International Journal of Engineering & Technology, 7 (4.10) (2018) 1105-1110

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

On generalizations of Soft Fuzzy G_δ Semi-Closed Sets

V. Visalakshi[1], D.Vidhya[2]*, T.Yogalakshmi[3]

[1] Department of Mathematics, SRM Institute of Science and Technology, Kattankulathur,Chennai.
[2] Department of Mathematics, School of Advanced Sciences, Kalasalingam Academy of Research and Education, Krishnan Koil.
[3] Department of Mathematics, School of Advanced Sciences, Vellore Institute of Technology, Vellore.
**Corresponding author E-mail: vidhya.d85@gmail.com.*

Abstract

In this paper, soft fuzzy G_δ semi closed set is introduced. Using soft fuzzy G_δ semi closed set, generalized soft fuzzy G_δ semi-continuous function, generalized soft fuzzy G_δ semi-connected space, generalized soft fuzzy F_σ semi-compact space are introduced and studied. In this connection, properties and characterizations are established. Also soft fuzzy T-F_σ space is introduced and studied. Examples and counter examples are provided wherever necessary.

Keywords: *gsfG_δs-closed; gsfG_δs-continuous; gsfG_δs-connected space; gsfG_δs-extremally disconnected space; gsfF_σs-compact space.*

AMS MSC2010: *54A40, 03E72*.

1. Introduction

The fundamental concepts of uncertainity fuzzy sets was introduced by Zadeh in his classical paper[12]. Fuzzy sets have applications in many fields such as information[6] and control [7]. In mathematics, topology provided the most natural framework for the concepts of fuzzy sets to flourish. Fuzzy topological spaces are introduced and many of its applications are discussed by Chang [3]. The concept of soft fuzzy topological space is introduced by [8]. Various properties of soft fuzzy topological space was discussed by the authors D. Vidhya [9], V.Visalakshi[10], T. Yogalakshmi [11]. V.Visalakshi [9] introduced the concept of soft fuzzy $G_\delta$set. Jin Han Park and Jin Keun Park [5] introduced and studied the concepts of generalized fuzzy semi-continuous, gfs-irresolute, strongly gfs-continuous and perfectly gfs-continuous functions. He also discussed gfs-connected space, gfs-extremally disconnected space and gfs- compact space. By using the concept of generalized fuzzy semi closed sets [5] and, the concept of generalized soft fuzzy $G_\delta$semi closed set is introduced. The concepts of generalized soft fuzzy G_δ semi-continuous functions, gsfG_δs-irresolute functions, strongly gsfG_δs-continuous functions, perfectly gsfG_δs-continuous functions are introduced and interrelations among these functions are discussed with suitable examples. The concepts of gsfG_δs-connected space, gsfG_δs-extremally disconnected space, gsfF_σs-compact space and soft fuzzy T-F_σ space are established.

2. Preliminaries

Definition: 2.1[9]
Let (X, T) be any soft fuzzy topological space and (λ,M) be a soft fuzzy set in (X, T). (λ,M) is said to be soft fuzzy G_δ set if (λ,M) = $\bigwedge_{i=1}^{\infty} (\lambda_i,M_i)$ where each $\lambda_i \in T$, $i\in I$, $M_i \in X$. (λ,M) is said to be soft fuzzy F_σ set if (λ,M) = $\bigvee_{i=1}^{\infty} ((1,X)-(\lambda_i,M_i))$where each $\lambda_i \in T$, $i\in I$.

3. Soft fuzzy semi closed sets

Definition: 3.1
Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. A mapping *k* : (X, T) → (Y, S) is said to be soft fuzzy continuous if the inverse image of every soft fuzzy closed set in (Y, S) is soft fuzzy closed in (X, T).

Definition: 3.2
A soft fuzzy topological space X is said to be soft fuzzy connected if it has no proper soft fuzzy closed and soft fuzzy open set.

Definition: 3.3
A soft fuzzy set (λ,M) in a soft fuzzy topological space (X, T) is said be soft fuzzy semi-closed (in short, sfs-closed) if $(\lambda_1,M) \sqsupseteq$ int (cl (λ_1,M))). Afuzzy set (λ_1,M) in a soft fuzzy topological space (X, T) is said be soft fuzzy semi-open (in short, sfs-open) $(\lambda_1,M) \sqsubseteq$ cl (int (λ_1,M)).

Definition :3.4
Let(λ_1M) be any soft fuzzy set in asoft fuzzy topological space(X, T). Soft fuzzy semi closure of λ is defined as
sfscl (λ_1,M) = $\sqcap\{(\mu_1,N) / (\mu_1,N) \sqsupseteq (\lambda_1,M)$ and (μ_1,N) is sfs-closed}.

Definition: 3.5
Let(λ_1,M) be any soft fuzzy set in a soft fuzzy topological space(X, T).Soft fuzzy semi interior of (λ_1,M) is defined as

sfsint (λ_1,M) = $\sqcup$ {(μ_1,N) / (μ_1,N) $\sqsubseteq$ (λ_1,M) and (μ_1,N) is sfs-open}.

Definition : 3.6
A soft fuzzy set (λ_1,M) in a soft fuzzy topological space (X, T) is called generalized soft fuzzy semi closed (in short, gsfs-closed) if sfscl (λ_1M) $\sqsubseteq$(μ_1,N) whenever (λ_1,M) $\sqsubseteq$(μ_1,N) and (μ_1,N) is soft fuzzy open.A soft fuzzy set (λ_1,M) is called generalized soft fuzzy semi open (in short, gsfs-open) if its complement (1,X)–(λ_1,M) is gsfs-closed.

Definition : 3.7
Let (X, T) and (Y, S) be any two fuzzy topological spaces. A function *k* : (X, T) $\to$ (Y, S) is called generalized soft fuzzy semi-continuous (in short, gsfs-continuous) if the inverse image of every soft fuzzy closed set in (Y, S) is gsfs-closed in (X, T).

Definition :.3.8
Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. A function *f* : (X, T) $\to$ (Y, S) is said to be strongly gsfs-continuous if the inverse image of every gsfs-open set in (Y, S) is soft fuzzy open in (X, T).

Definition : 3.9
Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. A function *f* : (X, T) $\to$ (Y, S) is said to be perfectly gsfs-continuous if the inverse image of every gsfs-open set in (Y, S) is both soft fuzzy open and soft fuzzy closed in (X, T).

Definition : 3.10
A soft fuzzy topological space (X, T) is said to be generalized soft fuzzy semi connected (in short, gsfs-connected) if the only soft fuzzy sets which are both gsfs-closed and gsfs-open are (0,ϕ) and (1,X).

Definition: 3.11
Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. A mapping*k* : (X, T) $\to$ (Y, S) is called soft fuzzy G_δ-irresolute ifk^{-1} (μ_1,N) is soft fuzzy G_δ set in (X, T) for each soft fuzzy G_δ set (μ_1,N)in (Y, S).

4. Generalized soft fuzzy g$_\delta$semi closed sets

Definition :4.1
A soft fuzzy set (λ_1,M) in a soft fuzzy topological space (X, T) is called generalized soft fuzzy G_δ semi closed (in short, gsfG_δs-closed) if sfscl (λ_1) $\sqsubseteq\mu_1$ whenever $\lambda_1\sqsubseteq\mu_1$ and μ_1 is soft fuzzy G_δ.

A soft fuzzy set λ_1 is called generalized soft fuzzy F_σ semi open (in short, gsfF_σs-open) if its complement (1,X) – (λ_1,M) is gsfG_δs-closed.

Example : 4.1
Let X = [0, 1] and T = {(0,ϕ), (1,X), (λ_n,M)} where λ_n(x) = $\frac{5n^2}{7(n^2-1)}$ (n = 2, 3, ...),$\forall$ x $\in$ X, M={0,1}. Then (λ_2,M) *is gsfG$_\delta$s-closed in (X, T).*

Definition : 4.2
Let (X, T) be any soft fuzzy topological space. Let (λ,M) be any soft fuzzy set, generalized soft fuzzy G_δ semi closure of λ is defined as gsfG_δs-cl (λ_1,M) = $\sqcap${(μ,N) / (μ,N) $\sqsupseteq$ (λ,M) and (μ,N) is gsfG_δs-closed}.

Definition : 4.3
Let (X, T) be any soft fuzzy topological space. Let (λ_1,M) be any soft fuzzy set, generalized soft fuzzy F_σ semi interior of (λ ,M)is defined as
gsfF_σs-int (λ_1,M) = $\sqcup${(μ_1,N) / (μ_1,N) $\sqsubseteq$ (λ_1,N) and (μ_1,N) is gsfF_σs-open}.

Property : 4.1
Every gsfG_δs-closed set is gsfs-closed.

Remark : 4.1
Every gsfs-closed set need not be gsfG_δs-closed as shown in the following example.

Example : 4.2
Let X = [0,1] and T = {(0,ϕ), (1,X), (λ_n,M)} where λ_n(x) = $\frac{2n+3}{5n+1}$(n = 1, 2, 3,......), $\forall$ x$\in$ X, M={0,1}. Then (λ_1,M)*is gsfs-closed but notgsfG$_\delta$s-closed in (X,T).*

Remark : 4.2
The intersection of any two gsfG_δs-closed sets need not be gsfG_δs-closed as seen in the following example.

Example :4.3
Let X = {a, b, c} and T = {(0,ϕ), (1,X), (λ,M)}where λ(a) = 1, λ(b) = 0, λ(c) = 0, M={a}. Define fuzzy sets (λ_1,M_1) and (λ_2,M_2) in (X, T) as follows: λ_1(a) = 1, λ_1(b) = 1, λ_1(c) = 0 ; M_1={a,b} andλ_2(a) = 1, λ_2(b) = 0, λ_2(c) = 1; M_2={a,c}. Then (λ_1,M_1) and (λ_2,M_2) *are gsfG$_\delta$s-closed but* (λ_1,M_1) $\sqcap$(λ_2,M_2) *is not gsfG$_\delta$s-closed in (X, T).*

Remark : 4.3
The union of any two gsfF_σs-open sets need not be gsfF_σs-open as seen in the following example.

Example : 4.4
Let X = {a, b, c} and T = T = {(0,ϕ), (1,X), (λ,M)} whereλ(a) = 1, λ(b) = 0, λ(c) = 0; M={a}. Define fuzzy sets λ_1 and λ_2 as follows: λ_1(a) = 0, λ_1(b) = 0, λ_1(c) = 1 ; M_1={a,b} ,λ_2(a) = 0, λ_2(b) = 1, λ_2(c) = 0,M_2={b}. Then (λ_1,M_1) and (λ_2,M_2) *are gsfF$_\sigma$s-open but* (λ_1,M_1) $\sqcup$ (λ_2,M_2) *is not gsfF$_\sigma$s-open in (X, T).*

Property : 4.2
Let (X,T) be any soft fuzzy topological space. A soft fuzzy set(λ,M) in (X,T) is gsfF_σs-open if and only if (δ,N) $\sqsubseteq$sfsint (λ,M) whenever (δ,N) is soft fuzzy F_σ in (X, T) and(δ,N) $\sqsubseteq$(λ,M).

Proof :
Assume that (λ,M) is a gsfF_σs-open set in (X, T). Let (δ,N) be soft fuzzy F_σ in (X, T) such that (δ,N) $\sqsubseteq$ (λ,M). Then, (1,X)–(δ,N) is soft fuzzy G_δ and (1,X)–(λ,M) $\sqsubseteq$ (1,X)–(δ,N). Since (1,X)–(λ,M) is gsfG_δs-closed, sfscl (1,X)–(λ,M) $\sqsubseteq$ (1,X)–(δ,N). That is, (1,X)–sfsint (λ,M) $\sqsubseteq$ (1,X)–(δ,N). Hence (δ,N) $\sqsubseteq$ sfsint (λ,M).Conversely, suppose that (λ,M) is a soft fuzzy set such that (δ,N) $\sqsubseteq$sfsint (λ,M) whenever (δ,N) is soft fuzzy F_σ and (δ,N) $\sqsubseteq$(λ,M). Let (1,X)–(λ,M) $\sqsubseteq$(μ,F), (μ,F) is a soft fuzzy G_δ set. Then, (1,X)–(μ,F) $\sqsubseteq$(λ,M). By assumption,(1,X)–(μ,M) $\sqsubseteq$sfsint (λ,M). That is, (1,X)–sfsint (λ,M) $\sqsubseteq$(μ,F). Which implies sfscl ((1,X)–(λ,M)) $\sqsubseteq$ (μ,F). Thus (1,X)–(λ,M) is a gsfG_δs-closed set in (X, T). Hence (λ,M) is a gsfF_σs-open set in(X, T).

Property : 4.3

Let (X,T) be any soft fuzzy topological space. Let (λ_1,M_1), (λ_2,M_2)be soft fuzzy sets in a soft fuzzy topological space (X, T). Then the following hold :

(i) If $(\lambda_1,M_1)$is gsfG_δs-closed in (X, T) and $(\lambda_1,M_1) \sqsubseteq (\lambda_2,M_2) \sqsubseteq$sfscl (λ_1,M_1), then (λ_2,M_2) is gsfG_δs-closed.

(ii) If $(\lambda_1,M_1)$is gsfF_σs-open in (X, T) and sfsint $(\lambda_1,M_1) \sqsubseteq (\lambda_2,M_2) \sqsubseteq (\lambda_1,M_1)$, then (λ_2,M_2) is gsfF_σs-open.

Property : 4.4

Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. If (λ,M) is a gsfG_δs-closed set in (X, T) and if $k : (X, T) \to (Y, S)$ is soft fuzzy G_δ irresolute and strongly sfs-closed, then $f(\lambda,M)$ is gsfG_δs-closed in (Y, S).

Proof :

Let (μ,N) be any soft fuzzy G_δ set in (Y, S) such that$k(\lambda,M) \sqsubseteq (\mu,N)$. Then $(\lambda,M) \sqsubseteq k^{-1}(\mu,N)$. Since (λ,M) is gsfG_δs-closed and$k^{-1}(\mu,N)$ is soft fuzzy G_δ, sfscl $(\lambda,M) \sqsubseteq k^{-1}(\mu,N)$. That is, k (sfscl $(\lambda,M)) \sqsubseteq (\mu,N)$. (3.1)

Since f is strongly sfs-closed, k (sfscl (λ,M)) is soft fuzzy closed in (Y, S). Now,$k(\lambda,M) \sqsubseteq k$ (sfscl (λ,M)). Thus, sfscl ($k(\lambda,M)) \sqsubseteq$sfscl ($k$(sfscl (λ,M))). Since k (sfscl (λ,M)) is soft fuzzyclosed in (Y, S), sfscl ($k(\lambda,M)) \le k$ (sfscl (λ,M)). By (3.1), sfscl ($k(\lambda,M)) \sqsubseteq (\mu,N)$.Hence $k(\lambda,M)$ is gsfG_δs-closed in (Y, S).

5. Generalizations of soft fuzzy g_δ semi-continuous functions

Definition : 5.1

Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. A function $k : (X, T) \to (Y, S)$ is called generalized soft fuzzy G_δ semi-continuous (in short, gsfG_δs-continuous) if the inverse image of every soft fuzzy closed set in (Y,S) is gsfG_δs-closed in (X, T).

Property : 5.1

Let (X, T) and (Y, S) be any two soft fuzzy topological spaces and $f : (X, T) \to (Y, S)$ be gsfG_δs-continuous then f is gsfs-continuous.

Remark : 5.1

Every gsfs-continuous function need not be gsfG_δs-continuous as shown in the following example.

Example : 5.1

Let X = [0, 1], T = $\{(0,\phi), (1,X), (\lambda_n,M)\}$ where $\lambda_n(x) = \frac{2n+3}{5n+1}$ (n = 1, 2, 3, ...), $\forall$ x$\in$ X, M={0,1} and Y = {a}, S=$\{(0,\phi),(1,Y), (\mu,Y)\}$ where $\mu(a) = 0.62$. Let $k : (X, T) \to (Y, S)$ be the identity function. Then$k$ is gsfs-continuous but not gsfG_δs-*continuous.* Since $k^{-1}((1,Y)–(\mu,Y))$ is not gsfG_δs-closed in (X, T) for the soft fuzzy closed set $((1,Y)–(\mu,Y))$ in (Y, S).

Property : 5.2

Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. For a function$k : (X, T) \to (Y, S)$, the following statements are equivalent :

(a) k is gsfG_δs-continuous.

(b) The inverse image of every soft fuzzy open set in (Y, S) is gsfF_σs-open in (X, T).

Property : 5.3

Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. If $k : (X,T) \to (Y,S)$ is gsfG_δs-continuous, then k (gsfG_δs-cl (λ,M)) $\sqsubseteq$sfcl ($f(\lambda,M)$) for any fuzzy set(λ,M) in (X, T).

Proof :

Let (λ,M) be any fuzzy set in (X, T). Then, $(\lambda,M) \sqsubseteq k^{-1}(k(\lambda,M)) \sqsubseteq k^{1}$(sfcl ($f(\lambda,M)$)). Thus, $(\lambda,M) \sqsubseteq k^{-1}$(sfcl ($k(\lambda,M)$)). Since k is gsfG_δs-continuous,k^{-1}(sfcl ($k(\lambda,M)$)) is gsfG_δs-closed in (X, T). Now, gsfG_δs-cl (λ,M) gsfG_δs-cl (k^{1}(sfcl ($k(\lambda,M)$))). Sincek^{1}(sfcl ($k(\lambda,M)$)) is gsfG_δs-closed in (X, T), gsfG_δs-cl $(\lambda,M) \sqsubseteq k^{-1}$(sfcl ($k(\lambda,M)$)).Hence,k (gsfG_δs-cl $(\lambda,M)) \sqsubseteq$sfcl ($f(\lambda,M)$).

Property : 5.4

Let (X, T), (Y, S) and (Z, R) be any three soft fuzzy topologicalspaces. If $k : (X, T) \to (Y, S)$ is gsfG_δs-continuous and $h : (Y, S) \to (Z, R)$ is soft fuzzy continuous, then the composition $hok : (X, T) \to (Z, R)$ is gsfG_δs-continuous.

Remark : 5.2

Composition of two gsfG_δs-continuous functions need not be gsfG_δs-continuous as shown in the following example.

Example : 5.2

Let X = [0, 1], T = $\{(0,\phi), (1,X), (\lambda_n,N)\}$ where $\lambda_n(x) = \frac{2n+3}{5n+1}$ (n = 1, 2, 3, ...), $\forall$ x $\in$ X, N={0.5} . Y = {p}, S = $\{(0,\phi), (1,Y),(\mu_1,Y), (\mu_2,Y)\}$ where $\mu_1(p) = 0.75$, $\mu_2(p) = 0.52$. Z = {p}, R = $\{(0,\phi), (1,Z),(\delta,Z)\}$ where $\delta(p) = 0.62$. Let $k : (X, T) \to (Y, S)$ and $h : (Y, S) \to (Z, R)$ be identity functions. Then k and h are gsfG_δs-continuous, but the composition is not gsfG_δs-continuous**.** Since $(hok)^{-1}((1,Z)–(\delta,Z))$ is not gsfG_δs-closed in (X, T) for the soft fuzzy closed set $((1,Z)–(\delta,Z))$ in (Z, R).

Definition : 5.2

A soft fuzzy topological space (X, T) is called soft fuzzy T-F_σ space if every gsfF_σs-open set is soft fuzzy open. Equivalently every gsfG_δs-closed set is soft fuzzy closed.

Property : 5.5

Let (X, T), (Y, S) and (Z, R) be any three soft fuzzy topological spaces and $k : (X, T) \to (Y, S)$ be soft fuzzy continuous and $h : (Y, S) \to (Z, R)$ be gsfG_δs-continuous. If (Y, S) is soft fuzzy T-F_σ space, then the composition $gof : (X, T) \to (Z, R)$ is soft fuzzy continuous.

Remark : 5.3

The above Property : 5.5 is not valid if (Y, S) is not soft fuzzy T-$F_\sigma$as shown in the following example.

Example : 5.3

Let X ={a} and T=$\{(0,\phi), (1,X), (\lambda_1,X), (\lambda_2,X), (\lambda_3,X)\}$ where $\lambda_1(a) = 0.67$, $\lambda_2(a) = 0.20$, $\lambda_3(a) = 0.50$. Y ={p} and S = $\{(0,\phi), (1,Y), (\mu_1,Y), (\mu_2,Y)\}$ where $\mu_1(p) = 0.67$, $\mu_2(p) = 0.20$. Z = {p} and R = $\{(0,\phi), (1,Z),(\delta,Z)\}$ where $\delta(p) = 0.40$. Let $k : (X, T) \to (Y, S)$ and $h : (Y, S) \to (Z, R)$ be the identity functions. Then k*is soft fuzzy continuous and h is* gsfG_δ*s-continuous, but the composition hok is not soft fuzzy continuous*. Since $(hok)^{-1}((1,Z)–\delta,Z))$ is not soft fuzzy closed in (X, T), for the soft fuzzy closed set $((1,Z)–\delta,Z))$ in (Z, R).

Property : 5.6

Let (X, T), (Y, S) and (Z, R) be any three soft fuzzy topological spaces and k : (X, T) → (Y, S) be gsfs-continuous and h : (Y, S) → (Z, R) be gsfG_δs-continuous. If (Y, S) is soft fuzzy T-F_σ space, then the composition *hok* : (X, T) → (Z, R) is gsfs-continuous.

Remark : 5.4

The above Property : 5.6 is not valid if (Y, S) is not soft fuzzy T-$F_\sigma$as shown in the following example.

Example : 5.4

Let X ={a, b, c} and T_1 ={(0,ϕ), (1,X), (λ_1,N)}, T_2 = {((0,ϕ), (1,X),(λ_2,M), (λ_3,P)}, T_3 ={(0,ϕ), (1,X),(λ_4,R)} where λ_1(a) =1,λ_1(b)=1, λ_1(c)= 0, N={a, b}; λ_2(a)= 0, λ_2(b) = 1, λ_2(c) = 1, M={b, c}; λ_3(a)= 1, λ_3(b) = 0, λ_3(c) = 0, P={a}; λ_4(a) = 1,λ_4(b) = 0, λ_4(c) = 1, R={a,c}. Let *f* : (X, T_1) → (X, T_2) be a function defined by *f* (a)= *f* (c)=c, *f* (b) = b and *g* : (X, T_2) → (X, T_3) be the identity function. Then *k is* g*sfs-continuous and h is* g*sfG*$_\delta$*s-continuous*. But the composition *hok is not* g*sfs-continuous***.** Since $(hok)^{-1}$ ((1,X)–(λ_4,R)) is not gsfs-closed in (X, T_1), for the soft fuzzy closed set ((1,X)–(λ_4,R)) in(X, T_3).

Definition : 5.3

Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. A function *k* : (X, T) → (Y, S) is called gsfG_δs-irresolute if the inverse image of every gsfG_δs-closed set in (Y, S) is gsfG_δs-closed in (X, T).

Property : 5.7

Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. If *k* : (X, T) → (Y, S) is gsfG_δs-irresolute then *k* is gsfG_δs-continuous.

Remark : 5.5

Every gsfG_δs-continuous function need not be gsfG_δs-irresolute as shown in the following example.

Example : 5.5

Let X = [0, 1], T = {(0,ϕ), (1,X), (λ_n,N)} where λ_n(x) = $\frac{2n+3}{5n+1}$ (n = 1, 2, 3, ...), ∀ x ∈ X, N={1}. Y = {p}, S = {(0,ϕ), (1,Y), (μ_1,Y), (μ_2,Y)} where μ_1(p) = 0.64and μ_2(p) = 0.52. Let k : (X, T) → (Y, S) be the identity function. Then *k is* g*sfG*$_\delta$*s-continuous but not* g*sfG*$_\delta$*s-irresolute*, for the soft fuzzy set (λ,Y) defined by λ(p) = 0.38 which is gsfG_δs-closed in (Y, S) butf^{-1} (λ,Y) is not gsfG_δs-closed in (X, T).

Property : 5.8

Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. For a function *k* : (X, T) → (Y, S), the following statements are equivalent :

(a) *k* is gsfG_δs-irresolute.

(b) The inverse image of every gsfF_σs-open set in (Y, S) is gsfF_σs-open in (X, T).

Property : 5.9

Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. If *k* : (X, T)→(Y, S) is gsfG_δs-irresolute, then *k* (gsfG_δs-cl (λ,M)) $\sqsubseteq$ gsfG_δs-cl (*k* (λ,M)) for all (λ,M) in (X, T).

Property : 5.10

Let (X, T), (Y, S) and (Z, R) be any three soft fuzzy topological spaces and *k :* (X, T) → (Y, S), *h*:(Y, S) → (Z, R) be functions.

(i) If *k* and h are gsfG_δs-irresolute, then the composition *hok* is gsfG_δs-irresolute.

(ii) If k is gsfG_δs-irresolute and *h* is gsfG_δs-continuous then the composition *hok* is gsfG_δs-continuous.

Definition : 5.4

Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. A function *k*: (X, T) → (Y, S) is said to be strongly gsfG_δs-continuous if the inverse image of every gsfG_δs-closed set in (Y, S) is soft fuzzy closed in (X, T).

Definition : 5.5

Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. A function *k* : (X, T) → (Y, S) is said to be perfectly gsfG_δs-continuous if the inverse image of every gsfG_δs-closed set in (Y, S) is both soft fuzzy open and soft fuzzy closed in (X, T).

Property : 5.11

Let (X, T) and (Y, S) be any two soft fuzzy topological spaces and k : (X, T) → (Y, S) be strongly gsfs-continuous then *f* is strongly gsfG_δs-continuous.

Remark : 5.6

Strongly gsfG_δs-continuous function need not be strongly gsfs-continuous as shown in the following example.

Example : 5.6

Let X = [0, 1], T_1 = {(0,ϕ), (1,X), (λ,M)} where 0.64 ≤ λ(x) ≤ 1, M={1} and T_2 = {(0,ϕ), (1,X), (λ_n,N)} where λ_n(x) = $\frac{2n+3}{5n+1}$ (n = 2, 3, ...), ∀ x ∈ X, N={1}. Let *f* : (X, T_1) → (X, T_2) be the identity function. Then *k is strongly* g*sfG*$_\delta$*s-continuous but not strongly* g*sfs-continuous***,** for the soft fuzzy set (μ,L) defined by μ (x) = 0.55, L={0,1} which is gsfs-closed in (X, T_2) but k^{-1} (μ,L) is not soft fuzzy closed in (X, T_1).

Property : 5.12

Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. A function k : (X, T) → (Y, S) is strongly gsfG_δs-continuous if and only if the inverse image of every gsfF_σs-open set in (Y, S) is soft fuzzy open in (X, T).

Property : 5.13

Let (X, T), (Y, S) and (Z, R) be any three soft fuzzy topological spaces and*k* : (X, T) → (Y, S) , *h* : (Y, S) → (Z, R) be the functions.

(i) If *k* is strongly gsfG_δs-continuous, *h* is gsfG_δs-continuous then *gof* is soft fuzzy continuous.

(ii) If *k*is strongly gsfG_δs-continuous, *h* is gsfG_δs-irresolute then *hok* is strongly gsfG_δs-continuous.

Property : 5.14

Let (X, T) and (Y, S) be any two soft fuzzy topological spaces and *k* : (X, T) → (Y, S).

(i) If *k* is perfectly gsfs-continuous then *k* is perfectly gsfG_δs -continuous.

(ii) If *k* is perfectly gsfG_δs-continuous then *k* is strongly gsfG_δs-continuous.

Remark : 5.7

Every **s**trongly gsfG_δs-continuous function need not be perfectly gsfG_δs-continuous as shown in the following example.

Example : 5.7

Let X = {a, b, c}, T = {(0,ϕ), (1,X), (λ,M)} and Y = {p, q, r} , S = {(0,ϕ), (1,Y), (μ_1,N_1), (μ_2,N_2)} where λ(a) = 0, λ(b) = 0, λ(c) = 1, M={c}; μ_1(p) = 0, μ_1(q) = 0, μ_1(r) = 1, N_1={r}; μ_2(p) = 1, μ_2(q) = 1, μ_2(r) = 0, N_2={p,q}. Let *k*: (X, T) → (Y, S) be defined by k(a) = p,*k* (b) = p, k(c) = q. Then *k is strongly* g*sfG*$_\delta$*s-continuous but not perfectly* g*sfG*$_\delta$*s-continuous.*

Property : 5.15

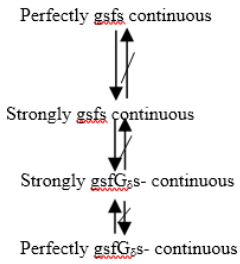
Let (X, T) and (Y, S) be any two soft fuzzy topolocical spaces. A function k : (X, T) → (Y, S) is perfectly gsfG_δs-continuous if and only if the inverse image of every gsfF_σs-open set in (Y, S) is both soft fuzzy open and soft fuzzy closed in (X, T).

Property : 5.16

Let (X, T), (Y, S) and (Z, R) be any three soft fuzzy topological spaces and *k* : (X, T) → (Y, S), *h* : (Y, S) →(Z, R) be the functions. If *k* is perfectly gsfG_δs-continuous, *h* is gsfG_δs-irresolute then *hok* is perfectly gsfG_δs-continuous.

Remark : 5.8

From the results proved above, the following implications are obtained :



6. Generalized soft fuzzy $g_\delta$semi-connected space

Definition : 6.1

A soft fuzzy topological space (X, T) is said to be generalized soft fuzzy G_δ semi connected (in short, gsfG_δs-connected) if the only soft fuzzy sets which are both gsfG_δs-closed and gsfF_σs-open are (0,ϕ) and (1,X).

Example : 6.1

Let X = [0, 1] and T = {(0,ϕ), (1,X), (λ_n,M)} where $\lambda_n(x) = \frac{5n^2}{7(n^2-1)}$ (n = 2, 3, ...),∀ x ∈ X, M={0,1}. There exists no proper soft fuzzy set in (X, T)which is both gsfG_δs-closed and gsfF_σs-open**.** Hence *(X, T) is gsfG_δs-connected.*

Property :6.1

Every gsfs-connected space is gsfG_δs-connected.

Proof :

Let (X, T) be a gsfs-connected space and suppose that (X, T) is not gsfG_δs-connected. Then there exists a proper soft fuzzy set (λ,M) of (X, T) which is both gsfF_σs-open and gsfG_δs-closed. Since every gsfF_σs-open set is gsfs-open and every gsfG_δs-closed set is gsfs-closed, (λ,M) is both gsfs-closed and gsfs-open. Hence (X, T) is not gsfs-connected. Contradiction. Hence every gsfs-connected space is gsfG_δs-connected.

Property : 6.2

Let (X, T) and (Y, S) be any two soft fuzzy topological spaces and k :(X,T) → (Y,S)

(i) If k is gsfG_δs-continuous surjection and (X, T) is gsfG_δs-connected, then (Y, S) is soft fuzzy connected.
(ii) If *k* is gsfG_δs-irresolute surjection and (X, T) is gsfG_δs-connected, then (Y, S) is gsfG_δs-connected.
(iii) If *k* is strongly gsfG_δs-continuous surjection and (X, T) is soft fuzzy connected, then (Y, S) is gsfG_δs-connected.

Property : 6.3

Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. If *k* : (X, T) → (Y, S) is gsfG_δs-irresolute surjection, (X, T) is soft fuzzy T-F_σ and soft fuzzy connected, then (Y, S) is gsfG_δs-connected.

Proof :

Suppose that (Y, S) is not gsfG_δs-connected. Then there exists proper soft fuzzy set (λ,M) which is both gsfG_δs-closed and gsfF_σs-open in (Y, S). Since k is gsfG_δs-irresolute surjection, f^{-1} (λ,M) is both gsfG_δs-closed and gsfF_σs-open in (X, T). Since (X, T) is soft fuzzy T-F_σ, k^{-1} (λ,M) is both soft fuzzy closed and soft fuzzy open in (X, T). Thus (X, T) is not soft fuzzy connected. Contradiction. Hence (Y, S) is gsfG_δs-connected.

7. Generalized soft fuzzy g_δ semi-extremally disconnected space and generalized soft fuzzy $f_\sigma$semi-compact space

Definition : 7.1

A soft fuzzy topological space (X, T) is said to be generalized soft fuzzy G_δ semi extremally disconnected (in short, gsfG_δs-extremally disconnected) if gsfG_δs-cl (λ,M) is gsfF_σs-open, whenever (λ,M) is gsfF_σs-open.

Property : 7.1

Let (X, T) be any soft fuzzy topological space and (X, T) be gsfG_δs-extremally disconnected space. Then the following statements are hold.

(i) For each gsfG_δs-closed set (λ,M), gsfF_σs-int (λ,M) is gsfG_δs-closed.
(ii) For each gsfF_σs-open set (λ,M).

gsfG_δs-cl(λ,M) + gsfG_δs-cl((1,X)– gsfG_δs-cl(λ,M)) = (1,X).

Proof :

(i) Let (λ,M) be any gsfG_δs-closed set in (X, T). Then (1,X)–(λ,M) is gsfF_σs-open in (X, T). Since (X, T) is gsfG_δs-extremally disconnected space, gsfG_δs-cl ((1,X)–(λ,M)) is gsfF_σs-open. Now, gsfG_δs-cl ((1,X)–(λ,M)) = (1,X)– gsfF_σs-int (λ,M) is gsfF_σs-open . Hence gsfF_σs-int (λ,M) is gsfG_δs-closed.

(ii) Let (λ,M) be any gsfF_σs-open set in (X, T). Since (1,X)– gsfG_δs-cl(λ,M)= gsfF_σs-int((1,X)–(λ,M)),
gsfG_δs-cl ((1,X)– gsfG_δs - cl (λ,M)) = gsfG_δs - cl (gsfF_σs -int ((1,X)–(λ,M)). Thus,
gsfG_δs-cl(λ,M)+ gsfG_δs-cl ((1,X)– gsfG_δs-cl(λ,M))=
gsfG_δs-cl(λ,M)+ gsfG_δs-cl(gsfF_σs-int((1,X)–(λ,M)).
Since (λ,M) is gsfF_σs-open, (1,X)–(λ,M) is gsfG_δs-closed. By (i), gsfF_σs-int((1,X)–(λ,M)) is gsfG_δs-closed. Hence, gsfG_δs-cl(gsfF_σs-int ((1,X)–(λ,M)))= gsfF_σs-int ((1,X)–(λ,M)).
Therefore,
gsfG_δs-cl(λ,M)+ gsfG_δs-cl ((1,X)– gsfG_δs-cl(λ,M))
= gsfG_δs-cl (λ,M)+ gsfF_σs-int ((1,X)–(λ,M)).
= gsfG_δs-cl(λ,M) +(1,X)– gsfG_δs-cl(λ,M).
= (1,X).
Hence, gsfG_δs-cl(λ,M) + gsfG_δs-cl ((1,X)– gsfG_δs-cl (λ,M)) =(1,X).

Definition : 7.2

A collection {(λ_i,M_i)}$_{i\in J}$ of soft fuzzy sets of a soft fuzzy topological space (X, T) is called gsfF_σs- open cover of (X, T), if (λ_i,M_i)′s (i∈J) are gsfF_σs-open sets of (X, T).

Definition : 7.3

A soft fuzzy topological space (X, T) is called gsfF_σs-compact if every gsfF_σs-open cover of (X, T) has a finite subcover.

Property : 7.2

Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. If *k* : (X, T) → (Y, S) is gsfG_δs-continuous bijective function and (X, T) is gsfF_σs-compact, then (Y, S) is soft fuzzy compact.

Proof :

Let $\{(\lambda_i,M_i)\}_{i\in J}$ be a collection of soft fuzzy open sets in (Y, S) such that

$(1,Y) \sqsubseteq \sqcup_{i=1}^{n}\{(\lambda_i,M_i)\}$(6.4)

Since f is gsfG$_\delta$s-continuous bijective function and each $(\lambda_i,M_i)$is soft fuzzy open in (Y, S), $f^{-1}(\lambda_i,M_i)$ is gsfF$_\sigma$s-open in (X, T). From (6.4), $(1,X) \sqsubseteq \sqcup_{i=1}^{n} f^{-1}(\lambda_i,M_i)$. Now, $\{ f^{-1}(\lambda_i,M_i)\}_{i\in J}$ is a gsfF$_\sigma$s-open cover of (X, T). Since (X, T) is gsfF$_\sigma$s-compact, there exists a finite subset F of J such that $(1,X) \sqsubseteq \sqcup_{i=1}^{n} f^{-1}(\lambda_i,M_i)$. Then, f $(1,X) \sqsubseteq \sqcup_{i=1}^{n} (\lambda_i,M_i)$.That is, $(1,Y) \sqsubseteq \sqcup_{i=1}^{n} (\lambda_i,M_i)$. Hence (Y, S) is soft fuzzy compact.

Property : 7.3

Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. If $k : (X, T) \rightarrow (Y, S)$ is gsfG$_\delta$s-irresolute bijective function and (X, T) is gsfF$_\sigma$s-compact, then (Y, S) is gsfF$_\sigma$s-compact.

Property : 7.4

Let (X, T) and (Y, S) be any two soft fuzzy topological spaces. If k: (X, T) $\rightarrow$ (Y, S) is strongly gsfG$_\delta$s-continuous bijective function and (X, T) is soft fuzzy compact, then (Y, S) is gsfF$_\sigma$s-compact.

References

[1] Azad KK,” On fuzzy semi continuity, fuzzy almost continuity and fuzzy weakly continuity”, *J. Math. Anal. Appl*., Vol.82 (1981), pp.14 - 32.

[2] Balasubramanian G, “Maximal fuzzy topologies”, *KYBERNETIKA*, Vol.31 (1995), pp.459 – 464.

[3] Chang CL, “Fuzzy topological spaces”, *J. Math. Anal. Appl*., Vol.24 (1968), pp.182 - 190.

[4] Fatteh UV and Bassan DS, “Fuzzy connectedness and its stronger forms”, *J. Math. Anal. Appl*., Vol.111 (1985), pp.449 - 464.

[5] Jin Han Park and Jin Kuen Park, “Generalized fuzzy semi-closed sets and generalizations of fuzzy continuous function”, *International Review of Fuzzy Mathematics*, Vol. 1, No. 1 (2006), pp. 89 - 101.

[6] Smets P,” The degree of belief in a fuzzy event”, *Inform. Sci*., Vol.25(1981), pp. 1-19.

[7] Sugeno M, “An introductory survey of fuzzy control”, *Inform. Sci.*,Vol. 36(1985), pp. 59-83.

[8] Ismail U Tiryaki,” Fuzzy sets over the poset I”, *Hacet. J. Math. Stat*., Vol. 37, No.2(2008), pp. 143-166.

[9] Vidhya D, Roja E and Uma MK, “On soft fuzzy B open sets”, *Journal of fuzzy Mathematics*, Vol. 21, No. 2(2013), 333-348

[10] Visalakshi V, Uma MK, Roja E, “On soft fuzzy G_δ pre continuity in soft fuzzy topological space”, *Annals of Fuzzy Mathematics and Informatics*, Vol. 8, No. 6(2014), pp 921-939.

[11] Yogalakshmi T, “Disconnectedness in soft fuzzy centred sys Tems”, *International journal of pure and applied mathematics*”, Vol. 115, No.9 (2017), pp.223 – 229.

[12] Zadeh LA,” Fuzzy sets”, *Information and Control*, Vol.8(1965), pp 338-353.