

Educational Software for Stress Analysis of Idealized Closed Thin Walled Sections

J.S. Mohamed Ali*, Hasna Nur Fadhila, Burhani M. Burhani, Miah Mohammed Riyadh

Dept. of Mechanical Engineering, International Islamic University Malaysia, Kuala Lumpur, Malaysia

*Corresponding author E-mail: jaffar@iium.edu.my

Abstract

Aerospace structures are typically semi-monocoque structures that are made up of thin-walled closed section reinforced with stiffeners. Stress analysis of such closed thin-walled structures which are statically indeterminate is tedious and time consuming. An educational software which can aid students in carrying out stress analysis of such idealized thin-walled closed sections has been developed. The software enables students to select different types of wing torsion box sections with stiffeners, which may be subjected to bending, shear or torsional loads and evaluate the resulting stresses on it. The software allows the student to idealize a selected twin spar unsymmetrical wing section with multiple booms under multiple loads. Results from this software have been validated against the results in the literature. The software has been developed using MATLAB with graphical user interface (GUI) which is very user friendly.

Keywords: Aircraft Structures; Educational Software; Idealized thin walled sections; MATLAB; Stress Analysis.

1. Introduction

In this computer age, modern tools such as educational softwares and games have greatly enhanced the teaching-learning process. In order to provide effective and efficient engineering programs and curriculums, the teaching methods should be continuously improved [1]. It is very important that the students are able to apply a set of knowledge and skills to perform a set of crucial tasks within the learning environment. Thus, in order to fulfill the learning requirements, the use of educational software combined with traditional lectures can be an economical and efficient alternative [2]. In Mechanical Engineering for lower level courses, students have been equipped with educational tools such as MDSolids, a software developed by Philpot [3] for a course on "Mechanics of Materials". With the help of these softwares, the students can research with a variety of problems within a short amount of time, giving the students the necessary confidence in the subject. With the software as a step by step guidance in hand, the students can check their mistakes and correct themselves. This will save the student's time as well as that of the lecturers. Moreover, this software can also be used by the lecturers in order to frame projects which are short term.

Programming software has been created before to help in the education of less complicated courses such as Mechanics of Solids [4], [5], Dynamics, Calculus and Statics. DYNASOFT is an educational package which is web based and aids in the learning process of structural dynamics [6]. Similarly, SECav is an educational software which supports the learning and teaching process of numerical calculus courses [7].

Students are in need of educational software when it comes to complicated courses such as Aircraft Structures, Advanced Mechanics of Materials, Machine Design, etc. The students are left with home assignments and tedious exercises where they are unable to do proper validation of their results. The analysis involved

in these advanced courses are complex and involves a lot of hand calculations which could be tedious to a student, thus depriving the student a chance to do many exercises needed to master the subject. Consequently, a software which will be related for such complicated subjects will be exceptionally beneficial to students, as such program will be favorable to traditional teaching and learning methods as well as assist the students in validation.

Aerospace Engineers are offered with advanced courses such as Aircraft Structures which involve the stress analysis of thin-walled structures subject to shear, bending as well as torsion. Similar software for thin-walled sections are available such as GT Shear, which involves the analysis of shear and bending stress of thin walled open sections [8]. Moreover, a software for the analysis of typical idealized thin walled open sections is also available to students [9]. Both of these software are limited to thin walled open sections and there are no software developed for the analysis of thin walled closed sections which are more tedious as they are statically indeterminate structures.

Thus the aim of this project is to develop a program which can readily and swiftly perform calculations in order to calculate stresses on typical idealized closed thin-walled sections which is under the influence of shear, bending as well as torsional loads. The software can also do idealization of closed cross-sections and then perform stress analysis of the idealized section. This work is an extension of a previous work which has been done for idealized thin-walled open sections [9]. The software has been developed in such a way that any type of cross section can be auto-generated from just the correct inputs. The software can be applied on different types of symmetric or unsymmetrical closed cross sections which are used in aircraft structures with different structural loadings. MATLAB GUI has been used for the development of this software.

2. Methodology

A closed thin-walled section is made from a group of either flat or curved elements. The ratio of the thickness to the cross-sectional dimensions should be at least one tenth for the section to be considered as thin-walled. A typical airfoil section forms a torsion box of a wing which resists all the loads on the wing and may have a curved web in the leading edge. The spar cap, skin and the stringers present in such torsion box will resist all the loads on the wing. The skin when it is effective in bending, then the analysis becomes tedious and lengthy because of the integration involved in finding the properties of the section. Thus idealization of the section leads to simplification in the analysis as discussed in the following section.

2.1. Structural Idealization

Stress analysis of thin walled sections is a lengthy, complicated and tedious process. Therefore, a simplification is required to be made to reduce the complexity in analysis but without loss in accuracy. The simplified idealized structure, when subjected to certain loading conditions should behave nearly the same as the actual structure. In a typical wing section, as the stringers and spar flanges are small and hence the direct stress variations across them due to bending are small, it can be replaced by concentrations of areas known as booms. The direct stress is constant over the booms and it is located along the mid-line of the skin (Figure 1).



Fig. 1: Idea of Idealization

Although the skin carries some of the direct stress, it is mainly responsible for resisting shear stress while the stringers and the spar flanges carry most of the direct stresses. As a result, one can logically assume that all the direct stresses are carried by the booms while the skin is effective only in shear. Moreover the direct stress carrying capability of the skin can still be brought in by adding statically equivalent areas to the existing boom areas. As shown in Figure 2, the direct stress carrying capability of the panel with thickness t_D which is effective in bending can be replaced with equivalent boom areas and the skin which is ineffective in bending (i.e. $t_D = 0$) which means that the skin is effective only in taking shear. The equivalent boom areas are dependent on the stress distribution on the panel, so a statically equivalent areas B_1 and B_2 leads to very minimal error.

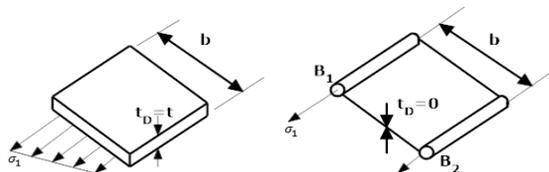


Fig. 2: Idealization of a panel

By equating the moment and forces for the panel and idealized section, one can obtain the statically equivalent boom areas [10] as

$$B_1 = \frac{tb}{6} \left(2 + \frac{\sigma_2}{\sigma_1} \right) \quad \text{and} \quad B_2 = \frac{tb}{6} \left(2 + \frac{\sigma_1}{\sigma_2} \right) \quad (1)$$

The developed software uses these equations to idealize any given symmetric/unsymmetric sections by giving proper weightage of boom areas based on the stress distribution. The software developed will be able to idealize a section that is subjected to both M_x and M_y . It can generate multiple boom sections placing it on the location of the stringer area.

2.2. Stress Analysis

A typical torsion box of a wing will be an idealized closed section that may be unsymmetrical with multiple boom areas and subjected to internal loads such as bending moments (M_x and M_y) shear forces (S_x and S_y) and torsional moment (T) resulting from the external forces lift and drag. Thus the software developed should be capable to evaluate stresses due to each individual component of these internal loads. Figure 3 below shows such typical idealized closed section subjected to internal loads.

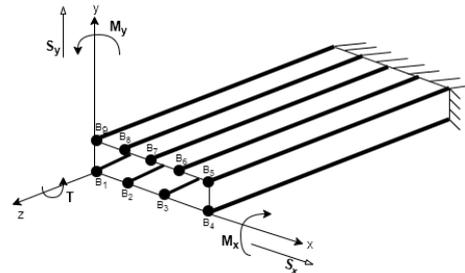


Fig. 3: A typical idealized thin walled closed section with internal loads

The software has been developed only for stress analysis of thin walled sections which are already idealized, and hence for non-idealized sections, the user first needs to idealize the given section before proceeding to the stress analysis. The software can idealize any single cell closed section and give directly boom areas as the output. The methods of calculations and theories have been referred from Megson [10].

2.2.1 Bending Stress

In an idealized section, the booms are singly responsible for the resistance of the bending moments and are responsible for producing direct bending stresses in the booms. The direct stresses (σ_z) with a general load (M_x and M_y) for such a section is given by the following equation:

$$\sigma_z = \left(\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \right) x + \left(\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \right) y \quad (2)$$

The coordinates (x, y) which are represented in equation (2) provides the location of any boom area on the cross-section. The sectional properties are referred to the axes Cxy in which the origin coincides with the centroid of the cross section area. The following equations gives the second moment of area, in which A is the boom area and I is the second moment of area of the cross section.

$$\begin{aligned} I_{xx} &= \int y^2 dB = \sum B y^2 \\ I_{yy} &= \int x^2 dB = \sum B x^2 \\ I_{xy} &= \int xy dB = \sum B xy \end{aligned} \quad (3)$$

2.2.2 Shear Stresses due to Shear load

The shear force on the section produces shear stresses which are resisted entirely by the skin of a cell. A shear force diagram can be used to obtain the shear force at any section which can be obtained by hand calculation or by using MDSolids [3]. After idealization, the cell is subjected to either or both shear loadings, S_x and S_y , whose line of action do not necessarily pass through the shear center. Therefore, the resulting shear flow distribution is due to the combined effects of shear and torsion.

Unlike the analysis of idealized open section which is straight forward since the shear stress/shear flow at the free edges are known (i.e. $q_{s,o} = 0$), the closed sections are statically indeterminate and hence the procedure is tedious.

The procedure followed to evaluate the shear stresses for a closed thin walled cell is as following:

Step 1: The cell is made statically determinate by ‘cutting’ a skin panel of the cell and the unknown closed section shear flow at the cut is called as $q_{s,o}$.

Step 2: The open section shear flow, represented by q_b on each panel is found by using (4), hence one needs to evaluate the section properties (centroid and moments of inertia) for a given cross section.

$$q_s = - \left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \right) \sum_{r=1}^n B_r x_r - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2} \right) \sum_{r=1}^n B_r y_r \quad (4)$$

Step 3: Use the Moment Equilibrium equation where, *External Moment = Internal Moment*

$$S_x \eta_0 - S_y \xi_0 = \oint q_b p ds + 2A q_{s,o} \quad (5)$$

where p is the moment arm of the force due to q_b about the moment centre. We can evaluate $q_{s,o}$ from the above equation as q_b is known.

Step 4: the final shear flows can be obtained from the following relation once $q_{s,o}$ is obtained:

$$q = q_b + q_{s,o} \quad (6)$$

and as a result, the shear stresses on any web or skin can be obtained by (7).

$$\tau = q/t \quad (7)$$

2.2.3 Shear Centre

The shear center of a cross section is a point which produces no twisting effects when shear loads are applied. In other words, it is the point through which there will be no torsion and will only be subjected to bending. Aeronautical structures, particularly wing and tail the shear center location need to be known to avoid twisting and hence aero-elasticity problems in these structures.

To locate the shear center of a cross section, the resulting shear flow is evaluated by applying an arbitrary shear load which passes through the shear center. The shear center for a section lies on the axis which has an axis of symmetry. The position of the shear center (ξ_s , η_s) has to be calculated for asymmetric thin-walled sections. In order to find ξ_s , an arbitrary shear load such as S_y has to be applied in such a way so that the applied load passes through the shear center. As a result, there is a shear flow distribution in the given cross-section as shown by (4). The procedure for evaluating the shear center of a closed thin walled idealized section is as follows:

Step 1: The cell is made statically determinate by ‘cutting’ a skin panel of the cell and call the unknown closed section shear flow at the cut as $q_{s,o}$.

Step 2: Find the open section shear flow q_b on each panel using (4), hence one needs to evaluate the section properties (centroid and moments of inertia) for a given cross section.

Step 3: The unknown shear flow at the cut, $q_{s,o}$, is evaluated using the equation (8) from the condition that load passing through the shear center produce no twisting,

$$q_{s,o} = - \frac{\oint (q_b/Gt) ds}{\frac{\oint ds}{Gt}} \quad (8)$$

Step 4: Once $q_{s,o}$ is obtained, the resulting shear flows q for each panel can be computed by adding q_b and $q_{s,o}$ from equation (6).

Step 5: Finally, by using the moment equilibrium equation, *External Moment = Internal Moment* we find the values for ξ_s and η_s .

2.2.4 Shear Stress due to Torsion

The shear flow due to torsion is constant around the cell as it just depends on the enclosed area of the closed cell. Although, the calculation of shear flow for torsion of a single cell is very simple and straightforward, it has been included in the software for the sake of completion. The shear flow due to torsion is evaluated around the cell panels by the following equation (9).

$$q = \frac{T}{2A} \quad (9)$$

3. Results and Discussion

MATLAB Graphical User Interface has been used for the programming of the components of stress analysis which has been discussed in the previous section. The steps followed for the software is as illustrated in the flow chart in Figure 4. The user is given an option for selecting the type of section, whether it is idealized or non-idealized from the first window. For the analysis to proceed, the user then enters the dimensions as input. If the section is not idealized, the software prompts the user to input the number of stringers as well as the dimensions to give the corresponding idealized boom areas as output. The section properties are evaluated and displayed once the user enters all the necessary dimension inputs after idealization. Proceeding further, it prompts the user to select the type of analysis to be done with the corresponding input for loading. In the end, the evaluated output stresses are displayed in both tabulated and graphical forms. The input and output windows are based on GUI which results in an absolute user friendly environment for the students to work. A user manual with solved examples provided with the software makes it further simpler for the users to follow. Figure 5 shows typical input/output windows for the program. The option of having a curved or straight web at the leading edge of the wing is also included in the analysis of this closed section.

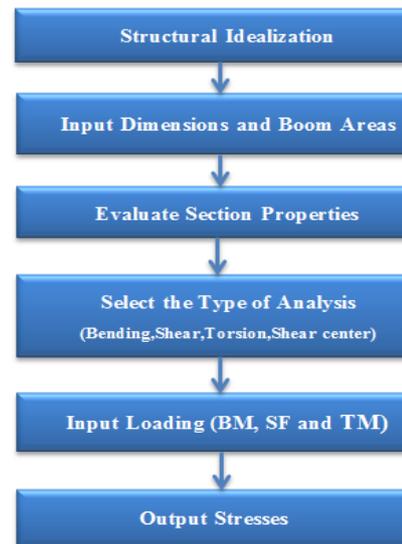


Fig. 4: Flow Chart for Stress Analysis

4. Validation of Results

Validation has been done by checking the individual components of the software. The accuracy of the developed software has been ensured by comparing the evaluated values with literature. Several case studies have been considered in order to illustrate the potential of the software.

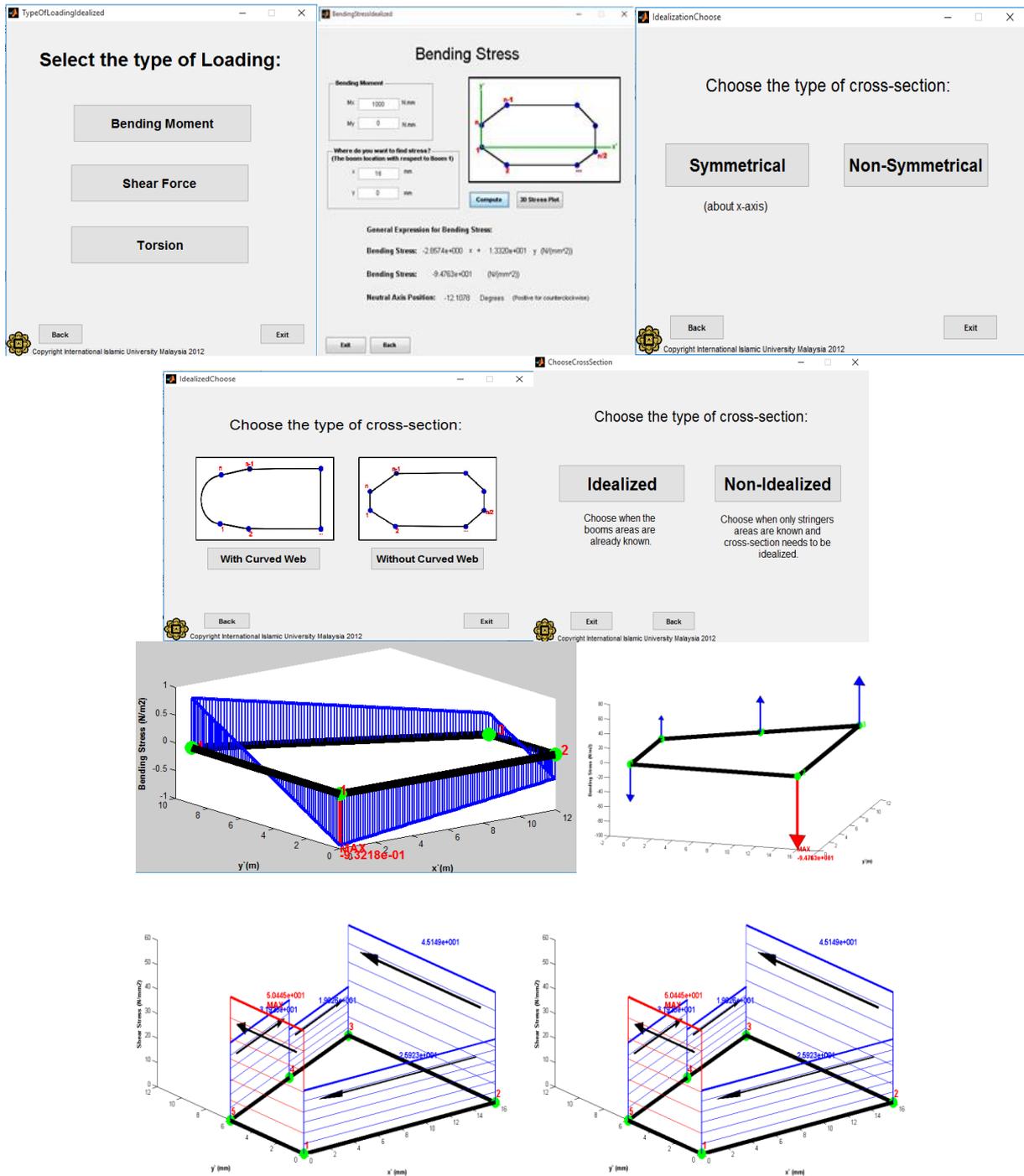


Fig. 5: Typical Input/output windows for closed section stress analysis

4.1. Idealization of an unsymmetrical box section

In this section a problem from Curtis [11] is considered to validate the idealization part of the software. In this section, the unsymmetrical section shown in Figure 6 is idealized into an arrangement of direct stress carrying booms positioned at the four corners and panels which are assumed to carry only shear stresses. The section has a uniform thickness of 0.1 inches. A vertical shear force acts at point 4 which has a positive magnitude of 1000 lb. It is assumed that the 1000 lb force which acts at point 4 is accompanied by a bending moment of $M_x=1000$ in-lb. The magnitude of the bending moments can be chosen arbitrarily as only ratios of stresses appear as shown in equation (1). It can be seen from the results presented in Table 1, that the software is able to predict the boom areas exactly as reported by the literature.

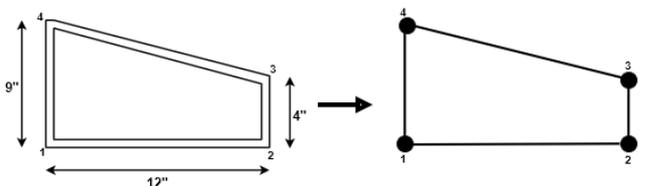


Fig. 6: Idealization of an unsymmetrical box section

Table 1: Boom Areas after idealization

Boom	Boom Area (software results)	Boom Area (Curtis [11])
1	64.67	64.70
2	90.92	91.20
3	99.70	99.40
4	67.13	67.10

4.2. Effect of Idealization

The maximum shear flow is found for the idealized section in Figure 6 which is subjected to a vertical shear force at point 4 that has a positive magnitude of 1000 lb. The magnitude of the maximum shear flow obtained by the software (and also reported in the literature [11]) was found to be 104 lb/in and it occurs on the vertical web 2-3. The corresponding results for the non-idealized section after lengthy analysis as reported by Curtis [11] is 113 lb/in. Thus, it can be seen that the maximum shear flow for any non-idealized section can be obtained very fast and with a reasonable accuracy by idealizing the section.

As a second case study, the advantage of idealization is demonstrated by finding the position of the shear center for both the idealized and non-idealized box beam section as shown in Figure 7 (Megson [10]). In this case $t_1=t_2=t_3=0.64\text{mm}$, $t_4=0.36\text{mm}$, $a=240\text{mm}$, $b=80\text{mm}$. The area of each stringer is 100mm^2 .

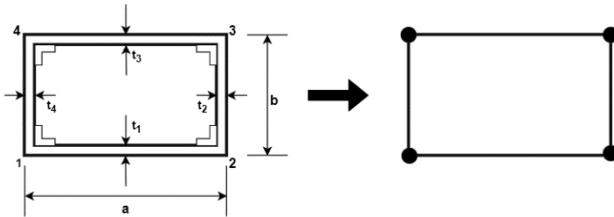


Fig. 7: Shear centre for idealized box section

The shear center for idealized section obtained from the software was 141.3 mm as against the exact value of 142.5 mm for non-idealized section [10]. The results are very close and thus it proves that idealization simplifies the analysis without great loss of accuracy. Thus the developed software can be used to idealize and analyze any unsymmetrical closed section under any loads. Thus idealizing a section is a faster and less tedious alternative to the lengthy tedious stress analysis of a non-idealized section.

4.3. Validation of bending stress and shear flow for a typical unsymmetrical wing section with lift and drag

A typical wing section with the curved web at the leading edge is considered from Curtis [11]. It's an idealized cantilevered box beam with an upward-directed, spanwise distributed load acting along one of the top stringers and an aft-directed distributed load acting along the middle of the leading edge. The leading edge is semi-circular as shown in Figure 8.

Table 2 gives the area and position of the boom areas, based on which the section properties are evaluated as shown in Table 3. With the shear forces and the bending moment evaluated at any specific point along the axis of the beam, these can be given as an input to the software to evaluate the stresses at that point. The results for shear flow in the webs and the bending stress on the stringers at $z=150$ are given in Table 4 and Table 5 respectively.

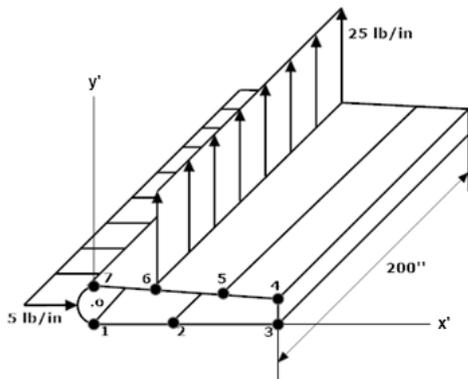


Fig. 8: Idealized cantilever box beam with uniform distributed loading

Table 2: Boom Areas and locations for idealized closed curved section

Stringer	x' (in)	y' (in)	Area (in^2)
1	0	0	2
2	12	0	1
3	24	0	0.5
4	24	6	0.5
5	16	7	0.75
6	8	8	0.75
7	0	9	2

Table 3: Section Properties of the idealized closed curved section

Section Properties	Software values	Literature values [11]
Centroid z (in)	7.2	7.2
Centroid y (in)	4.3	4.3
I_{zz} (in^4)	126.1	126.1
I_{yy} (in^4)	571.2	571.5
I_{zy} (in^4)	-28.2	-28.2
ξ_s (in)	5.414	5.414
η_s (in)	3.999	3.997

Table 4: Shear flow of an idealized closed curved section

Web	Shear flow (lb/in) Software values	Shear flow (lb/in) [11]
1-2	40.2657	40.29
2-3	157.371	157.38
3-4	199.048	199.05
4-5	149.6052	149.61
5-6	69.5364	69.55
6-7	-16.437	-16.472
7-1	-261.4453	-261.45

Table 5: Bending Stress of an idealized closed curved section

Stringer	Bending Stress (lb/in^2) Software values	Bending Stress (lb/in^2) [11]
1	11314	11314
2	8782.9	8783
3	6251.6	6252
4	-7416.4	-7416
5	-8006.9	-8007
6	-8597.3	-8597
7	-9187.8	-9188

The results obtained from software for both bending stress and shear flow matches exactly with the literature [11]. Thus the software can be used to carry out stress analysis and do parametric analysis of any type of single cell unsymmetrical torsion box with its corresponding loads. One should realize that, even though, the above problem is an idealized section, it may still take a considerable amount of time to solve this problem completely. If the student ends up with the wrong results after long effort, he may not repeat and hence loose interest in the subject. Hence such educational software available to the students will motivate them and make them gain confidence when they can compare their results step by step, with the software.

This software has been used extensively for the course on Aircraft Structures at International Islamic University Malaysia (IIUM) and it was very much appreciated and well received by the students. The student's feedback shows that they were greatly impressed and benefitted by the software and it has increased their interest in this course. Some students have volunteered themselves to further develop this software by adding additional modules.

5. Conclusion

An educational software for the stress analysis of idealized thin walled closed sections have been developed using a user-friendly MATLAB GUI. Through many case studies, the software has been validated and proven to be working well for any general shape of the section and for any loading. Moreover, the students can also generate sections of their interest and solve for different loading cases to increase their understanding on this topic.

The software has been tested in the classroom and very promising

results were found. It has been especially appealing to the students due to its user-friendly environment and it provides them a step by step solution procedure in solving difficult problems, thus correcting their mistakes by comparing with the software and learning independently without the help of lecturers. The student's feedback shows that they were greatly impressed and benefitted by the software and it has increased their interest in this course. The software which has been developed has proven to be an effective teaching and learning process for courses on Aircraft Structures.

Acknowledgement

The authors of this paper would like to acknowledge the Research Management Centre, International Islamic University Malaysia for the support of this work.

References

- [1] Cvetic B and Ilic OR, "Using educational software to enhance the teaching & learning in computer integrated manufacturing: Material requirements planning module," *Comput. Appl. Eng. Educ.*, vol. 23, no. 5, pp. 682–693, 2015.
- [2] Fu TT, "Applications of computer simulation in mechanism teaching," *Comput. Appl. Eng. Educ.*, vol. 11, no. 3, pp. 156–165, 2003.
- [3] Philpot TA, "MDSolids: Software to bridge the gap between lectures and homework in mechanics of materials," *Int. J. Eng. Educ.*, vol. 16, no. 5, pp. 401–407, 2000.
- [4] Beamboy, from <http://www.engineering.com/Default.aspx?grm2id=7&tabid=74>.
- [5] BeamAnalyzer, from <http://www.engineering.com/Default.aspx?grm2id=7&tabid=74>.
- [6] Panagiotopoulos CG and Manolis GD, "A web-based educational software for structural dynamics," *Comput. Appl. Eng. Educ.*, vol. 24, no. 4, pp. 599–614, 2016.
- [7] Ascheri ME and Pauletti C, "SECa Q . Educational Software for Numerical Calculus," pp. 1–2, 2017.
- [8] Thomas D and Gramoll KC, "GT Shear - An Interactive Graphic Program for Shear and Bending Stresses of Thin Walled Structures," 1994.
- [9] Mohamed Ali JS, Fadhila HN, and Aziz NAB, "Educational Software for Stress Analysis of Idealized Thin Walled Open Sections," *Appl. Mech. Mater.*, vol. 315, pp. 339–343, 2013.
- [10] Megson THG, "Aircraft Structures for Engineering Students," fourth ed. Oxford: Elsevier Ltd, 2007.
- [11] Curtis HD, "Fundamentals of Aircraft Structural Analysis," McGraw Hill., 1996, pp. 226-229.