International Journal of Engineering & Technology, 7 (4.10) (2018) 1041-1045

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

Characterizations of $(\gamma\beta)^{\alpha}(I,K)$-Continuous Mappings in Ideal Topological Spaces

N.Kalaivani [1]*,R.Narmada Devi[2] and T.Gunasekar[3]

[123]*Department of Mathematics*
Vel Tech Rangarajan Dr.Sakunthala R&D Institute of Science and Technology
Chennai,India
**Corresponding author E-mail:kalaivani.rajam@gmail.com*

Abstract

In this paper the notion of $(\gamma\beta)^{\alpha}(I,K)$ -Continuous Mappings is introduced and some of their properties are studied. Further the concept of contra $(\gamma\beta)^{\alpha}(I,K)$ -Continuous Mappings and(γ^{α}, β)-I-Continuous Mappings are introduced and properties are analyzed.

Keywords: *$(\gamma\beta)^{\alpha}(I,K)$ -Continuous Mappings; contra $(\gamma\beta)^{\alpha}(I,K)$ -Continuous Mappings; $(\gamma^{\alpha},\beta)-I$ -Continuous Mappings (2010)AMS Mathematics Subject Classification: 54C05, 54C08.*

1. Introduction

The α -open sets, operation on topological spaces, $\tau_{\alpha-\gamma}$, $\tau_{\alpha-\gamma-I}$, $\tau_{\alpha-\gamma-I}$ interior and $\tau_{\alpha-\gamma-I}$ closure operators are introduced respectively by Njastad [1], Kasahara [3], Ogata [4], Kalaivani [5] and Kalaivani et al [6].

In this paper, the notion of $(\gamma\beta)^{\alpha}(I,K)$ -Continuous Mappings, properties are studied. Further the contra $(\gamma\beta)^{\alpha}(I,K)$ -Continuous Mappings is introduced and their properties are analyzed. Also the $(\gamma^{\alpha},\beta)-I$ -Continuous Mappings, their properties are studied.

2. Preliminaries

The concepts of ideal[2], its properties[7], $A^{*}(I)$ -local function [8], $X=X^{*}$ [9], $\tau\cap I=\phi$ [10], $\beta(I,\tau)$ [11], Kuratowski closure operator, Kuratowski* closure operator, γ -semi-open set[12], (α - γ , β)-continuous mappings[13] are defined, studied and discussed earlier.

Dontechev and Przemki[14], Reilly and Vamanamurthy[15], Tong[16] and Maximilian Ganster and Ivan Reilly [17] studied about α-continuity and decompositions of continuity.

Here f_M denotes the mapping $f:(X,\tau,I)\to(Y,\sigma,K)$ and TX_I, TY_K denotes the ideal topological spaces (X,τ,I) , (Y,σ,K).Then CM denotes continuous mapping, C denotes continuity, OS denotes open set, CS denotes closed set, inv denotes inverse image, M denotes mapping, IM denotes the identity mapping and $(\gamma\beta)^{\alpha}(I,K)$ - CM denotes the $(\gamma\beta)^{\alpha}(I,K)$ -Continuous Mapping.

3. $(\gamma\beta)^{\alpha}(I,K)$ -Continuous Mappings

Definition 3.1. A mapping f_M is said to be a $(\gamma\beta)^{\alpha}(I,K)$ - CM if for each $\beta^{\alpha}K$ -OS,G of TY_K , the inv $f_M^{-1}(G)$ is a $\gamma^{\alpha}I$ -OS in TX_I .

Theorem3.1. Let f_M be a mapping, then the following statements are equivalent:

(i) f_M is a $(\gamma\beta)^{\alpha}(I,K)$ -CM.

(ii) For each element $a\in TX_I$ and $Q\in\sigma_{\beta^{\alpha}K}$ containing $f_M(a)$, there exists $P\in\tau_{\gamma^{\alpha}I}$ containing a such that $f_M(P)\subseteq Q$.

(iii) The inv of each $\sigma_{\beta^{\alpha}K}$ -CS of TY_K is a $\tau_{\gamma^{\alpha}I}$ -CS in TX_I .

(iv) $cl_{\tau_{\gamma}}(\text{int}^{*}_{\tau_{\gamma}}(cl_{\tau_{\gamma}}(f_M^{-1}(D))))\subseteq f_M^{-1}(cl_{\sigma_{\beta}}(D))$ for each $D\subseteq TY_K$.

(v) $f_M(cl_{\tau_{\gamma}}(\text{int}^{*}_{\tau_{\gamma}}(cl_{\tau_{\gamma}}(C))))\subseteq f_M(C)$ for each $C\in TX_I$.

Proof. $(i)\Rightarrow(ii)$ Let $a\in TX_I$ and Q be any $\beta^{\alpha}K$ - OS of TY_K containing f_M (a). Let $H=f_M^{-1}(Q)$, then by Definition 3.1 , H is a $\tau_{\gamma^{\alpha}I}$ - OS containing a and $f_M(H)\subseteq Q$.

$(ii)\Rightarrow(iii)$ Let C be a $\beta^{\alpha}K$ - CS of TY_K . Set $S=TY_K-C$ then S is a $\beta^{\alpha}K$ - OS in TY_K .Let $b\in f_M^{-1}(S)$, by (ii) , there exists a $\gamma^{\alpha}I$ -

Copyright © 2018 Authors. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

OS, G of TX_I containing b such that $f_M(G)\subseteq S$. We obtain $b\in G\subseteq \text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(\text{int}_{\tau_\gamma}(G)))\subseteq \text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(\text{int}_{\tau_\gamma}(f_M^{-1}(S))))$ and hence $b\in f_M^{-1}(S)\subseteq \text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(\text{int}_{\tau_\gamma}(f_M^{-1}(S))))$. This shows that $f_M^{-1}(S)$) is a $\tau_{\gamma^\alpha I}$ -OS in TX_I . Hencewe obtain that,

$f_M^{-1}(C)=TX_I - f_M^{-1}(TY_K - C)=TX_I - f_M^{-1}(S)$ is a $\gamma^\alpha I$ -CS in TX_I .

$(iii)\Rightarrow(iv)$ Let M be any subset of TY_K .Since $cl_{\sigma_\beta}(M)$ is a CS in TY_K , by (iii) , $f_M^{-1}(cl_{\sigma_\beta}(M))$ is a $\gamma^\alpha I$ -CS and $TX_I - f_M^{-1}(cl_{\sigma_\beta}(M))$ is a $\gamma^\alpha I$ -OS. Thus $TX_I - f_M^{-1}(cl_{\sigma_\beta}(M))\subseteq \text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(\text{int}_{\tau_\gamma}(TX_I - f_M^{-1}(cl_{\sigma_\beta}(M)))))=$ $TX_I - cl_{\tau_\gamma}(\text{int}^*_{\tau_\gamma}(cl_{\tau_\gamma}(f_M^{-1}(cl_{\sigma_\beta}(M)))))$.

Hence $cl_{\tau_\gamma}(\text{int}^*_{\tau_\gamma}(cl_{\tau_\gamma}(f_M^{-1}(M))))\subseteq f_M^{-1}(cl_{\sigma_\beta}(M))$.

$(iv)\Rightarrow(v)$ Let L be any subset of TX_I . By (iv) , $cl_{\tau_\gamma}(\text{int}^*_{\tau_\gamma}(cl_{\tau_\gamma}(L)))\subseteq cl_{\tau_\gamma}(\text{int}^*_{\tau_\gamma}(cl_{\tau_\gamma}(f_M^{-1}(f_M(L)))))\subseteq f_M^{-1}(f_M(L))$ and hence $f_M(cl_{\tau_\gamma}(\text{int}^*_{\tau_\gamma}(cl_{\tau_\gamma}(L))))\subseteq cl_{\sigma_\beta}(f_M(L))$.

$(v)\Rightarrow(i)$ Let H be any $\beta^\alpha K$ -OS of TY_K . Then, by (v) $f_M(cl_{\tau_\gamma}(\text{int}^*_{\tau_\gamma}(cl_{\tau_\gamma}(f_M^{-1}(TY_K - H)))))\subseteq cl_{\sigma_\beta}(f_M(f_M^{-1}(TY_K - H)))$ $\subseteq cl_{\sigma_\beta}(TY_K - H)=TY_K - H$. Therefore, $cl_{\tau_\gamma}(\text{int}^*_{\tau_\gamma}(cl_{\tau_\gamma}(f_M^{-1}(TY_K - H))))\subseteq f_M^{-1}(TY_K - H)\subseteq TX_I - f_M^{-1}(H)$. Hence we obtain that $f_M^{-1}(H)\subseteq \text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(\text{int}_{\tau_\gamma}(f_M^{-1}(H))))$. This implies that $f_M^{-1}(H)$ is a $\gamma^\alpha I$ -OS.Hence, f_M is a $(\gamma\beta)^\alpha(I,K)$ - CM.

Remark 3.1. Every CM need not be a $(\gamma\beta)^\alpha(I,K)$ -CM: Illustrated by the following example.

Example 3.1. Let $X=\{d,e,f\},\tau=\{\phi,X,\{d\},\{e\},\{d,e\},\{d,f\}\}$ $\sigma=\{\phi,Y,\{d\},\{e\}\{d,e\},\{d,f\}\},I=\{\phi,\{d\},\{f\},\{d,f\}\}$ and $K=\{\phi,\{d\}\}$.

The γ operation on τ is given as follows: $\gamma(\mathsf{M})=cl(\mathsf{M})$, for $\mathsf{M}\in\tau$ .Then $\tau_{\gamma^\alpha I}=\{\phi,X,\{e\},\{d,f\}\}$.

We define β on σ as follows $\mathrm{H}^\beta=\begin{cases}\mathrm{H} & if\ e\in\mathrm{H}\\ cl(\mathrm{H}) & if\ e\notin\mathrm{H}\end{cases}$.

Then $\sigma_{\beta^\alpha K}=\{\phi,Y,\{e\},\{d,e\},\{d,f\}\}$.

Hence the IM f_M is a CM, but it is not a $(\gamma\beta)^\alpha(I,K)$ - CM, since $\{d,e\}\in\sigma_{\beta^\alpha K}$ and $f_M^{-1}(\{d,e\})=\{d,e\}\notin\tau_{\gamma^\alpha I}$.

Theorem 3.2. Let f_M be a M and I,K are ideals of TX_I , TY_K . If $I = K = \{\phi\}$ or I_n , then the concept of α - (γ,β) - C and $(\gamma\beta)^\alpha(I,K)$ - C are equivalent.

Proof. Follows from the 3.1.Theorem's Proof.

Theorem3.3. Let f_M be a M and $L\in\sigma$ .Then $f_M^{-1}(L^*_\gamma)=(f_M^{-1}(L))^*$ implies that $f_M^{-1}(cl^*_{\tau_\gamma}(L))\subseteq cl^*_{\tau_\gamma}(f_M^{-1}(L))$.

Proof. Since $f_M^{-1}(cl^*_{\tau_\gamma}(L))=f_M^{-1}(L\cup L^*_\gamma)=f_M^{-1}(L)\cup f_M^{-1}(L^*_\gamma)\subseteq$.

Remark 3.2. The converse of the Theorem 3.3 is not true . It is demonstrated by the example given below.

Example 3.2. Let $X=Y=\{1,2,3\}$, $\tau=\{\phi,X,\{1\},\{3\},\{1,2\},\{1,3\}\}$, $\sigma=\{\phi,Y,\{1\},\{3\},\{1,2\},\{1,3\}\}$, $I=\{\phi,\{1\},\{3\},\{1,3\}\}$ and $K=\{\phi,\{2\}\}$.

γ on τ is defined as follows : $F^\gamma=\begin{cases}\mathrm{F}\ if\ \mathrm{F}=\{1\}\\ F\cup\{3\}\ if\ \mathrm{F}\neq\{1\}\end{cases}$ for $F\in\tau$ .Then $\tau_{\gamma^\alpha I}=\{\phi,X,\{1\},\{3\},\{1,3\}\}$.

Also we define the operation β on σ : for every $\mathrm{F}\in\sigma$, $\mathrm{F}^\beta=\begin{cases}\mathrm{F}\ if\ \mathrm{F}=\{1\}\\ \mathrm{F}\cup\{3\}\ if\ \mathrm{F}\neq\{1\}\end{cases}$

Then $\sigma_{\beta^\alpha K}=\{\phi,Y,\{1\},\{3\},\{1,2\},\{1,3\}\}$

We define a mapping f_M as $f_M(\{1\})=\{1\}$; $f_M(\{2\})=\{2\}$; $f_M(\{3\})=\{3\}$.

For the subset $\{1,2\}\in\sigma$ we have $f_M^{-1}(\{1,2\})^*=\{1,2\}$ and($f_M^{-1}(\{1,2\}))^*=(\{1,2\})^*=\{2\}$.

Thus $f_M^{-1}(P^*)\subseteq(f_M^{-1}(P))^*$ for $P=\{1,2\}\in\sigma$. But $f_M^{-1}(cl^*_{\tau_\gamma}(P))\subseteq cl^*_{\tau_\gamma}(f_M^{-1}(P))$ for each $P\in\sigma$.

Lemma 3.1. Let $H,C\in TX_I$ .Then the following properties hold:

(i) $H\in\tau_{\gamma^\alpha I}$ if and only if there exists $U\in\tau$ such that $U\subseteq H\subseteq\text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(U))$

(ii) If $H\in\tau_{\gamma^\alpha I}$ and $H\subseteq C\subseteq\text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(H))$, then $C\in\tau_{\gamma^\alpha I}$.

Proof . (i) Let H be a $\gamma^\alpha I$ -OS, then $H\subseteq\text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(\text{int}_{\tau_\gamma}(H)))$ .Let $\mathsf{A}=\text{int}_{\tau_\gamma}(H)$ .Then, $\mathsf{A}\subseteq H\subseteq\text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(\mathsf{A}))$.

Conversely, let $\mathsf{A}\subseteq H\subseteq\text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(\mathsf{A}))$ for some $\mathsf{A}\in\tau$. Since $\mathsf{A}\subseteq H$, $\mathsf{A}\subseteq\text{int}_{\tau_\gamma}(H)$ and hence $cl^*_{\tau_\gamma}(\mathsf{A})\subseteq cl^*_{\tau_\gamma}(\text{int}_{\tau_\gamma}(H))$.

Hence we obtain that $H\subseteq\text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(\text{int}_{\tau_\gamma}(H))))$ .Then H is a $\gamma^\alpha I$ -OS.

(ii) Since H is a $\gamma^\alpha I$ -OS, there exists an OS, k such that $\kappa\subseteq H\subseteq\text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(\kappa))$ .We have $\kappa\subseteq H\subseteq C\subseteq\text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(H))$ implies that $\kappa\subseteq\text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(\kappa))$. Using the (i) result, $C\in\tau_{\gamma^\alpha I}$.

4. Contra $(\gamma\beta)^{\alpha}(I,K)$ -Continuous Mappings

Definition 4.1. A M f_M is said to be a contra $(\gamma\beta)^{\alpha}(I,K)$ -CM if the set $f_M^{-1}(\wp)$ is a $\gamma^{\alpha}I$ -CS in $\tau_{\gamma^{\alpha}I}$ for each $\beta^{\alpha}K$ -OS, $\wp$ of $\sigma_{\beta^{\alpha}K}$.

Definition 4.2. A M f_M is said to be a $(\gamma\beta)^{\alpha}(I,K)$ -CM if the set $f_M^{-1}(\mathrm{S})$ is a $\gamma^{\alpha}I$ -OS in $\tau_{\gamma^{\alpha}I}$ for each $\beta^{\alpha}K$ -OS, S of $\sigma_{\beta^{\alpha}K}$.

Remark 4.1. The concept of contra $(\gamma\beta)^{\alpha}(I,K)$ C and $(\gamma\beta)^{\alpha}(I,K)$ C are independent as shown by the following two examples.

Example 4.1. Let $X=\{k,l,m\}, \tau=\{\phi,X,\{k\},\{l\},\{k,l\},\{k,m\}\}$ and $I=\{\phi,\{k\}\}$.

The γ on τ is described as: $\mathrm{B}^{\beta}=\begin{cases} B \text{ if } l\in B \\ cl(B) \text{if } l\notin B \end{cases}$

Then $\tau_{\gamma^{\alpha}I}=\{\phi,X,\{k,l\},\{k,m\}\}$

Let $Y=\{k,l,m\}, \sigma=\{\phi,Y,\{k\},\{l\},\{k,l\},\{k,m\}\}$ and $K=\{\phi,\{k\},\{m\},\{k,m\}\}$. β on σ is given as follows: for $\square\in\sigma$, $\beta(\square)=cl(\square)$.Then $\sigma_{\beta^{\alpha}K}=\{\phi,Y,\{l\},\{k,m\}\}$.

The M f_M is defined as $f_M(\{k\})=\{m\}; f_M(\{l\})=\{k\}; f_M(\{m\})=\{l\}$. Then for every $P\in\sigma_{\beta^{\alpha}K}$, $f_M^{-1}(P)$ is a $\gamma^{\alpha}I$ -CS in TX_I. Hence f_M is a contra $(\gamma\beta)^{\alpha}(I,K)$ -CM. But $f_M^{-1}(\{l\})=\{m\}$ is not a $\gamma^{\alpha}I$ -OS in TX_I. Hence f_M is not a $(\gamma\beta)^{\alpha}(I,K)$ -CM.

Example 4.2. Let $X=\{p,q,r\}$, $\tau=\{\phi,X,\{p\},\{r\},\{p,q\},\{p,r\}\}$ and $I=\{\phi,\{q\}\}$.

Then γ on τ is defined for $D\in\tau$ as follows:

$D^{\gamma}=\begin{cases} D \text{ if } D=\{p\} \\ D\cup\{r\} \text{if } D\neq\{p\} \end{cases}$

Then $\tau_{\gamma^{\alpha}I=}\{\phi,X,\{p\},\{r\},\{p,q\},\{p,r\}\}$

Let $Y=\{p,q,r\}$, $\sigma=\{\phi,Y,\{p\},\{q\},\{p,q\},\{p,r\}\}$ and $K=\{\phi,\{p\},\{r\},\{p,r\}\}$.

We also define β on σ as follows: $\beta(\mathrm{E})=cl(\mathrm{E})$ for every $\mathrm{E}\in\sigma$.Then $\sigma_{\beta^{\alpha}K}=\{\phi,Y,\{q\},\{p,q\},\{p,r\}\}$.

We define f_M as $f_M(\{p\})=\{q\}; f_M(\{q\})=\{r\}; f_M(\{r\})=\{p\}$. Then for every $E\in\sigma_{\beta^{\alpha}K}$, $f_M^{-1}(E)$ is a $\gamma^{\alpha}I$ -OS in TX_I. Hence f_M is a $(\gamma\beta)^{\alpha}(I,K)$ -CM. But $f_M^{-1}(\{q\})=\{p\}$; $f_M^{-1}(\{p,q\})=\{p,r\}$ are not $\gamma^{\alpha}I$ -CS in TX_I.Hence f_M is not a contra $(\gamma\beta)^{\alpha}(I,K)$ -CM.

Theorem 4.1. A mapping f_M is a contra $(\gamma\beta)^{\alpha}(I,K)$ - CM if and only if the inv of each $\beta^{\alpha}K$ -CS in TY_K is a$\gamma^{\alpha}I$-OS in TX_I.

Proof. Proof follows from the Definition 4.1.

Theorem 4.2. Let f_M and $g:(Y,\sigma,K)\to(Z,\varsigma,W)$ be any two mappings.

(i) $g\circ f$ is a contra $(\gamma\beta)^{\alpha}(I,W)$ -CM , if g is $(\gamma\beta)^{\alpha}(K,W)$ -CM and f_M is contra $(\gamma\beta)^{\alpha}(I,K)$ -CM.

(ii) $g\circ f$ is a contra $(\gamma\beta)^{\alpha}(I,W)$ -CM , if g is a contra $(\gamma\beta)^{\alpha}(K,W)$ -CM and f_M is an $(\gamma\beta)^{\alpha}(I,K)$ - CM.

Proof. Proof follows from the Definition 4.1.

.

Definition 4.3. A mapping f_M is called a perfectly contra $(\gamma\beta)^{\alpha}(I,K)$ -CM if the inv of each $\beta^{\alpha}K$ -OS in TY_K is a $\gamma^{\alpha}I$ - clopen set in TX_I

Example 4.4. Let $X=\{1,2,3\}, \tau=\{\phi,X,\{1\},\{2\},\{1,2\},\{1,3\}\}$, and $I=\{\phi,\{1\}\}$.

γ on τ as follows: $\mathrm{B}^{\gamma}=\begin{cases} \mathrm{B} \text{ if } 2\in B \\ cl(B) \text{if } 2\notin B \end{cases}$

Then $\tau_{\gamma^{\alpha}I=}\{\phi,X,\{2\},\{1,2\},\{1,3\}\}$

Let $Y=\{1,2,3\}, \sigma=\{\phi,Y,\{1\},\{2\},\{1,2\},\{1,3\}\}$ and $K=\{\phi,\{1\},\{3\},\{1,3\}\}$ β on σ as follows: $\beta(P)=cl(P)$, for $P\in\sigma$ Then $\sigma_{\beta^{\alpha}K}=\{\phi,Y,\{2\},\{1,3\}\}$.

We define the M f_M as $f_M(\{1\})=\{3\}; f_M(\{2\})=\{2\}; f_M(\{3\})=\{1\}$.

Then for $\mathrm{H}\in\sigma_{\beta^{\alpha}K}$, $f_M^{-1}(H)$ is a $\gamma^{\alpha}I$ -clopen set in TX_I. Hence f_M is a perfectly contra $(\gamma\beta)^{\alpha}(I,K)$ -CM.

Remark 4.3. Every perfectly contra $(\gamma\beta)^{\alpha}(I,K)$ -CM is a contra $(\gamma\beta)^{\alpha}(I,K)$ -CM and $(\gamma\beta)^{\alpha}(I,K)$ -CM.

Remark 4.4.A contra $(\gamma\beta)^{\alpha}(I,K)$ -CM may not be perfectly contra $(\gamma\beta)^{\alpha}(I,K)$ -CM. The example given below illustrates this remark.

Let $X=\{1,2,3\}$, $\tau=\{\phi,X,\{1\},\{2\},\{1,2\},\{1,3\}\}$ and $I=\{\phi,\{1\}\}$.

γ on τ as follows: $B^{\gamma}=\begin{cases} \mathrm{B} \text{ if } 2\in B \\ cl(B) \text{if } 2\notin B \end{cases}$

Then $\tau_{\gamma^{\alpha}I=}\{\phi,X,\{2\},\{1,2\},\{1,3\}\}$.

Let $Y=\{1,2,3\}, \sigma=\{\phi,Y,\{1\},\{2\},\{1,2\},\{1,3\}\}$ and $K=\{\phi,\{1\},\{3\},\{1,3\}\}$. β on σ as follows: $\beta(P)=cl(P)$, for $P\in\sigma$ and we obtain that $\sigma_{\beta^{\alpha}K}=\{\phi,Y,\{2\},\{1,3\}\}$

We define M f_M as $f_M(\{1\})=\{3\}; f_M(\{2\})=\{1\}; f_M(\{3\})=\{2\}$.

Then for every $\mathbf{Z}\in\sigma_{\beta^{\alpha}K}$, $f_M^{-1}(\mathbf{Z})$ is a $\gamma^{\alpha}I$ -CS in TX_I. Hence

f_M is a contra $(\gamma\beta)^{\alpha}(I,K)$ -CM. But $f_M^{-1}(\{2\})=\{3\}$ is not a $\gamma^{\alpha}I$ -OS in TX_I. Hence f_M is not a $(\gamma\beta)^{\alpha}(I,K)$ -CM.

Theorem 4.3. For the M f_M ,statements described below are equivalent:

(i) f_M is a perfectly contra $(\gamma\beta)^{\alpha}(I,K)$ -CM

(ii) f_M is contra $(\gamma\beta)^{\alpha}(I,K)$ -CM and $(\gamma\beta)^{\alpha}(I,K)$ -CM.

Proof. Follows from the Definitions 4.1, 4.2 and 4.3.

5. (γ^{α},β) - I -Continuous Mappings

Definition 5.1.A subset A of an ideal topological space is called a $I\beta$ -OS if $A\subseteq cl_{\tau_\gamma}(\text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(A)))$

Definition 5.2. A M f_M is defined as a (γ^{α},β) -I-CM if for every $I\beta$ -open set V of TY_K , $f_M^{-1}(V)$ is a $\gamma^{\alpha}I$ -OS in TX_I.

Theorem 5.1. If a M f_M is a (γ^{α},β) - I -CM, then f_M is an (γ^{α},β) -CM.

Proof. Immediate from the above illustrated Theorem 4.1.

Theorem 5.2. Let f_M be a M. Statements given below are same:

(i) f_M be a (γ^{α},β) - I -CM.

(ii) For each b and each $I\beta$ -OS, $H\subseteq TY_K$ containing $f_M(b)$, there exists $G\in\tau_{\alpha_\gamma}$ such that $b\in G$, $f_M(G)\subseteq H$.

(iii) The inv of each $I\beta$ -CS in TY_K is a $\gamma^{\alpha}I$ -CS.

(iv) $cl_{\tau_\gamma}(\text{int}^*_{\tau_\gamma}(cl_{\tau_\gamma}(f_M^{-1}(B))))\subseteq f_M^{-1}(cl_{\sigma_\beta}(B))$ for $B\subseteq TY_K$.

(v) $f_M(cl_{\tau_\gamma}(\text{int}^*_{\tau_\gamma}(cl_{\tau_\gamma}((\text{A})))))\subseteq cl_{\sigma_\beta}(f_M(\text{A}))$ for $\text{A}\subseteq TX_I$.

Proof . Proof is analogous to the proof of the 4.1.Theorem.

Corollary 5.1. Let f_M be a (γ^{α},β) - I - CM, then

(i) $f_M(cl^*_{\tau_\gamma}(\Upsilon))\subseteq cl_{\sigma_\beta}(f_M(\Upsilon))$ for each $\Upsilon\in\gamma$ - $PIO(X)$.

(ii) $cl^*_{\tau_\gamma}((f_M^{-1}(\Im))\subseteq f_M^{-1}(cl_{\sigma_\beta}(\Im))$ for each $\Im\in\gamma$ - $PIO(Y)$.

Proof. Proof is analogous to the proof to the corollary 4.1.

s

Definition 5.3.A subset □ of an ideal TS is said to be a γI -open set with respect to the operation γ on τ if

$\square\subseteq\text{int}_{\tau_\gamma}(cl^*_{\tau_\gamma}(\square))\cup cl^*_{\tau_\gamma}(\text{int}_{\tau_\gamma}(\square))$.

Definition 5.4. A M f_M is called a (γ,β^{α}) - I -open mapping if the image of each γI -OS in TX_I is a $\beta^{\alpha}I$ -OS of TY_I .

Theorem 5.3. A M f_M is said to be a (γ,β^{α}) - I -open mapping if and only if for each subset $\aleph\subseteq TY_I$ and each γI -CS, F of TX_I containing $f_M^{-1}(\aleph)$, there exists a $\beta^{\alpha}I$ -CS, $H\subseteq TY_I$ containing W such that $f_M^{-1}(H)\subseteq F$.

Proof. Let $\wp=TY_I-f_M(TX_I-\Im)$ .Since $f_M^{-1}(\aleph)\subseteq\Im$,s.Since f_M is a (γ,β^{α}) - I - open mapping, then $\wp$ is a $\beta^{\alpha}I$ - CS and $f_M^{-1}(\wp)=TX_I-f_M^{-1}(f_M(TX_I-\Im))\subseteq TX_I-(TX_I-\Im)=\Im$.
Conversely, let ℓ be any γI -OS of TX_I and $\aleph=TY_I-f_M(\ell)$.
Then, $f_M^{-1}(\aleph)=TX_I-f_M^{-1}(f_M(\ell))\subseteq TX_I-\ell$ and $TX_I-\ell$ is a γI -CS. There exists a $\beta^{\alpha}I$ - CS,$\mathfrak{H}$ of TY_I containing $\aleph$ such that $f_M^{-1}(\wp)\subseteq TX_I-\ell$. Then, $f_M^{-1}(\wp)\cap\ell=\phi$ and $\wp\cap f_M(\ell)=\phi$.
Therefore, $TY_I-f_M(\ell)\supseteq\wp\supseteq\aleph=TY_I-f_M(\ell)$ and $f_M(\ell)$ is a $\beta^{\alpha}I$ - OS in TY_I . This implies that f_M is a (γ,β^{α}) - I -open mapping.

Corollary 5.1. If f_M is a (γ,β^{α}) - I - open mapping, then these properties hold:

(i) $f_M^{-1}(cl_{\sigma_\beta}(\text{int}_{\sigma_\beta}(cl_{\sigma_\beta}(\aleph))))\subseteq cl_{\tau_\gamma}(f_M^{-1}(\aleph))$ for $\aleph\subseteq TY_I$.

(ii) $f_M^{-1}(cl^*_{\sigma_\beta}(\text{O}))\subseteq cl_{\tau_\gamma}(f_M^{-1}(\text{O}))$ for γ -preopen set O of TY_I .

Proof. (i) Let $\aleph$ be any subset of TY_I , then $cl_{\tau_\gamma}(f_M^{-1}(\aleph))$ is a γI - CS in TX_I . By Theorem 4.3, there exists a $\beta^{\alpha}I$ -CS,□ $\subseteq TY_I$ containing $\aleph$ such that $f_M^{-1}(\square)\subseteq cl_{\tau_\gamma}(f_M^{-1}(\aleph))$. Since $TY_I-\square$ is a $\beta^{\alpha}I$ - OS, $f_M^{-1}(TY_I-\square)\subseteq f_M^{-1}(\text{int}_{\sigma_\beta}(cl^*_{\sigma_\beta}(\text{int}_{\sigma_\beta}(TY_I-\square))))$
and we obtain that

$$f_M^{-1}(cl_{\sigma_\beta}(\text{int}^*_{\sigma_\beta}(cl_{\sigma_\beta}(\aleph))))\subseteq f_M^{-1}(cl_{\sigma_\beta}(\text{int}^*_{\sigma_\beta}(cl_{\sigma_\beta}(\square))))\subseteq f_M^{-1}(\square)\subseteq cl_{\tau_\gamma}(f_M^{-1}(\aleph))$$

Therefore $f_M^{-1}(cl_{\sigma_\beta}(\text{int}^*_{\sigma_\beta}(cl_{\sigma_\beta}(\aleph))))\subseteq cl_{\tau_\gamma}(f_M^{-1}(\aleph))$

(ii) Let O be any γ - preopen set of TY_I . By using (i) result,

$$f_M^{-1}(cl^*_{\sigma_\beta}(\text{O}))\subseteq f_M^{-1}(cl_{\sigma_\beta}(\text{O}))\subseteq f_M^{-1}(cl_{\sigma_\beta}(\text{int}_{\sigma_\beta}(cl_{\sigma_\beta}(\text{O}))))\subseteq f_M^{-1}(cl_{\sigma_\beta}(\text{int}^*_{\sigma_\beta}(cl_{\sigma_\beta}(\text{O}))))$$

References

[1] O. Njastad, On some classes of nearly open sets, Pacific J. Math 15,(1965),961- 970.

[2] H.H.Corson and E.Michael, Metrizability of certain countable unions, Illinois J.Math.8(1964)351-360.

[3] S.Kasahara, Operation- compact spaces, Math. Japonica, 24 (1979), 97.

[4] H. Ogata, Operations on topological spaces and associated topology, Math.Japonica, 36(1) (1991),175 - 184.

[5] N.Kalaivani and G.Sai Sundara Krishnan, Operation approaches on α-γ-open sets in topological spaces, Int.J.of Math. Analysis, Vol.7, 2013, no.10, 491-498.

[6] N.Kalaivani, A-I-EL-Maghrabi and G.Sai Sundara Krishnan, Operation approaches on α-γ-I-open sets and on α-γ-I-continuous functions in topological spaces, J. of the Egyptian Mathematical Society [Production and hosting by Elsevier], vol.22, pp. 265-271.

[7] T.R.Hamlett and Dragan Jankovic, Ideals in General Topology, Proceedings of the conference General topology and Applications,(1988),115.

[8] K.Kuratowski, Topology,Vol.I, Academic Press,New York,(1966).

[9] E.Hayashi, Topologies defined by local properties, Math.Ann.,156(1964),205-215.

[10] P.Samuels, A topology formed from a given topology and ideal, J. London Math. Soc.(2),10(1975),409-416.

[11] D.Jankovic and T.R.Hamlett, New topologies from old via ideals, Amer.Math.Monthly,97(1990),295-310.

[12] G.Sai Sundara Krishnan and K.Balachandran, On γ-semi-open sets in topological spaces, Bull.Cal.Math.Soc., 98, (6),(2006), 517-530.
[13] N.Kalaivani,D.Saravanakumar and G.Sai Sundara Krishnan, On (α-γ,β)-continuous functions and α-(γ,β)-continuous functions in topological spaces, International Journal of Mathematical Sciences and Applications,vol.1, (2012), 341-347.
[14] J.Dontchev and M.Przemki,On the various Decompositions of continuous and some weakly continuous functions, Acta Math. Hungar.,71(1-2)(1996),109-120.
[15] I.L.Reilly and M.K.Vamanamurthy, On α-continuity in topological spaces, Acta Math.Hungar.45(1985)27-32.
[16] J.Tong, A decomposition of continuity, Acta Math. Hugar., 48(1986)11-15.
[17] Maximilian Ganster and Ivan Reilly, A Decomposition of Continuity, Acta Math.Hungar.56(3-4)(1990)299-301.