International Journal of Engineering & Technology, 7 (4.10) (2018) 1100-1104

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

Interval Valued Intuitionistic Fuzzy Weak Bi-Ideals of Gamma Near-Rings

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Abstract

In this paper, we introduce the concept of interval valued intuitionistic fuzzy weak bi-ideals of Γ-near-rings is a generalized concept of interval valued fuzzy weak bi-ideals of Γ- near-rings. We also characterize some results and illustrate our examples.

Keywords: *Γ-near-rings, intuitionistic fuzzy set, fuzzy weak bi-ideals, interval valued fuzzy weak bi-ideals.*

1. Introduction

Zadeh [22] introduced the concept of fuzzy sets in 1965 and [23] generalized it also into fuzzy subsets of interval value.Atanassov [1] introduced the fuzzy sets of the intuitionist as a generalization of the notion of fuzzy sets. The near ring was introduced by Pilz [15]. The gamma ring was introduced by Satyanarayana [16] in 1984.Tamizh chelvam et al.[17] the concept of bi-ideals of near-ring was applied to Γ-near-rings. Kim et al.[10] was proposed to the idea of fuzzy ideals of near-rings . In 1998, Fuzzy ideals in Gamma- near-rings was proposed by Jun et al.[9].The notion of fuzzy bi-ideals of near-rings was introduced by Manikantan [11] and discussed. Meenakumari et al.[13] studied the fuzzy bi-ideals in gamma -near-rings. The concept of weak bi-ideals of near-rings was introduced by Yong Uk Cho et al. [21]. Thillaigovindan at al.[18] studied the interval valued fuzzy quasi-ideals of semi-groups. Chinnadurai et al. [6] studied the fuzzy weak bi-ideals of Γ-near rings. Thillaigovindan et al.[19] worked on interval valued fuzzy ideals of near-rings.

2. Preliminaries

Definition 2.1 [15] The algebraic system (R,+,·)is called near ring if its satisfies (R,+) is a group not necessarily abelian and (R,·) is a semigroup and distributive law:
(x+z)·y=x·y+z·y valid for all x,y,z∈R. We use the word near-ring to mean right near-ring . We denote xy instead of x·y.

Definition 2.2[16] A Γ- near-ring is a triple (M,+,Γ) where
(i) (M,+) is a group,
(ii) Γ is a nonempty set of binary operators on M such that for each α∈Γ,(M,+,α) is a near-ring,
(iii) xα(yβz)=(xαy)βz for all x,y,z∈M.

Definition 2.3[13] A Γ-near-ring M is said to be zero-symmetric if its satisfy the condition xα0 = 0 for all x∈M and α∈Γ.
Throughout this paper M denotes a zero-symmetric right Γ-near-ring with atleast two elements.

Definition 2.4[16] A subset A of a Γ-near-ring M is called a left(resp. right) ideal of M if (A,+) is a normal subgroup of (M,+), (i.e) x-y∈A for all x,y∈A and y+x-y∈A for x∈A, y∈M. uα(x+v)-uαv∈A (resp. xαu∈A) for all x∈A, α∈Γ and u,v∈M.

Definition 2.5[16] Given two subsets A and B of M, we define AΓB={aαb|a∈A,b∈B and α∈Γ} and define another operation * of M define by AΓ*B={a γ (a'+b) - aγa' |a, a'∈A, γ∈Γ, b∈B}.

Definition 2.6[17] A subgroup B of (M, +) is called a bi-ideal of a near ring M if BΓMΓB ⊆ B.

Definition 2.7[6] A subgroup H of (M, +) is said to be a weak bi-ideal of M if HΓHΓH ⊆ H.
The characteristic function of M is denoted by **M**.

Definition 2.8[23] If X be any set. A mapping $\eta: X \rightarrow D[0,1]$ is called an i.v fuzzy subset of X, $\tilde{\eta}(x) = [\eta^-(x), \eta^+(x)]$, where $\eta^-(x)$ and $\eta^+(x)$ are fuzzy subsets of X such that
$\eta^-(x) \leq \eta^+(x)$ for all x∈X

Definition 2.9[18] By an interval number $\tilde{a}$, we mean an interval $[a^-, a^+]$ such that $0 \leq a^- \leq a^+ \leq 1$ and where a^- is the lower limits and a^+ is the upper limits of $\tilde{a}$. D[0,1] is the set of all closed subintervals of [0,1] We also identify the interval [a,a] by the number a∈[0,1]. $\tilde{a}_i = [a_i^-, a_i^+], \tilde{b}_i = [b_i^-, b_i^+] \in D[0,1], j \in \Omega$,

$$\max{}^i\{\tilde{a}_j, \tilde{b}_j\} = [\max\{a_j^-, b_j^-\}, \max\{a_j^+, b_j^+\}],$$
$$\min{}^i\{\tilde{a}_j, \tilde{b}_j\} = [\min\{a_j^-, b_j^-\}, \max\{a_j^+, b_j^+\}],$$
$$\inf{}^i\tilde{a}_j = [\cap_{j\in\Omega} a_j^-, \cap_{j\in\Omega} a_j^+],$$
$$\sup{}^i\tilde{a}_j = [\cup_{j\in\Omega} a_j^-, \cup_{j\in\Omega} a_j^+].$$

and let $\tilde{a} \leq \tilde{b} \Leftrightarrow a^- \leq b^-$ and $a^+ \leq b^+$,
$\tilde{a} = \tilde{b} \Leftrightarrow a^- = b^-$ and $a^+ = b^+$,
$\tilde{a} < \tilde{b} \Leftrightarrow \tilde{a} \leq \tilde{b}$ and $\tilde{a} \neq \tilde{b}$,
$k\tilde{a} = [ka^-, ka^+]$, whenever $0 \leq k \leq 1$.

Definition 2.10[18] Let $\tilde{\eta}$ be an i.v fuzzy subset of X, $[t_1, t_2] \in D[0,1]$ .Then $\tilde{U}(\tilde{\eta}: [t_1, t_2]) = \{x \in X | \tilde{\eta}(x) \geq [t_1, t_2]\}$ is called upper level subset of $\tilde{\eta}$.

Definition 2.11[8,14,19] If $\tilde{\eta}$ and $\tilde{\lambda}$ are i.v fuzzy subsets of M .Now, $\tilde{\eta} \cap \tilde{\lambda}, \tilde{\eta} \cup \tilde{\lambda}, \tilde{\eta} + \tilde{\lambda}$, and $\tilde{\eta} * \tilde{\lambda}$ are fuzzy subsets of M, defined as follows.For x,y,z $\in$M.

$$(\tilde{\eta} \cap \tilde{\lambda})(x) = \min^i\{\tilde{\eta}(x), \tilde{\lambda}(x)\}.$$

$$(\tilde{\eta} \cup \tilde{\lambda})(x) = \max^i\{\tilde{\eta}(x), \tilde{\lambda}(x)\}.$$

$$(\tilde{\eta} + \tilde{\lambda})(x) = \begin{cases} \sup^i_{x=y+z}\{\min^i\{\tilde{\eta}(y), \tilde{\lambda}(z)\}\} & \text{if } x \text{ is expressible as } x = y+z \\ 0 & \text{otherwise.} \end{cases}$$

$$(\tilde{\eta} * \tilde{\lambda})(x) = \begin{cases} \sup^i_{x=y\alpha z}\{\min^i\{\tilde{\eta}(y), \tilde{\lambda}(z)\}\} & \text{if } x \text{ is expressible as } x = y\alpha z \\ 0 & \text{otherwise.} \end{cases}$$

Definition 2.12[1] An intuitionistic fuzzy set (briefly IFS) A in a non-empty set X is $A = \{x, (\mu_A(x), \eta_A(x)) : x \in X\}$ where the functions $\mu_A : X \to [0,1]$ and $\eta_A : X \to [0,1]$ define the degree of membership and the degree of non-membership of $x \in X$ to the set A,, $0 \leq \mu_A(x) + \eta_A(x) \leq 1$,and $A = (\mu_A, \eta_A)$.

Definition 2.13[6] An i.v fuzzy subset $\tilde{\eta}$ in M is called an i.v fuzzy left (resp. right) ideal of M if $\tilde{\eta}$ is an i.v fuzzy normal divisor with respect to the addition,
$\tilde{\eta}(u\alpha(x+v) - u\alpha v) \geq \tilde{\eta}(x)$, (resp. $\tilde{\eta}(x\alpha u) \geq \tilde{\eta}(x)$ for all $x, u, v \in M$ and $\alpha \in \Gamma$. The condition of definition 2.12 means that $\tilde{\eta}$ satisfies: $\tilde{\eta}(x - y) \geq \min^i\{\tilde{\eta}(x), \tilde{\eta}(y)\}$,
$\tilde{\eta}(y + x - y) \geq \tilde{\eta}(x)$, for all $x, y \in M$
Note that $\tilde{\eta}$ is an i.v fuzzy left (resp. right) ideal of M, $\tilde{\eta}(0) \geq \tilde{\eta}(x)$ for all $x \in M$, 0 is the zero element of M.

Definition 2.14 An i.v fuzzy subset $\tilde{\eta}$ of M is called an i.v fuzzy bi-ideal of M if $\tilde{\eta}(x - y) \geq \min^i\{\tilde{\eta}(x), \tilde{\eta}(y)\}$ for all $x, y \in \tilde{\eta}(x\alpha y\beta z) \geq \min^i\{\tilde{\eta}(x), \tilde{\eta}(z)\}$ for $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

3. I.V intuitionistic fuzzy weak bi-ideals of Γ-near-rings.

Definition 3.1 An i.v fuzzy set $\tilde{\eta}$ of M is called an i.v intuitionistic fuzzy weak bi-ideal of M, if
$\tilde{\eta}(x - y) \geq \min^i\{\tilde{\eta}(x), \tilde{\eta}(y)\}$,
$\tilde{\lambda}(x - y) \leq \max^i\{\tilde{\lambda}(x), \tilde{\lambda}(y)\}$,
$\tilde{\eta}(x\alpha y\beta z) \geq \min^i\{\tilde{\eta}(x), \tilde{\eta}(y), \tilde{\eta}(z)\}$,
$\tilde{\eta}(x\alpha y\beta z) \leq \max^i\{\tilde{\lambda}(x), \tilde{\lambda}(y), \tilde{\lambda}(z)\}$ for all $x, y, z \in M$ and $\alpha, \beta \in \Gamma$.

Example 3.2 Let $M = \{0, a, b, c\}$be an gamma near-ring with "+" and $\Gamma = \{\gamma\}$ be a non-empty set of binary operations as shown in the following tables:

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

γ	0	a	b	c
0	0	0	0	0
a	a	a	a	a
b	0	0	b	b
c	a	a	c	c

$\tilde{\eta}(0) = [0.7, 0.8], \tilde{\eta}(a) = [0.5, 0.6]$, $\tilde{\eta}(b) = \tilde{\eta}(c) = [0.1, 0.2]$ and $\tilde{\lambda}(0) = [0.1, 0.2], \tilde{\lambda}(a) = [0.3, 0.4]$,
$\tilde{\lambda}(b) = \tilde{\lambda}(c) = [0.7, 0.8]$.Then $B = (\tilde{\eta}_B, \tilde{\lambda}_B)$ is an i.v intuitionistic fuzzy weak bi-ideal of M.

Theorem 3.3 Let $B = (\tilde{\eta}_B, \tilde{\lambda}_B)$ be an i.v intuitionistic fuzzy subgroup of M.Prove that$B$ is an i.v intuitionistic fuzzy weak bi-ideal of M iff$\tilde{\eta} * \tilde{\eta} * \tilde{\eta} \subseteq \tilde{\eta}$ and $\tilde{\lambda} * \tilde{\lambda} * \tilde{\lambda} \supseteq \tilde{\lambda}$.

Proof. *Let* $\tilde{\eta}$ is an i.v fuzzy weak bi-ideal of M and $x, y, z, y_1, y_2 \in M$ and $\alpha, \beta \in \Gamma$ such that $x = y\alpha z$ and $y = y_1\beta y_2$.

$$\begin{aligned}(\tilde{\eta} * \tilde{\eta} * \tilde{\eta})(x) &= \sup^i_{x=y\alpha z}\{\min^i\{(\tilde{\eta} * \tilde{\eta})(y), \tilde{\eta}(z)\}\} \\ &= \sup^i_{x=y\alpha z}\{\min^i\{\sup^i_{y=y_1\beta y_2}\min^i\{\tilde{\eta}(y_1), \tilde{\eta}(y_2)\}, \tilde{\eta}(z)\}\} \\ &= \sup^i_{x=y\alpha z}\sup^i_{y=y_1\beta y_2}\{\min^i\{\min^i\{\tilde{\eta}(y_1), \tilde{\eta}(y_2)\}, \tilde{\eta}(z)\}\} \\ &= \sup^i_{x=y_1\beta y_2\alpha z}\{\min^i\{\tilde{\eta}(y_1), \tilde{\eta}(y_2), \tilde{\eta}(z)\}\}\end{aligned}$$

Since $\tilde{\eta}$ *is an i.v fuzzy weak bi* − *ideal of* M.

$$\begin{aligned}\tilde{\eta}(y_1\beta y_2\alpha z) &\geq \min^i\{\tilde{\eta}(y_1), \tilde{\eta}(y_2), \tilde{\eta}(z)\} \\ &\leq \sup^i_{x=y_1\beta y_2\alpha z}\tilde{\eta}(y_1\beta y_2\alpha z) = \tilde{\eta}(x).\end{aligned}$$

$$\begin{aligned}(\tilde{\lambda} * \tilde{\lambda} * \tilde{\lambda})(x) &= \inf^i_{x=y\alpha z}\{\max^i\{(\tilde{\lambda} * \tilde{\lambda})(y), \tilde{\lambda}(z)\}\} \\ &= \inf^i_{x=y\alpha z}\{\max^i\{\inf^i_{y=y_1\beta y_2}\max^i\{\tilde{\lambda}(y_1), \tilde{\lambda}(y_2)\}, \tilde{\lambda}(z)\}\} \\ &= \inf^i_{x=y\alpha z}\inf^i_{y=y_1\beta y_2}\{\max^i\{\max^i\{\tilde{\lambda}(y_1), \tilde{\lambda}(y_2)\}, \tilde{\lambda}(z)\}\} \\ &= \inf^i_{x=y_1\beta y_2\alpha z}\{\max^i\{\tilde{\lambda}(y_1), \tilde{\lambda}(y_2), \tilde{\lambda}(z)\}\}\end{aligned}$$

Since $\tilde{\lambda}$*is an i.v fuzzy weak bi* − *ideal of* M .

$$\begin{aligned}\tilde{\lambda}(y_1\beta y_2\alpha z) &\leq \max^i\{\tilde{\lambda}(y_1), \tilde{\lambda}(y_2), \tilde{\lambda}(z)\} \\ &\geq \inf^i_{x=y_1\beta y_2\alpha z}\tilde{\lambda}(y_1\beta y_2\alpha z) \\ &= \tilde{\lambda}(x).\end{aligned}$$

If x cannot as $x = y\alpha z$, then $(\tilde{\eta} * \tilde{\eta} * \tilde{\eta})(x) = \tilde{0} \leq \tilde{\eta}(x)$.
Conversely, assume that $\tilde{\eta} * \tilde{\eta} * \tilde{\eta} \subseteq \tilde{\eta}$. For $x', x, y, z \in M$ and $\alpha, \beta, \alpha_1, \beta_1 \in \Gamma$. Let x' be such that $x' = x\alpha y\beta z$.

$$\begin{aligned}\tilde{\eta}(x\alpha y\beta z) = \eta(x') &\geq (\tilde{\eta} * \tilde{\eta} * \tilde{\eta})(x') \\ &= \sup^i_{x'=p\alpha_1 q}\{\min^i\{(\tilde{\eta} * \tilde{\eta})(p), \tilde{\eta}(q)\}\} \\ &= \sup^i_{x'=p\alpha_1 q}\{\min^i\{\sup^i_{p=p_1\beta_1 p_2}\min^i\{\tilde{\eta}(p_1), \tilde{\eta}(p_2)\}, \tilde{\eta}(q)\}\} \\ &= \sup^i_{x'=p_1\beta_1 p_2\alpha_1 q}\{\min^i\{\tilde{\eta}(p_1), \tilde{\eta}(p_2), \tilde{\eta}(q)\}\} \\ &\geq \min^i\{\tilde{\eta}(x), \tilde{\eta}(y), \tilde{\eta}(z)\}.\end{aligned}$$

$$\begin{aligned}\tilde{\lambda}(x\alpha y\beta z) = \eta(x') &\leq (\tilde{\lambda} * \tilde{\lambda} * \tilde{\lambda})(x') \\ &= \inf^i_{x'=p\alpha_1 q}\{\max^i\{(\tilde{\lambda} * \tilde{\lambda})(p), \tilde{\lambda}(q)\}\} \\ &= \inf^i_{x'=p\alpha_1 q}\{\max^i\{\inf^i_{p=p_1\beta_1 p_2}\max^i\{\tilde{\lambda}(p_1), \tilde{\lambda}(p_2)\}, \tilde{\lambda}(q)\}\} \\ &= \inf^i_{x'=p_1\beta_1 p_2\alpha_1 q}\{\max^i\{\tilde{\lambda}(p_1), \tilde{\lambda}(p_2), \tilde{\lambda}(q)\}\} \\ &\leq \max^i\{\tilde{\lambda}(x), \tilde{\lambda}(y), \tilde{\lambda}(z)\}.\end{aligned}$$

Lemma 3.4 Every i.v intuitionistic fuzzy ideal of M is an i.v intuitionistic fuzzy bi-ideal of M.

Proof. Let B be an i.v intuitionistic fuzzy ideal of M. Then $\tilde{\eta} * M * \tilde{\eta} \subseteq \tilde{\eta} * M * M \subseteq \tilde{\eta} * M \subseteq \tilde{\eta}, \tilde{\lambda} * M * \tilde{\lambda} \supseteq \tilde{\lambda} * M * M \supseteq \tilde{\lambda} * M \supseteq \tilde{\lambda}$.
since B be an i.v intuitionistic fuzzy ideal of M. This implies that $\tilde{\eta} * M * \tilde{\eta} \subseteq \tilde{\eta}$ and $\tilde{\lambda} * M * \tilde{\lambda} \supseteq \tilde{\lambda}$.

Theorem 3.5 Every i.v intuitionistic fuzzy bi-ideal of M is an i.v intuitionistic fuzzy weak bi-ideal of M.
Proof. Assume that B is an i.v intuitionistic fuzzy bi-ideal of M. Then $\tilde{\eta} * M * \tilde{\eta} \subseteq \tilde{\eta}$ and $\tilde{\lambda} * M * \tilde{\lambda} \supseteq \tilde{\lambda}$. We have
$\tilde{\eta} * \tilde{\eta} * \tilde{\eta} \subseteq \tilde{\eta} * M * \tilde{\eta}$ and $\tilde{\lambda} * \tilde{\lambda} * \tilde{\lambda} \supseteq \tilde{\lambda} * M * \tilde{\lambda}$. This implies that $\tilde{\eta} * \tilde{\eta} * \tilde{\eta} \subseteq \tilde{\eta} * M * \tilde{\eta} \subseteq \tilde{\eta}$ and $\tilde{\lambda} * \tilde{\lambda} * \tilde{\lambda} \supseteq \tilde{\lambda} * M * \tilde{\lambda} \supseteq \tilde{\lambda}$.

Theorem 3.6 Every i.v intuitionistic fuzzy ideal of M is an i.v intuitionistic fuzzy weak bi-ideal of M.

Proof. By Lemma 3.4, every i.v intuitionistic fuzzy ideal of M is an i.v fuzzy bi-ideal of M. By Theorem 3.5, Thus B is an i.v intuitionistic fuzzy weak bi-ideal of M.
However the converse of the Theorems 3.5 and 3.6 is not true by the following example.

Example 3.7 Let $M = \{0, a, b, c\}$ be a non-emptyset with binary operation + and $\Gamma = \{\alpha, \beta\}$.

+	0	a	b	c
0	0	a	b	c

a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

α	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	0	0	b	b
c	0	a	b	c

$\tilde{\eta}(0) = [0.7,0.8], \tilde{\eta}(a) = [0.3,0.4] = \tilde{\eta}(b)$ and $\tilde{\eta}(c) = [0.5,0.6]$. $\tilde{\lambda}(0) = [0.1,0.2], \tilde{\lambda}(a) = [0.5,0.6] = \tilde{\lambda}(b)$, $\tilde{\lambda}(c) = [0.3,0.4]$.Thus $\tilde{\eta}$ is an i.v fuzzy weak bi-ideal .Now,$\tilde{\eta}$ is not an i.v fuzzy ideal and bi-ideal,

$\tilde{\eta}(b\gamma c) = \tilde{\eta}(b) = [0.3,0.4] \ngeq [0.5,0.6] = \tilde{\eta}(c), \tilde{\lambda}(a\alpha(0+0) - a\alpha 0) \leq \tilde{\lambda}(0) = [0.5,0.6] \nleq [0.1,0.2]$

and $\tilde{\lambda}(c\gamma a\gamma c) = \tilde{\eta}(a) = [0.5,0.6] \nleq [0.3,0.4] = \min^i\{\tilde{\eta}(c), \tilde{\eta}(c)\}$.

Theorem 3.8 Let $\{\tilde{\eta}_i | i \in \Omega\}$ be family of i.v intuitionistic fuzzy weak bi-ideals of M, then $\bigcap_{i\in\Omega} \tilde{\eta}_i$ is also an i.v intuitionistic fuzzy weak bi-ideal of M.

Proof. Let $x, y, z \in M$, $\alpha, \beta \in \Gamma$ and $\tilde{\eta} = \bigcap_{i\in\Omega} \tilde{\eta}_i$.
$\tilde{\eta}(x) = \bigcap_{i\in\Omega} \tilde{\eta}_i(x) = (\inf^i_{i\in\Omega}\tilde{\eta}_i)(x) = \inf^i_{i\in\Omega}\tilde{\eta}_i(x)$.Now

$$\begin{aligned}\tilde{\eta}(x-y) &= \inf^i_{i\in\Omega}\tilde{\eta}_i(x-y)\\ &\geq \inf^i_{i\in\Omega}\min^i\{\tilde{\eta}_i(x), \tilde{\eta}_i(y)\}\\ &= \min^i\{\inf^i_{i\in\Omega}\tilde{\eta}_i(x), \inf^i_{i\in\Omega}\tilde{\eta}_i(y)\}\\ &= \min^i\left\{\bigcap_{i\in\Omega}\tilde{\eta}_i(x), \bigcap_{i\in\Omega}\tilde{\eta}_i(y)\right\}\\ &= \min^i\{\tilde{\eta}(x), \tilde{\eta}(y)\}.\end{aligned}$$

$$\begin{aligned}\tilde{\lambda}(x-y) &= \sup^i_{i\in\Omega}\tilde{\lambda}_i(x-y)\\ &\leq \sup^i_{i\in\Omega}\max^i\{\tilde{\lambda}_i(x), \tilde{\lambda}_i(y)\}\\ &= \max^i\{\sup^i_{i\in\Omega}\tilde{\lambda}_i(x), \sup^i_{i\in\Omega}\tilde{\lambda}_i(y)\}\\ &= \max^i\left\{\bigcap_{i\in\Omega}\tilde{\lambda}_i(x), \bigcap_{i\in\Omega}\tilde{\lambda}_i(y)\right\}\\ &= \max^i\{\tilde{\lambda}(x), \tilde{\lambda}(y)\}.\end{aligned}$$

$$\begin{aligned}\tilde{\eta}(x\alpha y\beta z) &= \inf^i_{i\in\Omega}\tilde{\eta}_i(x\alpha y\beta z)\\ &\geq \inf^i_{i\in\Omega}\min^i\{\tilde{\eta}_i(x), \tilde{\eta}_i(y), \tilde{\eta}_i(z)\}\\ &= \min^i\{\inf^i_{i\in\Omega}\tilde{\eta}_i(x), \inf^i_{i\in\Omega}\tilde{\eta}_i(y), \inf^i_{i\in\Omega}\tilde{\eta}_i(z)\}\\ &= \min^i\left\{\bigcap_{i\in\Omega}\tilde{\eta}_i(x), \bigcap_{i\in\Omega}\tilde{\eta}_i(y), \bigcap_{i\in\Omega}\tilde{\eta}_i(z)\right\}\\ &= \min^i\{\tilde{\eta}(x), \tilde{\eta}(y), \tilde{\eta}(z)\}.\end{aligned}$$

$$\begin{aligned}\tilde{\lambda}(x\alpha y\beta z) &= \sup^i_{i\in\Omega}\tilde{\lambda}_i(x\alpha y\beta z)\\ &\leq \sup^i_{i\in\Omega}\max^i\{\tilde{\lambda}_i(x), \tilde{\lambda}_i(y), \tilde{\lambda}_i(z)\}\\ &= \max^i\{\sup^i_{i\in\Omega}\tilde{\lambda}_i(x), \sup^i_{i\in\Omega}\tilde{\lambda}_i(y), \sup^i_{i\in\Omega}\tilde{\lambda}_i(z)\}\\ &= \max^i\left\{\bigcap_{i\in\Omega}\tilde{\lambda}_i(x), \bigcap_{i\in\Omega}\tilde{\lambda}_i(y), \bigcap_{i\in\Omega}\tilde{\lambda}_i(z)\right\}\\ &= \max^i\{\tilde{\lambda}(x), \tilde{\lambda}(y), \tilde{\lambda}(z)\}.\end{aligned}$$

Theorem 3.9 An intuitionistic fuzzy set B is an i.v intuitionistic fuzzy weak bi-ideal of M if f $\tilde{U}(\tilde{\eta}_B: [s_1, s_2])$ and $\tilde{L}(\tilde{\lambda}_B: [t_1, t_2])$ is a weak bi-ideal of M, for all $[t_1, t_2], [s_1, s_2] \in D[0,1]$.

Proof. Assume that $B = (\tilde{\eta}_B, \tilde{\lambda}_B)$ is an i.v fuzzy weak bi-ideal of M. Let $[t_1, t_2] \in D[0,1]$ such that $x, y \in \tilde{U}(\tilde{\eta}: [t_1, t_2])$.
$\tilde{\eta}(x-y) \geq \min^i\{\tilde{\eta}(x), \tilde{\eta}(y)\} \geq \min^i\{[t_1, t_2], [t_1, t_2]\} = [t_1, t_2]$.
Thus $x - y \in \tilde{U}(\tilde{\eta}: [t_1, t_2])$.

Let $x, y, z \in \tilde{U}(\tilde{\eta}: [t_1, t_2])$ and $\alpha, \beta \in \Gamma$.
We have $\tilde{\eta}(x\alpha y\beta z) \geq \min^i\{\tilde{\eta}(x), \tilde{\eta}(y), \tilde{\eta}(z)\} \geq \min^i\{[t_1, t_2], [t_1, t_2], [t_1, t_2]\} = [t_1, t_2]$.
$x\alpha y\beta z \in \tilde{U}(\tilde{\eta}: [t_1, t_2])$. Hence $\tilde{U}(\tilde{\eta}: [t_1, t_2])$ is a weak bi-ideal of M. Now $[s_1, s_2] \in D[0,1]$,
$x, y \in M\ \tilde{\lambda}(x-y) \leq \max^i\{\tilde{\lambda}(x), \tilde{\lambda}(y)\} \leq \max^i\{[s_1, s_2], [s_1, s_2]\} = [s_1, s_2]$
$\tilde{\eta}(x\alpha y\beta z) \leq \max^i\{\tilde{\lambda}(x), \tilde{\lambda}(y), \tilde{\lambda}(z)\} \leq \max^i\{[s_1, s_2], [s_1, s_2], [s_1, s_2]\} = [s_1, s_2]$.
Therefore $x\alpha y\beta z \in \tilde{L}(\tilde{\lambda}: [s_1, s_2])$. Hence $\tilde{L}(\tilde{\eta}: [s_1, s_2])$ is a weak bi-ideal of M. Conversely, assume $\tilde{U}(\tilde{\eta}: [t_1, t_2])$ and $\tilde{L}(\tilde{\lambda}: [s_1, s_2])$ are weak bi-ideal of M, for all $[t_1, t_2], [s_1, s_2] \in D[0,1]$ .Let $x, y \in M$. Suppose $\tilde{\eta}(x-y) < \min^i\{\tilde{\eta}(x), \tilde{\eta}(y)\}$.Choose $[0,0] < [t_1, t_2] \leq [1,1]$, $\tilde{\eta}(x-y) < [t_1, t_2] < \min^i\{\tilde{\eta}(x), \tilde{\eta}(y)\}$ .Thus $\tilde{\eta}(x) > [t_1, t_2], \tilde{\eta}(y) > [t_1, t_2]$ and $\tilde{\eta}(x-y) < [t_1, t_2]$ .Then we have $x, y \in \tilde{U}(\tilde{\eta}: [t_1, t_2])$,but $x - y \notin \tilde{U}(\tilde{\eta}: [t_1, t_2])$ a contradict.
Thus $\tilde{\eta}(x-y) \geq \min^i\{\tilde{\eta}(x), \tilde{\eta}(y)\}$.Choose
$[0,0] > [t_1, t_2] \geq [1,1]$, $\tilde{\eta}(x-y) > [s_1, s_2] > \max^i\{\tilde{\lambda}(x), \tilde{\lambda}(y)\}$.
Thus $\tilde{\lambda}(x) > [s_1, s_2], \tilde{\lambda}(y) > [s_1, s_2]$ and $\tilde{\lambda}(x-y) < [s_1, s_2]$.
Then we have $x, y \in \tilde{L}(\tilde{\lambda}: [s_1, s_2])$,but $x - y \notin \tilde{L}(\tilde{\lambda}: [s_1, s_2])$ a contradiction. Thus $\tilde{\lambda}(x-y) \leq \max^i\{\tilde{\lambda}(x), \tilde{\lambda}(y)\}$.If there exist $x, y, z \in M$ and $\alpha, \beta \in \Gamma$ such that
$\tilde{\eta}(x\alpha y\beta z) < \min^i\{\tilde{\eta}(x), \tilde{\eta}(y), \tilde{\eta}(z)\}$ .Choose $[t_1, t_2]$ such that $\tilde{\eta}(x\alpha y\beta z) < [t_1, t_2] < \min^i\{\tilde{\eta}(x), \tilde{\eta}(y), \tilde{\eta}(z)\}$. Then
$\tilde{\eta}(x) > [t_1, t_2], \tilde{\eta}(y) > [t_1, t_2], \tilde{\eta}(z) > [t_1, t_2]$ and and but $x\alpha y\beta z \notin \tilde{U}(\tilde{\eta}: [t_1, t_2])$, which is a contradiction. Now Choose $[s_1, s_2]$ such that $\tilde{\lambda}(x\alpha y\beta z) > [s_1, s_2] < \max^i\{\tilde{\lambda}(x), \tilde{\lambda}(y), \tilde{\lambda}(z)\}$.
Then $\tilde{\lambda}(x) < [s_1, s_2], \tilde{\lambda}(y) < [s_1, s_2], \tilde{\lambda}(z) < [s_1, s_2]$ and $[s_1, s_2]$ s.t $\tilde{\eta}(x\alpha y\beta z) > [s_1, s_2]$. $x, y, z \in \tilde{U}(\tilde{\eta}: [t_1, t_2]), x, y, z \in \tilde{L}(\tilde{\lambda}: [s_1, s_2])$ and but $x\alpha y\beta z \notin \tilde{U}(\tilde{\eta}: [s_1, s_2])$, which is a contradiction. Therefore B is an i.v intuitionistic fuzzy weak bi-ideal of M.

Theorem 3.10 Let B be i.v intuitionistic fuzzy weak bi-ideal of M then $M_{\tilde{\eta},\tilde{\lambda}} = \{x \in M | \tilde{\eta}(x) = \tilde{\eta}(0), \tilde{\lambda}(x) = (0)\}$ is a weak bi-ideal of M.

Proof. Let $x, y \in M_{(\tilde{\eta},\tilde{\lambda})}$.Then
$\tilde{\eta}(x) = \tilde{\eta}(0), \tilde{\eta}(y) = \tilde{\eta}(0), \tilde{\lambda}(x) = 0, \tilde{\lambda}(y) = 0,$

$$\begin{aligned}\tilde{\eta}(x-y) &\geq \min^i\{\tilde{\eta}(x), \tilde{\eta}(y)\}\\ &= \min^i\{\tilde{\eta}(0), \tilde{\eta}(0)\} = \tilde{\eta}(0),\end{aligned}$$

and

$$\begin{aligned}\tilde{\lambda}(x-y) &\leq \max^i\{\tilde{\lambda}(x), \tilde{\lambda}(y)\}\\ &= \max^i\{\tilde{\lambda}(0), \tilde{\lambda}(0)\} = \tilde{\lambda}(0).\end{aligned}$$

So $\tilde{\eta}(x-y) = \tilde{\eta}(0), \tilde{\lambda}(x-y) = \tilde{\lambda}(0)$.
Thus $x - y \in M_{\tilde{\eta}}$, $x - y \in M_{\tilde{\lambda}}$. For every $x, y, z \in M_{\tilde{\eta}}$ and

$$\begin{aligned}\tilde{\eta}(x\alpha y\beta z) &\geq \min^i\{\tilde{\eta}(x), \tilde{\eta}(y), \tilde{\eta}(z)\}\\ &= \min^i\{\tilde{\eta}(0), \tilde{\eta}(0), \tilde{\eta}(0)\} = \tilde{\eta}(0)\end{aligned}$$

$$\begin{aligned}\tilde{\lambda}(x\alpha y\beta z) &\leq \max^i\{\tilde{\lambda}(x), \tilde{\lambda}(y), \tilde{\lambda}(z)\}\\ &= \max^i\{\tilde{\lambda}(0), \tilde{\lambda}(0), \tilde{\lambda}(0)\} = \tilde{\lambda}(0).\end{aligned}$$

Thus $x\alpha y\beta z \in M_{\tilde{\eta}}$, $x\alpha y\beta z \in M_{\tilde{\lambda}}$. Hence $M_{(\tilde{\eta},\tilde{\lambda})}$ is a weak bi-ideal of M.

4. Homomorphism of interval valued intuitionistic fuzzy weak bi-ideals of Γ-near-rings

Definition 4.1 Let $f: M \to S$. Let $\tilde{\eta}$ and $\tilde{\delta}$ be i.v fuzzy subsets of M and S respectively. Then $f(\tilde{\eta})$, the image of $\tilde{\eta}$ under f is an i.v fuzzy subset of S, defined by

$$f(\tilde{\eta})(y) = \begin{cases}\sup^i_{x\in f^{-1}(y)} \tilde{\eta}(x) & if \quad f^{-1}(y) \neq \varphi\\ 0 & otherwise\end{cases}$$

The pre-image of $\tilde{\eta}$ under f is an i.v fuzzy subset of M defined by $f^{-1}(\tilde{\delta}(x)) = \tilde{\delta}(f(x))$, and $f^{-1}(y) = \{x \in M | f(x) = y\}$.

Definition 4.2 [10] Let M and S be Γ-near-rings. A map $\theta: M \to S$ is called a homomorphism if
$\theta(x+y) = \theta(x) + \theta(y)$ and $\theta(x\alpha y) = \theta(x)\alpha\theta(y)$ for all $x, y \in M$ and $\alpha \in \Gamma$.

Theorem 4.3 Let $f: M \to S$ be a homomorphism. If $B = (\tilde{\delta}_B, \tilde{\lambda}_B)$ is an i.v intuitionistic fuzzy weak bi-ideal of S, then $f^{-1}(B) = [f^{-1}(\tilde{\delta}_B), f^{-1}(\tilde{\lambda}_B)]$ is an i.v intuitionistic fuzzy weak bi-ideal of M.

Proof. Let $x, y, z \in M$ and $\alpha, \beta \in \Gamma$. Then

$$\begin{aligned}
f^{-1}(\tilde{\delta})(x-y) &= \tilde{\delta}(f(x-y)) \\
&= \tilde{\delta}(f(x)-f(y)) \\
&\geq \min^i\{\tilde{\delta}(f(x)), \tilde{\delta}(f(y))\} \\
&= \min^i\{f^{-1}(\tilde{\delta}(x)), f^{-1}(\tilde{\delta}(y))\}. \\
\text{and} f^{-1}(\tilde{\lambda})(x-y) &= \tilde{\lambda}(f(x-y)) \\
&= \tilde{\lambda}(f(x)-f(y)) \\
&\leq \max^i\{\tilde{\lambda}(f(x)), \tilde{\lambda}(f(y))\} \\
&= \max^i\{f^{-1}(\tilde{\lambda}(x)), f^{-1}(\tilde{\lambda}(y))\}. \\
f^{-1}(\tilde{\delta})(x\alpha y\beta z) &= \tilde{\delta}(f(x\alpha y\beta z)) \\
&= \tilde{\delta}(f(x)\alpha f(y)\beta f(z)) \\
&\geq \min^i\{\tilde{\delta}(f(x)), \tilde{\delta}(f(y)), \tilde{\delta}(f(z))\} \\
&= \min^i\{f^{-1}(\tilde{\delta}(x)), f^{-1}(\tilde{\delta}(y)), f^{-1}(\tilde{\delta}(z))\}. \\
\text{and} f^{-1}(\tilde{\lambda})(x\alpha y\beta z) &= \tilde{\lambda}(f(x\alpha y\beta z)) \\
&= \tilde{\lambda}(f(x)\alpha f(y)\beta f(z)) \\
&\leq \min^i\{\tilde{\lambda}(f(x)), \tilde{\lambda}(f(y)), \tilde{\lambda}(f(z))\} \\
&= \max^i\{f^{-1}(\tilde{\lambda}(x)), f^{-1}(\tilde{\lambda}(y)), f^{-1}(\tilde{\lambda}(z))\}.
\end{aligned}$$

Converse of the Theorem 4.3 by strengthening the condition on f as follows.

Theorem 4.4 Let $f: M \to S$ be onto homomorphism . Let $B = (\tilde{\delta}, \tilde{\lambda}_B)$ be an i.v intuitionistic fuzzy subset of S. If $f^{-1}(B)[f^{-1}(\tilde{\delta}_B), f^{-1}(\tilde{\lambda}_B)]$ is an i.v fuzzy weak bi-ideal of M, then $B = (\tilde{\delta}, \tilde{\lambda}_B)$ is an i.v intuitionistic fuzzy weak bi-ideal of S.

Proof. Let $x, y, z \in S$. Then $f(a) = x$, $f(b) = y$ and $f(c) = z$ for some $a, b, c \in M$ and $\alpha, \beta \in \Gamma$.

$$\begin{aligned}
\tilde{\delta}(x-y) &= \tilde{\delta}(f(a)-f(b)) \\
&= \tilde{\delta}(f(a-b)) \\
&= f^{-1}(\tilde{\delta})(a-b) \\
&\geq \min^i\{f^{-1}(\tilde{\delta})(a), f^{-1}(\tilde{\delta})(b)\} \\
&= \min^i\{\tilde{\delta}(f(a)), \tilde{\delta}(f(b))\} \\
&= \min^i\{\tilde{\delta}(x), \tilde{\delta}(y)\}. \\
\tilde{\lambda}(x-y) &= \tilde{\lambda}(f(a)-f(b)) \\
&= \tilde{\lambda}(f(a-b)) \\
&= f^{-1}(\tilde{\lambda})(a-b) \\
&\leq \max^i\{f^{-1}(\tilde{\lambda})(a), f^{-1}(\tilde{\lambda})(b)\} \\
&= \max^i\{\tilde{\lambda}(f(a)), \tilde{\lambda}(f(b))\} \\
&= \max^i\{\tilde{\lambda}(x), \tilde{\lambda}(y)\}. \\
\tilde{\delta}(x\alpha y\beta z) &= \tilde{\delta}(f(a)\alpha f(b)\beta f(c)) \\
&= \tilde{\delta}(f(a\alpha b\beta c)) \\
&= f^{-1}(\tilde{\delta})(a\alpha b\beta c) \\
&\geq \min^i\{f^{-1}(\tilde{\delta})(a), f^{-1}(\tilde{\delta})(b), f^{-1}(\tilde{\delta})(c)\} \\
&= \min^i\{\tilde{\delta}(f(a)), \tilde{\delta}(f(b)), \tilde{\delta}(f(c))\} \\
&= \min^i\{\tilde{\delta}(x), \tilde{\delta}(y), \tilde{\delta}(z)\}. \\
\tilde{\lambda}(x\alpha y\beta z) &= \tilde{\lambda}(f(a)\alpha f(b)\beta f(c)) \\
&= \tilde{\lambda}(f(a\alpha b\beta c)) \\
&= f^{-1}(\tilde{\lambda})(a\alpha b\beta c) \\
&\leq \max^i\{f^{-1}(\tilde{\lambda})(a), f^{-1}(\tilde{\lambda})(b), f^{-1}(\tilde{\lambda})(c)\} \\
&= \max^i\{\tilde{\lambda}(f(a)), \tilde{\lambda}(f(b)), \tilde{\lambda}(f(c))\} \\
&= \max^i\{\tilde{\lambda}(x), \tilde{\lambda}(y), \tilde{\lambda}(z)\}.
\end{aligned}$$

Theorem 4.5 Let $f: M \to S$ be an onto homomorphism. If $\tilde{\eta}$ is an i.v fuzzy weak bi-ideal of M, then $f(\tilde{\eta})$ is an i.v fuzzy weak bi-ideal of S.

Proof. Since $f(\tilde{\eta})(x') = \sup^i_{f(x)=x'}(\tilde{\eta}(x))$,
for $x' \in S$ and hence $f(\tilde{\eta})$ is nonempty. Let $x', y' \in S$ and
$\{x | x \in f^{-1}(x'-y')\} \supseteq \{x - y | x \in f^{-1}(x') \text{and} y \in f^{-1}(y')\}$
$\{x | x \in f^{-1}(x'y')\} \supseteq \{x\alpha y | x \in f^{-1}(x') \text{and} y \in f^{-1}(y')\}$.

$$\begin{aligned}
f(\tilde{\eta})(x'-y') &= \sup^i_{f(z)=x'-y'}\{\tilde{\eta}(z)\} \\
&\geq \sup^i_{f(x)=x', f(y)=y'}\{\tilde{\eta}(x-y)\} \\
&\geq \sup^i_{f(x)=x', f(y)=y'}\{\min^i\{\tilde{\eta}(x), \tilde{\eta}(y)\}\} \\
&= \min^i\{\sup^i_{f(x)=x'}\{\tilde{\eta}(x)\}, \sup^i_{f(y)=y'}\{\tilde{\eta}(y)\}\} \\
&= \min^i\{f(\tilde{\eta})(x'), f(\tilde{\eta})(y')\}. \\
f(\tilde{\lambda})(x'-y') &= \inf^i_{f(z)=x'-y'}\{\tilde{\lambda}(z)\} \\
&\leq \inf^i_{f(x)=x', f(y)=y'}\{\tilde{\lambda}(x-y)\} \\
&\leq \inf^i_{f(x)=x', f(y)=y'}\{\max^i\{\tilde{\lambda}(x), \tilde{\lambda}(y)\}\} \\
&= \max^i\{\inf^i_{f(x)=x'}\{\tilde{\lambda}(x)\}, \inf^i_{f(y)=y'}\{\tilde{\lambda}(y)\}\} \\
&= \max^i\{f(\tilde{\lambda})(x'), f(\tilde{\lambda})(y')\}. \\
f(\tilde{\eta})(x'\alpha y'\beta z') &= \sup^i_{f(h)=x'\alpha y'\beta z'}\{\tilde{\eta}(h)\} \\
&\geq \sup^i_{f(x)=x', f(y)=y', f(z)=z'}\{\tilde{\eta}(x\alpha y\beta z)\} \\
&\geq \sup^i_{f(x)=x', f(y)=y', f(z)=z'}\{\min^i\{\tilde{\eta}(x), \tilde{\eta}(y), \tilde{\eta}(z)\}\} \\
&= \min^i\{\sup^i_{f(x)=x'}\{\tilde{\eta}(x)\}, \sup^i_{f(y)=y'}\{\tilde{\eta}(y)\}, \sup^i_{f(z)=z'}\{\tilde{\eta}(z)\}\} \\
&= \min^i\{f(\tilde{\eta})(x'), f(\tilde{\eta})(y'), f(\tilde{\eta})(z')\}. \\
f(\tilde{\lambda})(x'\alpha y'\beta z') &= \inf^i_{f(h)=x'\alpha y'\beta z'}\{\tilde{\lambda}(h)\} \\
&\leq \inf^i_{f(x)=x', f(y)=y', f(z)=z'}\{\tilde{\lambda}(x\alpha y\beta z)\} \\
&\leq \inf^i_{f(x)=x', f(y)=y', f(z)=z'}\{\max^i\{\tilde{\lambda}(x), \tilde{\lambda}(y), \tilde{\lambda}(z)\}\} \\
&= \max^i\{\inf^i_{f(x)=x'}\{\tilde{\lambda}(x)\}, \inf^i_{f(y)=y'}\{\tilde{\lambda}(y)\}, \inf^i_{f(z)=z'}\{\tilde{\lambda}(z)\}\} \\
&= \max^i\{f(\tilde{\lambda})(x'), f(\tilde{\lambda})(y'), f(\tilde{\lambda})(z')\}.
\end{aligned}$$

Theorem 4.5*Let $f: M \to S$ be an onto homomorphism. If $\tilde{\eta}$ is an i.v fuzzy weak bi-ideal of M, then $f(\tilde{\eta})$ is an i.v fuzzy weak bi-ideal of S.*

Proof. Since $f(\tilde{\eta})(x') = \sup^i_{f(x)=x'}(\tilde{\eta}(x))$,
for $x' \in S$ and hence $f(\tilde{\eta})$ is nonempty. Let $x', y' \in S$ and
$\{x | x \in f^{-1}(x'-y')\} \supseteq \{x - y | x \in f^{-1}(x') \text{and} y \in f^{-1}(y')\}$
$\{x | x \in f^{-1}(x'y')\} \supseteq \{x\alpha y | x \in f^{-1}(x') \text{and} y \in f^{-1}(y')\}$.

$$\begin{aligned}
f(\tilde{\eta})(x'-y') &= \sup^i_{f(z)=x'-y'}\{\tilde{\eta}(z)\} \\
&\geq \sup^i_{f(x)=x', f(y)=y'}\{\tilde{\eta}(x-y)\} \\
&\geq \sup^i_{f(x)=x', f(y)=y'}\{\min^i\{\tilde{\eta}(x), \tilde{\eta}(y)\}\} \\
&= \min^i\{\sup^i_{f(x)=x'}\{\tilde{\eta}(x)\}, \sup^i_{f(y)=y'}\{\tilde{\eta}(y)\}\} \\
&= \min^i\{f(\tilde{\eta})(x'), f(\tilde{\eta})(y')\}. \\
f(\tilde{\lambda})(x'-y') &= \inf^i_{f(z)=x'-y'}\{\tilde{\lambda}(z)\} \\
&\leq \inf^i_{f(x)=x', f(y)=y'}\{\tilde{\lambda}(x-y)\} \\
&\leq \inf^i_{f(x)=x', f(y)=y'}\{\max^i\{\tilde{\lambda}(x), \tilde{\lambda}(y)\}\} \\
&= \max^i\{\inf^i_{f(x)=x'}\{\tilde{\lambda}(x)\}, \inf^i_{f(y)=y'}\{\tilde{\lambda}(y)\}\} \\
&= \max^i\{f(\tilde{\lambda})(x'), f(\tilde{\lambda})(y')\}. \\
f(\tilde{\eta})(x'\alpha y'\beta z') &= \sup^i_{f(h)=x'\alpha y'\beta z'}\{\tilde{\eta}(h)\} \\
&\geq \sup^i_{f(x)=x', f(y)=y', f(z)=z'}\{\tilde{\eta}(x\alpha y\beta z)\} \\
&\geq \sup^i_{f(x)=x', f(y)=y', f(z)=z'}\{\min^i\{\tilde{\eta}(x), \tilde{\eta}(y), \tilde{\eta}(z)\}\} \\
&= \min^i\{\sup^i_{f(x)=x'}\{\tilde{\eta}(x)\}, \sup^i_{f(y)=y'}\{\tilde{\eta}(y)\}, \sup^i_{f(z)=z'}\{\tilde{\eta}(z)\}\} \\
&= \min^i\{f(\tilde{\eta})(x'), f(\tilde{\eta})(y'), f(\tilde{\eta})(z')\}. \\
f(\tilde{\lambda})(x'\alpha y'\beta z') &= \inf^i_{f(h)=x'\alpha y'\beta z'}\{\tilde{\lambda}(h)\} \\
&\leq \inf^i_{f(x)=x', f(y)=y', f(z)=z'}\{\tilde{\lambda}(x\alpha y\beta z)\} \\
&\leq \inf^i_{f(x)=x', f(y)=y', f(z)=z'}\{\max^i\{\tilde{\lambda}(x), \tilde{\lambda}(y), \tilde{\lambda}(z)\}\} \\
&= \max^i\{\inf^i_{f(x)=x'}\{\tilde{\lambda}(x)\}, \inf^i_{f(y)=y'}\{\tilde{\lambda}(y)\}, \inf^i_{f(z)=z'}\{\tilde{\lambda}(z)\}\} \\
&= \max^i\{f(\tilde{\lambda})(x'), f(\tilde{\lambda})(y'), f(\tilde{\lambda})(z')\}.
\end{aligned}$$

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