open in (X, T).

Property : 5.9
Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. If *k* : (X, T)→(Y, S) is gsfG$_\delta$s-irresolute, then *k* (gsfG$_\delta$s-cl (λ,M)) ⊑ gsfG$_\delta$s-cl (*k* (λ,M)) for all (λ,M) in (X, T).

Property : 5.10
Let (X, T), (Y, S) and (Z, R) be any three soft fuzzy topological spaces and *k :* (X, T) → (Y, S), *h*:(Y, S) → (Z, R) be functions.
(i) If *k* and h are gsfG$_\delta$s-irresolute, then the composition *hok* is gsfG$_\delta$s-irresolute.
(ii) If k is gsfG$_\delta$s-irresolute and *h* is gsfG$_\delta$s-continuous then the composition *hok* is gsfG$_\delta$s-continuous.

Definition : 5.4
Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. A function *k*: (X, T) → (Y, S) is said to be strongly gsfG$_\delta$s-continuous if the inverse image of every gsfG$_\delta$s-closed set in (Y, S) is soft fuzzy closed in (X, T).

Definition : 5.5
Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. A function *k* : (X, T) → (Y, S) is said to be perfectly gsfG$_\delta$s-continuous if the inverse image of every gsfG$_\delta$s-closed set in (Y, S) is both soft fuzzy open and soft fuzzy closed in (X, T).

Property : 5.11
Let (X, T) and (Y, S) be any two soft fuzzy topological spaces and k : (X, T) → (Y, S) be strongly gsfs-continuous then *f* is strongly gsfG$_\delta$s-continuous.

Remark : 5.6
Strongly gsfG$_\delta$s-continuous function need not be strongly gsfs-continuous as shown in the following example.

Example : 5.6
Let X = [0, 1], T_1 = {(0,ϕ), (1,X), (λ,M)} where 0.64 ≤ λ(x) ≤ 1, M={1} and T_2 = {(0,ϕ), (1,X), (λ_n,N)} where $\lambda_n(x) = \frac{2n+3}{5n+1}$ (n = 2, 3, ...), ∀ x ∈ X, N={1}. Let *f* : (X, T_1) → (X, T_2) be the identity function. Then *k is strongly gsfG$_\delta$s-continuous but not strongly gsfs-continuous***,** for the soft fuzzy set (μ,L) defined by μ (x) = 0.55, L={0,1} which is gsfs-closed in (X, T_2) but k^{-1} (μ,L) is not soft fuzzy closed in (X, T_1).

Property : 5.12
Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. A function k : (X, T) → (Y, S) is strongly gsfG$_\delta$s-continuous if and only if the inverse image of every gsfF$_\sigma$s-open set in (Y, S) is soft fuzzy open in (X, T).

Property : 5.13
Let (X, T), (Y, S) and (Z, R) be any three soft fuzzy topological spaces and*k* : (X, T) → (Y, S) , *h* : (Y, S) → (Z, R) be the functions.
(i) If *k* is strongly gsfG$_\delta$s-continuous, *h* is gsfG$_\delta$s-continuous then *gof* is soft fuzzy continuous.
(ii) If *k*is strongly gsfG$_\delta$s-continuous, *h* is gsfG$_\delta$s-irresolute then *hok* is strongly gsfG$_\delta$s-continuous.

Property : 5.14
Let (X, T) and (Y, S) be any two soft fuzzy topological spaces and *k* : (X, T) → (Y, S).
(i) If *k* is perfectly gsfs-continuous then *k* is perfectly gsfG$_\delta$s -continuous.
(ii) If *k* is perfectly gsfG$_\delta$s-continuous then *k* is strongly gsfG$_\delta$s-continuous.

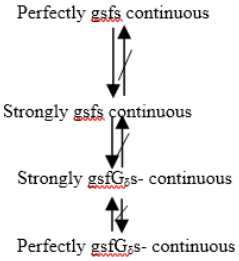
Remark : 5.7
Every **s**trongly gsfG$_\delta$s-continuous function need not be perfectly gsfG$_\delta$s-continuous as shown in the following example.

Example : 5.7
Let X = {a, b, c}, T = {(0,ϕ), (1,X), (λ,M)} and Y = {p, q, r} , S = {(0,ϕ), (1,Y), (μ_1,N_1), (μ_2,N_2)} where λ(a) = 0, λ(b) = 0, λ(c) = 1, M={c}; μ_1(p) = 0, μ_1(q) = 0, μ_1(r) = 1, N_1={r}; μ_2(p) = 1, μ_2(q) = 1, μ_2(r) = 0, N_2={p,q}. Let *k*: (X, T) → (Y, S) be defined by k(a) = p,*k* (b) = p, k(c) = q. Then *k is strongly gsfG$_\delta$s-continuous but not perfectly gsfG$_\delta$s-continuous.*

Property : 5.15
Let (X, T) and (Y, S) be any two soft fuzzy topolocical spaces. A function k : (X, T) → (Y, S) is perfectly gsfG$_\delta$s-continuous if and only if the inverse image of every gsfF$_\sigma$s-open set in (Y, S) is both soft fuzzy open and soft fuzzy closed in (X, T).

Property : 5.16
Let (X, T), (Y, S) and (Z, R) be any three soft fuzzy topological spaces and *k* : (X, T) → (Y, S), *h* : (Y, S) →(Z, R) be the functions. If *k* is perfectly gsfG$_\delta$s-continuous, *h* is gsfG$_\delta$s-irresolute then *hok* is perfectly gsfG$_\delta$s-continuous.

Remark : 5.8
From the results proved above, the following implications are obtained :



6. Generalized soft fuzzy g$_\delta$semi-connected space

Definition : 6.1
A soft fuzzy topological space (X, T) is said to be generalized soft fuzzy G$_\delta$ semi connected (in short, gsfG$_\delta$s-connected) if the only soft fuzzy sets which are both gsfG$_\delta$s-closed and gsfF$_\sigma$s-open are (0,ϕ) and (1,X).

Example : 6.1
Let X = [0, 1] and T = {(0,ϕ), (1,X), (λ_n,M)} where $\lambda_n(x) = \frac{5n^2}{7(n^2-1)}$ (n = 2, 3, ...),∀ x ∈ X, M={0,1}. There exists no proper soft fuzzy set in (X, T)which is both gsfG$_\delta$s-closed and gsfF$_\sigma$s-open**.** Hence *(X, T) is gsfG$_\delta$s-connected.*

Property :6.1
Every gsfs-connected space is gsfG$_\delta$s-connected.

Proof :
Let (X, T) be a gsfs-connected space and suppose that (X, T) is not gsfG$_\delta$s-connected. Then there exists a proper soft fuzzy set (λ,M) of (X, T) which is both gsfF$_\sigma$s-open and gsfG$_\delta$s-closed. Since every gsfF$_\sigma$s-open set is gsfs-open and every gsfG$_\delta$s-closed set is gsfs-closed, (λ,M) is both gsfs-closed and gsfs-open. Hence (X, T) is not gsfs-connected. Contradiction. Hence every gsfs-connected space is gsfG$_\delta$s-connected.

Property : 6.2
Let (X, T) and (Y, S) be any two soft fuzzy topological spaces and k :(X,T) → (Y,S)

(i) If k is gsfG$_\delta$s-continuous surjection and (X, T) is gsfG$_\delta$s-connected, then (Y, S) is soft fuzzy connected.
(ii) If *k* is gsfG$_\delta$s-irresolute surjection and (X, T) is gsfG$_\delta$s-connected, then (Y, S) is gsfG$_\delta$s-connected.
(iii) If *k* is strongly gsfG$_\delta$s-continuous surjection and (X, T) is soft fuzzy connected, then (Y, S) is gsfG$_\delta$s-connected.

Property : 6.3
Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. If *k* : (X, T) → (Y, S) is gsfG$_\delta$s-irresolute surjection, (X, T) is soft fuzzy T-F$_\sigma$ and soft fuzzy connected, then (Y, S) is gsfG$_\delta$s-connected.

Proof :
Suppose that (Y, S) is not gsfG$_\delta$s-connected. Then there exists proper soft fuzzy set (λ,M) which is both gsfG$_\delta$s-closed and gsfF$_\sigma$s-open in (Y, S). Since k is gsfG$_\delta$s-irresolute surjection, f^{-1} (λ,M) is both gsfG$_\delta$s-closed and gsfF$_\sigma$s-open in (X, T). Since (X, T) is soft fuzzy T-F$_\sigma$, k^{-1} (λ,M) is both soft fuzzy closed and soft fuzzy open in (X, T). Thus (X, T) is not soft fuzzy connected. Contradiction. Hence (Y, S) is gsfG$_\delta$s-connected.

7. Generalized soft fuzzy g$_\delta$ semi-extremally disconnected space and generalized soft fuzzy f$_\sigma$semi-compact space

Definition : 7.1
A soft fuzzy topological space (X, T) is said to be generalized soft fuzzy G$_\delta$ semi extremally disconnected (in short, gsfG$_\delta$s-extremally disconnected) if gsfG$_\delta$s-cl (λ,M) is gsfF$_\sigma$s-open, whenever (λ,M) is gsfF$_\sigma$s-open.

Property : 7.1
Let (X, T) be any soft fuzzy topological space and (X, T) be gsfG$_\delta$s-extremally disconnected space. Then the following statements are hold.

(i) For each gsfG$_\delta$s-closed set (λ,M), gsfF$_\sigma$s-int (λ,M) is gsfG$_\delta$s-closed.
(ii) For each gsfF$_\sigma$s-open set (λ,M).

gsfG$_\delta$s-cl(λ,M) + gsfG$_\delta$s-cl((1,X)– gsfG$_\delta$s-cl(λ,M)) = (1,X).

Proof :
(i) Let (λ,M) be any gsfG$_\delta$s-closed set in (X, T). Then (1,X)–(λ,M) is gsfF$_\sigma$s-open in (X, T). Since (X, T) is gsfG$_\delta$s-extremally disconnected space, gsfG$_\delta$s-cl ((1,X)–(λ,M)) is gsfF$_\sigma$s-open. Now, gsfG$_\delta$s-cl ((1,X)–(λ,M)) = (1,X)– gsfF$_\sigma$s-int (λ,M) is gsfF$_\sigma$s-open . Hence gsfF$_\sigma$s-int (λ,M) is gsfG$_\delta$s-closed.
(ii) Let (λ,M) be any gsfF$_\sigma$s-open set in (X, T). Since (1,X)–gsfG$_\delta$s-cl(λ,M)= gsfF$_\sigma$s-int((1,X)–(λ,M)),
gsfG$_\delta$s-cl ((1,X)– gsfG$_\delta$s - cl (λ,M)) = gsfG$_\delta$s - cl (gsfF$_\sigma$s -int ((1,X)–(λ,M)). Thus,
gsfG$_\delta$s-cl(λ,M)+ gsfG$_\delta$s-cl ((1,X)– gsfG$_\delta$s-cl(λ,M))=
gsfG$_\delta$s-cl(λ,M)+ gsfG$_\delta$s-cl(gsfF$_\sigma$s-int((1,X)–(λ,M)).
Since (λ,M) is gsfF$_\sigma$s-open, (1,X)–(λ,M) is gsfG$_\delta$s-closed. By (i), gsfF$_\sigma$s-int((1,X)–(λ,M)) is gsfG$_\delta$s-closed. Hence, gsfG$_\delta$s-cl(gsfF$_\sigma$s-int ((1,X)–(λ,M)))= gsfF$_\sigma$s-int ((1,X)–(λ,M)).
Therefore,
gsfG$_\delta$s-cl(λ,M)+ gsfG$_\delta$s-cl ((1,X)– gsfG$_\delta$s-cl(λ,M))
= gsfG$_\delta$s-cl (λ,M)+ gsfF$_\sigma$s-int ((1,X)–(λ,M)).
= gsfG$_\delta$s-cl(λ,M) +(1,X)– gsfG$_\delta$s-cl(λ,M).
= (1,X).
Hence, gsfG$_\delta$s-cl(λ,M) + gsfG$_\delta$s-cl ((1,X)– gsfG$_\delta$s-cl (λ,M)) =(1,X).

Definition : 7.2
A collection $\{(\lambda_i,M_i)\}_{i\in J}$ of soft fuzzy sets of a soft fuzzy topological space (X, T) is called gsfF$_\sigma$s- open cover of (X, T), if (λ_i,M_i)′s (i∈J) are gsfF$_\sigma$s-open sets of (X, T).

Definition : 7.3
A soft fuzzy topological space (X, T) is called gsfF$_\sigma$s-compact if every gsfF$_\sigma$s-open cover of (X, T) has a finite subcover.

Property : 7.2
Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. If *k* : (X, T) → (Y, S) is gsfG$_\delta$s-continuous bijective function and (X, T) is gsfF$_\sigma$s-compact, then (Y, S) is soft fuzzy compact.

Proof :

Let $\{(\lambda_i,M_i)\}_{i\in J}$ be a collection of soft fuzzy open sets in (Y, S) such that

$$(1,Y) \sqsubseteq \bigsqcup_{i=1}^{n} \{(\lambda_i,M_i)\} \quad (6.4)$$

Since f is gsfG$_\delta$s-continuous bijective function and each $(\lambda_i,M_i)$is soft fuzzy open in (Y, S), $f^{-1}(\lambda_i,M_i)$ is gsfF$_\sigma$s-open in (X, T). From (6.4), $(1,X) \sqsubseteq \bigsqcup_{i=1}^{n} f^{-1}(\lambda_i,M_i)$. Now, $\{ f^{-1}(\lambda_i,M_i)\}_{i\in J}$ is a gsfF$_\sigma$s-open cover of (X, T). Since (X, T) is gsfF$_\sigma$s-compact, there exists a finite subset F of J such that $(1,X) \sqsubseteq \bigsqcup_{i=1}^{n} f^{-1}(\lambda_i,M_i)$. Then, f $(1,X) \sqsubseteq \bigsqcup_{i=1}^{n} (\lambda_i,M_i)$.That is, $(1,Y) \sqsubseteq \bigsqcup_{i=1}^{n} (\lambda_i,M_i)$. Hence (Y, S) is soft fuzzy compact.

Property : 7.3

Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. If k : (X, T) $\rightarrow$ (Y, S) is gsfG$_\delta$s-irresolute bijective function and (X, T) is gsfF$_\sigma$s-compact, then (Y, S) is gsfF$_\sigma$s-compact.

Property : 7.4

Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. If k: (X, T) $\rightarrow$ (Y, S) is strongly gsfG$_\delta$s-continuous bijective function and (X, T) is soft fuzzy compact, then (Y, S) is gsfF$_\sigma$s-compact.

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