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Research paper

Interval-Valued Intuitionistic Fuzzy INK-Ideals of INK-Algebra

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Abstract

Mathematical structures of Interim valued IF INK-ideal on INK-algebras are presented. We established that every IF- (INK)-ideal of an INK algebra A can be executed as an level ideal (INK) of an iv IF(INK) of U. As far as the idea of homomorphism, we talked about the cartesian result of i-v IF(INK)- ideal.

Keywords: *INK-algebra, INK-ideal, fuzzy INK-ideal, IF INK-ideal, Interval-valued fuzzy INK-ideal, Interval-valued intuitionistic fuzzy INK-ideal*

1. Introduction

Several researchers have developed algebraic structures. Imai and Iseki (1966) proposed two algebraic structure BCI and BCI-algebras. Then Hu and Li (1983) are expanding Algebra is called BCH-algebra, which is a generalization of BCK- and BCI-algebras. J. Neggers and H. S. Kim (1999) introduced d-algebras and studied the relationship flanked by d-algebras and BCK-algebras. Conceptual Fs and i-v Fs are then introduced by Zadeh (1982). Zadeh also used his i-v Fs to construct an near reasoning system. In addition, Attanssov [1986] introduced the concept of Intuitive Fuzzy Sets (IF_S) and Interval-valued $IVIF_S$ as a generalization of ordinary FS. Atanassov and Gargov [1989] show that IF_S and IVF_S are equal probability generalizations of F_S. In this paper, we first introduce an i-v IF INK-ideal INK-algebra. Then we prove that each intuitionistic fuzzy INK ideal of INK algebra A can be executed as the ideal INK ideal of intuitionistic fuzzy INK-U. In relation to the concept of homomorphism, we study the Cartesian product of the interval. Pay attention to intuitionistic fuzzy INK-ideal.

2. Preliminaries

Definition 2.1*: A INK-algebra (U, ·, 0) is a nonvoid set U with a non-varying value '0' and a binary operation ' · ' fulfilling the additional adages*

i) $a \cdot 0 = a$

ii) $(b \cdot a) \cdot (b \cdot c) = a \cdot c \ \forall a, b, c \in U$.

In U we can regulate a binary association $\leq$ by $a \leq b$ *if and only if* $a \cdot b = 0$.

The nonvoid subset T of INK-algebra (U, ·,0) is called sub-algebra of U, and if it is $a \cdot b \in T$, $\forall a, b \in T$.

Let S be a nonvoid subset of U. Then S is called a INK-ideal of U if I- 1) $0 \in A$ *I-2)* $(c \cdot a) \cdot (a \cdot b) \in S$ *and* $y \in S \Leftrightarrow x \in S$, $\forall a, b, c \in U$.

Definition 2.4*: Let U be a universe set.* $\aleph_\S$ *be a Fs in U is a mapping* $\aleph_\S$*: U→ [0,1].*

Definition 2.2: *A Fs* $\aleph_\S$ *in a INK-algebra X is called a* fuzzy *subalgebra of U if,* $\aleph_\S(a \cdot b) \geq \min\{\aleph_\S(a), \aleph_\S(b)\}$, $\forall a, b \in U$.

Definition 2.3: *A IFs* χ *is in the nonvoid U, in the form =* $\{(U, \aleph_\chi(a), \sigma_\chi(a)): x \in U\}$ *where* $\aleph_\chi$*: U →* $[0, 1]$ *and* λ_χ*: U →* $[0,1]$ *means the degree of membership of each member* $x \in U$ *(ie,* $\aleph_\chi(a)$*) and the degree of non-membership (in particular,* $\sigma_\chi(a)$*),*$0 \leq \aleph_\chi(a) + \sigma_\chi(a) \leq 1$*),* $\forall x \in U$*. We use the character* $\chi = (U, \aleph_\chi, \sigma_\chi)$ *for IFs* $A = \{(U, \aleph_\chi(a), \sigma_\chi(a)) / a \in U\}$.

Definition 2.4: *The IFs* $\chi = (U, \aleph_\chi, \sigma_A)$ *in a INK-algebra U is called an IF INK-ideal of U, if*

1. $\aleph_\chi(0) \geq \aleph_\chi(a)$ *in addition* $\sigma_\chi(0) \leq \sigma_\chi(a)$
2. $\aleph_\chi(a) \geq \min\{\aleph_\chi(c \cdot a) \cdot (a \cdot b), \aleph_\chi(b)\}$,
3. $\sigma_\chi(a) \leq \max\{\sigma_\chi(c \cdot a) \cdot (a \cdot b), \sigma_\chi(b)\}$,

$\forall a, b, c \in U$.

3. I-v IF_S -ideals (INK)

Definition 3.1: *An i-v IFs in INK-algebra U is called an i-v IF INK-ideal of U if it satisfies,*

1. $\bar{\aleph}_\chi(0) \geq \overline{\aleph}_\chi(a)$, $\bar{\sigma}_\chi(0) \leq \bar{\sigma}_\chi(a)$,
2. $\overline{\aleph}_\chi(a) \geq \text{rmin}\{\overline{\aleph}_\chi((c \cdot a) \cdot (c \cdot b)), \overline{\aleph}_\chi(b)\}$
3. $\bar{\sigma}_\chi(a) \leq \text{rmax}\{\bar{\sigma}_\chi((c \cdot a) \cdot (c \cdot b)), \bar{\sigma}_\chi(b)\}$.

Example 3.2.
Discuss the following table of INK-Algebra: U = {0, p, q, r}.

*	0	p	q	r
0	0	p	q	r
p	p	0	r	q
q	q	r	0	p

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r	r	q	p	0

Let A be an *i-v* IF_S in U by

$$\bar{\mu}_A(x) = \begin{pmatrix} 0 & p & q & r \\ [0.6,0.8] & [0.4,0.5] & [0.3,0.4] & [0.3,0.4] \end{pmatrix}$$

$$\bar{\sigma}_A(x) = \begin{pmatrix} 0 & p & q & r \\ [0.3,0.4] & [0.5,0.7] & [0.5,0.7] & [0.4,0.6] \end{pmatrix}$$

Then routine calculations give that A is an i-v intuitionistic fuzzy INK-ideal of *U*.

Theorem 3.3. Let τ be i-v IF INK-ideal of U. If there exists a sequence $\{x_n\}$ in τ such that $\lim_{n\to\infty}\{xn\}$=[1,1], $\lim_{n\to\infty}\{xn\}$=[0,0]. Then $\bar{\mu}_\tau(0)$ = [1,1] and $\bar{\sigma}_\tau(0)$ = [0,0].

Proof. Since $\bar{\mu}_\tau(0) \geq \bar{\mu}_\tau(x)$ and $\bar{\sigma}_\tau(0) \leq \bar{\sigma}_\tau(x)$ $\forall x \in U$, We have $\bar{\mu}_\tau(0) \geq \bar{\mu}_\tau(x_n)$ and $\bar{\sigma}_\tau(0) \leq \bar{\sigma}_\tau(x_n)$, (n is positive integer) Note that

$[1,1] \geq \bar{\mu}_\tau(0) \geq \lim_{n\to\infty}\{xn\} = [1,1]$

$[0,0] \leq \bar{\sigma}_\tau(0) \leq \lim_{n\to\infty}\{xn\} = [0,0]$.

Hence $\bar{\mu}_\tau(0) = [1,1]$ and $\bar{\sigma}_\tau(0) = [0,0]$.

Theorem 3.4. An i-v IF_s $\tau = [\langle \mu_\tau^L, \mu_\tau^U \rangle, \langle \sigma_\tau^L, \sigma_\tau^U \rangle]$ in U is an i-v intuitionistic fuzzy ideal (INK) of U if and just if $\langle \mu_\tau^L, \mu_\tau^U \rangle$and $\langle \sigma_\tau^L, \sigma_\tau^U \rangle$ are IF_s of U.

Proof. Since $\mu_A^L(x)$

$\mu_\tau^L(0) \geq \mu_\tau^L(x)$; $\mu_\tau^U(0) \geq \mu_\tau^U(x)$

$\sigma_\tau^L(0) \leq \sigma_\tau^L(x)$; $\sigma_\tau^U(0) \leq \sigma_\tau^U(x)$,

Therefore $\bar{\mu}_\tau(0) \geq \bar{\mu}_\tau(x)$; $\bar{\sigma}_\tau(0) \leq \bar{\sigma}_\tau(x)$.

$\langle \mu_\tau^L, \mu_\tau^U \rangle$ and $\langle \sigma_\tau^L, \sigma_\tau^U \rangle$ are IFI of U.

Let x, y $\in$U, then

$\bar{\mu}_\tau(x) = [\mu_\tau^L(x), \mu_\tau^U(x)]$

$\geq$[min{$\mu_\tau^L(x\cdot y), \mu_\tau^L(y)$}, min{ $\mu_\tau^U(x\cdot y), \mu_\tau^U(y)$}]

= rmin {[$\mu_\tau^L(x\cdot y), \mu_\tau^U(x\cdot y)$], [$\mu_\tau^L(y), \mu_\tau^L(y)$]}

= rmin {$\bar{\mu}_\tau(x\cdot y), \bar{\mu}_\tau(y)$}and

$\bar{\sigma}_\tau(x) = [\sigma_\tau^L(x), \sigma_\tau^U(x)]$

$\leq$ [max{ $\sigma_\tau^L(x\cdot y), \sigma_\tau^L(y)$}, max{ $\sigma_\tau^U(x\cdot y), \sigma_\tau^U(y)$}]

= rmax{[$\sigma_\tau^L(x\cdot y), \sigma_\tau^U(x\cdot y)$], [$\sigma_\tau^L(y), \sigma_\tau^U(y)$]}

= rmax{$\bar{\sigma}_\tau(x\cdot y), \bar{\sigma}_\tau(y)$}.

Along these lines , τ is an i-v IFI of U. TUYT87T7Y89698689

Similarly, expect that A is an i-v IFI of U.

$[\mu_\tau^L(x), \mu_\tau^U(x)] = \bar{\mu}_\tau(x)$

$\geq$ rmin{ $\bar{\mu}_\tau(x\cdot y), \bar{\mu}_\tau(y)$}

= rmin {[$\mu_\tau^L(x\cdot y), \mu_\tau^U(x\cdot y)$], [$\mu_\tau^L(y), \mu_\tau^U(y)$]}

=[min {$\mu_\tau^L(x\cdot y), \mu_\tau^L(y)$}, min{ $\mu_\tau^U(x\cdot y), \mu_\tau^U(y)$}]

what's more $[\sigma_\tau^L(x), \sigma_\tau^L(x)] = \bar{\sigma}_\tau(x) \leq$ rmax{$\bar{\sigma}_\tau(x\cdot y), \bar{\sigma}_\tau(y)$}

= rmax {[$\sigma_\tau^L(x\cdot y), \sigma_\tau^U(x\cdot y)$], [$\sigma_\tau^L(y), \sigma_\tau^U(y)$]}

= [max {$\sigma_\tau^L(x\cdot y), \sigma_\tau^L(y)$}, min {$\sigma_\tau^U(x\cdot y), \sigma_\tau^L(y)$}].

It pursues that,

$\mu_\tau^L(x) \geq$ min {$\mu_\tau^L(x\cdot y), \mu_\tau^L(y)$},

$\sigma_\tau^L(x) \leq$ max {$\sigma_\tau^L(x\cdot y), \sigma_\tau^L(y)$}

and

$\mu_\tau^U(x) \geq$ min {$\mu_\tau^U(x\cdot y), \mu_\tau^U(y)$},

$\sigma_\tau^L(x) \leq$ max {$\sigma_\tau^L(x\cdot y), \sigma_\tau^L(y)$}.

Hence $\langle \mu_\tau^L, \mu_\tau^U \rangle$ and $\langle \sigma_\tau^L, \sigma_\tau^U \rangle$ are IFI of U.

Proposition 3.5. Every i-v IF INK-ideal of a INK-algebra U is an i-v IFI.

Definition 3.6. *An IF_s τ in U is called an interval-valued intuitionistic fuzzy INK-sub algebra of U if*

i) $\bar{\mu}_\tau(x\cdot y) \geq rmin\{\bar{\mu}_\tau(x), \bar{\mu}_\tau(y)\}$ *and*

ii) $\bar{\sigma}_\tau(x\cdot y) \leq rmax\{\bar{\sigma}_\tau(x), \bar{\sigma}_\tau(y)\}$, $\forall x, y \in U$.

Proposition 3.7. Every i-v IF INK-ideal of a INK-algebra U is i-v IF sub algebra of U.

4. Product of i-v intuitionistic fuzzy INK-ideal

Definition 4.1. *An IF_r ν on any set is a IF subset A with a membership function*

HVHVHJVHJVHJVH ΩRA: U×U→ [0, 1]

and non-membership function XHCHFT8698-=5654

ψ_A: U×U→ [0, 1].

Definition 4.2. *Let $\nu = [\langle \mu_\nu^L, \mu_\nu^U \rangle, \langle \upsilon_\nu^L, \upsilon_\nu^U \rangle]$ and $\omega = [\langle \mu_\omega^L, \mu_\omega^U \rangle, \langle \upsilon_\omega^L, \upsilon_\omega^U \rangle]$ be two i-v IFS in a set U. The product of $\nu \times \omega$ is defined by 6TTUI90L;*

$\nu \times \omega$ = {((x, y), ($\bar{\mu}_\nu \times \bar{\mu}_\omega$), ($\bar{\sigma}_\nu \times \bar{\sigma}_\omega$)); $\forall x, y \in U\times U$},

where $\nu \times \omega$: U×U→D [0,1].

Theorem 4.3. Let A = $[\langle \mu_\nu^L, \mu_\nu^U \rangle, \langle \upsilon_\nu^L, \upsilon_\nu^U \rangle]$ and B = $[\langle \mu_\omega^L, \mu_\omega^U \rangle, \langle \upsilon_\omega^L, \upsilon_\omega^U \rangle]$ be two i-v on the off chance that subsets in a set U, $\nu \times \omega$ is an I-v IF INK-ideal of U×U.

Proof. Let (x, y) ϵU×U, at that point by definition

($\bar{\mu}_\nu \times \bar{\mu}_\omega$), (0,0) = r min {($\bar{\mu}_\nu$(0), $\bar{\mu}_\nu$(0)}

= r min {[μ_ν^L(0), μ_ν^U(0)],[μ_ω^L(0), μ_ω^U(0)]}

= [min{μ_ν^L(0), μ_ω^L(0)},min{μ_A^U(0), μ_ω^U(0)}]

$\geq$ [min {μ_ν^L(x), μ_ω^L(y)}, min {μ_ν^U(x), μ_ω^U(y)}]

= rmin {[μ_ν^L(x), μ_ν^U(x)], [μ_ω^L(y), μ_ω^U(y)]}

= rmin {($\bar{\mu}_\nu$(x), $\bar{\mu}_\omega$(y)}

= ($\bar{\mu}_\nu \times \bar{\mu}_\omega$) (x, y)

in addition

($\bar{\sigma}_\nu \times \bar{\sigma}_B$)(0,0) = rmax{($\bar{\sigma}_\nu$(0), $\bar{\sigma}_\nu$(0)}

= rmax {[σ_ν^L (0), σ_ν^U (0)], [σ_ω^L(0), σ_ω^U(0)]}

= [max {σ_ν^L(0), σ_ω^L (0)}, max {σ_ν^U(0), σ_ω^U(0)}]

$\leq$ [max {σ_ν^L(x), σ_ω^L(y)}, max {σ_ν^U(x), σ_ω^U(y)}]

= rmax {[σ_ν^L(x), σ_ν^U(x)], [σ_ω^L(y), σ_ω^U(y)]}

= rmax{($\bar{\sigma}_\nu$(x), $\bar{\sigma}_\nu$(y)}

= ($\bar{\sigma}_\nu \times \bar{\sigma}_\omega$) (x, y)

Thusly (FI_2) holds.

($\bar{\mu}_\nu \times \bar{\mu}_\omega$) ((x, x')) = rmin {$\mu_\nu$(x), μ_ω(x')}

r min {rmin{$\bar{\mu}_\nu$((z· x)·(z· y)),$\bar{\mu}_\nu$(y)},

rmin {$\bar{\mu}_\omega$((z·x)·(z·y)), $\bar{\mu}_\omega$(y)}}

= r min{{ min{ μ_ν^L((z ·x)·(z·y)), μ_ν^L(y)},min{ μ_ν^U((z ·x) ·(z ·y)), μ_ν^U(y)}},min{μ_ω^L((z·x)·(z ·y)),μ_ω^L(y)},min{μ_ω^U(((z ·x).QWN (z·y)), μ_ω^U (y)}}}.

={min{min{μ_ν^L((z·x) ·(z ·y)), μ_ω^L((z ·x) ·(z ·y))},min{μ_ν^L(y), μ_ω^L(y)},min{min{μ_ν^U((z ·x) ·(z ·y)), μ_ω^U(((z·x) ·(z·y))}, min{μ_ν^U(y), μ_ω^U (y)}}}

= rmin{($\bar{\mu}_\nu \times \bar{\mu}_\omega$)(((z·x)·(z·y)),((z·x) ·(z·y))),($\bar{\mu}_\nu \times \bar{\mu}_\omega$) (y, y)}.

furthermore, ($\bar{\sigma}_\nu \times \bar{\sigma}_\omega$)((x, x)) = r max{ $\bar{\sigma}_\nu$(x), $\bar{\sigma}_\omega$(x)}

$\leq$ rmax{rmax{$\bar{\sigma}_\nu$((z· x)·(z· y)) $\bar{\sigma}_\nu$(y)},rmax{$\bar{\sigma}_\omega$((z^1·x^1)·(z^1 ·y^1)), $\bar{\sigma}_\omega$(y^1)}}

= r max {{min {σ_ν^L ((z ·x)·(z ·y)), σ_ν^L(y)},max{ σ_ν^U((z ·x) ·(z·y)), σ_ν^U(y)}}, max{σ_ω^L((z·x) ·(z·y)), σ_ω^L (y)}, max{σ_ω^U(((z·x) (z ·y)), σ_ω^U (y)}}}.

= {max {max{σ_ν^L((z· x)·(z ·y)), σ_ω^L((z ·x) ·(z ·y))},max{σ_ν^L(y), σ_ω^L(y^1)}, max {max {σ_ν^U ((z· x) · (z· y)), σ_ν^U(((z·x) · (z ·y))}, max {σ_ν^U(y), σ_ν^U (y)}}}

= rmax {($\bar{\sigma}_\nu \times \bar{\sigma}_\omega$) (((z· x)·(z· y)),((z·x)·(z·y))), ($\bar{\sigma}_\nu \times \bar{\sigma}_\omega$) (y,y)}.

Henceforth total the evidence.

Definition 4.4. *Let $\overline{\mu}_{\acute{\omega}}$ and $\overline{\sigma}_{\acute{\omega}}$ respectively be an i-v membership and non-membership function of every element $x \in U$ to the set $\acute{\omega}$. Then strongest i-v IFS relation on U, that is a membership function relation $\overline{\mu}_{\acute{\omega}}$ on $\overline{\sigma}_{\acute{\omega}}$ and $\overline{\mu}_{\acute{\nu}}$, $\overline{\sigma}_{\acute{\nu}}$ whose i-v membership and non-membership function of every element (x, y) $\in U \times U$ and defined by*

i) $\overline{\mu}_{\acute{\nu}}(x,y) = \text{rmin}\{\overline{\mu}_{\acute{\omega}}(x), \overline{\mu}_{\acute{\omega}}(y)\}$ and

ii) $\overline{\sigma}_{\acute{\nu}}(x,y) = \text{rmax}\{\overline{\sigma}_{\acute{\omega}}(x), \overline{\sigma}_{\acute{\omega}}(y)\}$.

Definition 4.5. *Let $\acute{\omega} = [\langle \mu_{\acute{\omega}}^{L}, \mu_{\acute{\omega}}^{U}\rangle, \langle \upsilon_{\acute{\omega}}^{L}, \upsilon_{\acute{\omega}}^{U}\rangle]$ be an i-v subset in a set U, then the strongest i-v IF relation on U that is a i-v A on $\acute{\omega}$ is $\acute{\nu}_{\acute{\omega}}$ and defined by*

$\acute{\nu}_{\acute{\omega}} = [\langle \mu_{\acute{\nu}_{\acute{\omega}}}^{L}, \mu_{\acute{\nu}_{\acute{\omega}}}^{U}\rangle, \langle \upsilon_{\acute{\nu}_{\acute{\omega}}}^{L}, \upsilon_{\acute{\nu}_{\acute{\omega}}}^{U}\rangle]$.

Theorem 4.6. Let $\acute{\omega} = [\langle \mu_{\acute{\nu}_{\acute{\omega}}}^{L}, \mu_{\acute{\nu}_{\acute{\omega}}}^{U}\rangle, \langle \upsilon_{\acute{\nu}_{\acute{\omega}}}^{L}, \upsilon_{\acute{\nu}_{\acute{\omega}}}^{U}\rangle]$ be an i-v subset in a set U and be the strongest i-v IF relation on U. at that point $\acute{\omega}$ is an i-v IF INK-ideal of U if and just if $\acute{\nu}_{\acute{\omega}}$ is an i-v IF INK-ideal of U×U.

Proof. Let B be an i-v IF INK-ideal of U.

$\overline{\mu}_{\acute{\nu}_{\acute{\omega}}}(0,0) = \text{rmin}\{\overline{\mu}_{\acute{\omega}}(0), \overline{\mu}_{\acute{\omega}}(0)\}$

$\geq \text{rmin}\{\overline{\mu}_{\acute{\omega}}(x), \overline{\mu}_{\acute{\omega}}(y)\}$

$= \overline{\mu}_{\acute{\nu}_{\acute{\omega}}}(x, y)$

and

$\overline{\sigma}_{\acute{\nu}_{\acute{\omega}}}(0,0) = \text{rmax}\{\overline{\sigma}_{\acute{\omega}}(0), \overline{\sigma}_{\acute{\omega}}(0)\}$

$\leq \text{rmax}\{\overline{\sigma}_{\acute{\omega}}(x), \overline{\sigma}_{\acute{\omega}}(y)\}$

$= \overline{\sigma}_{\acute{\nu}_{\acute{\omega}}}(x, y)$

On the other hand,

$\overline{\mu}_{\acute{\nu}_{\acute{\omega}}}(x, x') = \text{rmin}\{\overline{\mu}_{\acute{\omega}}(x), \overline{\mu}_{\acute{\omega}}(x')\}$

$\geq \text{rmin}\{\text{rmin}\{\overline{\mu}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \overline{\mu}_{\acute{\omega}}(y)\},$

$\text{rmin}\{\overline{\mu}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \overline{\mu}_{\acute{\omega}}(y^1)\}\}$

$= \text{rmin}\{\text{rmin}\{\overline{\mu}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)),$

$\overline{\mu}_{\acute{\omega}}((z\cdot x')\cdot(z\cdot y'))\},$

$\text{r min}\{\overline{\mu}_{\acute{\omega}}(y), \overline{\mu}_{\acute{\omega}}(y')\}\}$

$= \text{r min}\{\overline{\mu}_{\acute{\nu}_{\acute{\omega}}}((z\cdot x)\cdot(z\cdot y)), ((z\cdot x')\cdot(z\cdot y')), \overline{\mu}_{\acute{\nu}_{\acute{\omega}}}(y, y')\}$

$= \text{r min}\{\overline{\mu}_{\acute{\nu}_{\acute{\omega}}}(((z, z)\cdot(y, y))\cdot((z, z)\cdot(x, x))), \overline{\mu}_{\acute{\nu}_{\acute{\omega}}}(y, y')\}$

Also,

$\overline{\sigma}_{\acute{\nu}_{\acute{\omega}}}(x, x') = \text{rmax}\{\overline{\sigma}_{\acute{\omega}}(x), \overline{\sigma}_{\acute{\omega}}(x')\}$

$\leq \text{rmax}\{\text{rmax}\{\overline{\sigma}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \overline{\sigma}_{\acute{\omega}}(y)\},$

$\text{rmax}\{\overline{\sigma}_{\acute{\omega}}((z\cdot x')\cdot(z\cdot y')), \overline{\sigma}_{\acute{\omega}}(y')\}\}$

$= \text{rmax}\{\text{rmax}\{\overline{\sigma}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y))\ \overline{\sigma}_{\acute{\omega}}((z\cdot x')\cdot(z\cdot y'))\},$

$\text{r max}\{\overline{\sigma}_{\acute{\omega}}(y), \overline{\sigma}_{\acute{\omega}}(y^1)\}\}$

$= \text{rmax}\{\overline{\sigma}_{\acute{\nu}_{\acute{\omega}}}((z\cdot x)\cdot(z\cdot y)), ((z\cdot x')\cdot(z\cdot y')),$

$\overline{\sigma}_{\acute{\nu}_{\acute{\omega}}}(y, y^1)\}.$

$= \text{rmax}\{\overline{\sigma}_{\acute{\nu}_{\acute{\omega}}}(((z, z')\cdot(y, y'))\cdot((z, z')\cdot(x, x'))),$

$\overline{\sigma}_{\acute{\nu}_{\acute{\omega}}}(y, y^1)\},$

for all $(x, x'), (y, y'), (z, z')$ in $U \times U$.

Equally, let $\acute{\nu}_{\acute{\omega}}$ be an i-v IF-ideal (INK) of U×U.

$(x, x) \in U \times U$.

$\text{rmin}\{\overline{\mu}_{\acute{\omega}}(0), \overline{\mu}_{\acute{\omega}}(0)\} = \overline{\mu}_{\acute{\nu}_{\acute{\omega}}}(0,0)$

$\geq \overline{\mu}_{\acute{\nu}_{\acute{\omega}}}(x, x)$

$\text{rmin}\{\overline{\mu}_{\acute{\omega}}(0), \overline{\mu}_{\acute{\omega}}(0)\} = \text{r min}\{\overline{\mu}_{\acute{\omega}}(x), \overline{\mu}_{\acute{\omega}}(x)\}$

furthermore, $\text{rmax}\{\overline{\mu}_{\acute{\omega}}(0), \overline{\mu}_{\acute{\omega}}(0)\} = \overline{\mu}_{\acute{\nu}_{\acute{\omega}}}(0,0)$

$\leq \overline{\mu}_{\acute{\nu}_{\acute{\omega}}}(x, x)$

$\text{rmax}\{\overline{\mu}_{\acute{\omega}}(0), \overline{\mu}_{\acute{\omega}}(0)\} = \text{rmax}\{\overline{\mu}_{\acute{\omega}}(x), \overline{\mu}_{\acute{\omega}}(x)\}$.

$(x, x^1), (y, y^1), (z, z^1)$ in $U \times U$.

$\text{rmin}\{\overline{\mu}_{\acute{\omega}}(x, x')\} = \overline{\mu}_{\acute{\nu}_{\acute{\omega}}}\{(x, x')\}$

$\geq \text{rmin}\{\overline{\mu}_{\acute{\nu}_{\acute{\omega}}}(((z, z')\cdot(y, y'))\cdot((z, z')\cdot(x, x'))), \overline{\mu}_{\acute{\nu}_{\acute{\omega}}}(y, y')\}$

$= \text{rmin}\{\overline{\mu}_{\acute{\nu}_{\acute{\omega}}}((z\cdot x)\cdot(z\cdot y)), ((z\cdot x')\cdot(z\cdot y')), \overline{\mu}_{\acute{\nu}_{\acute{\omega}}}(y, y')\}$

$= \text{rmin}\{\text{rmin}\{\overline{\mu}_{\acute{\nu}_{\acute{\omega}}}((z\cdot x)\cdot(z\cdot y)), \overline{\mu}_{\acute{\nu}_{\acute{\omega}}}((z\cdot x')\cdot(z\cdot y'))\},$

$\text{r min}\{\overline{\mu}_{\acute{\nu}_{\acute{\omega}}}(y), \overline{\mu}_{\acute{\nu}_{\acute{\omega}}}(y')\}\}$

$= \text{r min}\{\text{r min}\{\overline{\mu}_{\acute{\nu}_{\acute{\omega}}}((z\cdot x)\cdot(z\cdot y)), \overline{\mu}_{\acute{\omega}}(y)\},$

$\text{r min}\{\overline{\mu}_{\acute{\omega}}((z^1\cdot x^1)\cdot(z^1\cdot y^1)), \overline{\mu}_{\acute{\omega}}(y^1)\}\}$

If $x^1 = y^1 = z^1 = 0$, then

$\text{rmin}\{\overline{\mu}_{\acute{\omega}}(x), \overline{\mu}_{\acute{\omega}}(0)\}$

$= \text{rmin}\{\text{r min}\{\overline{\mu}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \overline{\mu}_{\acute{\omega}}(y)\}, \overline{\mu}_{\acute{\omega}}(0)\}$

$\overline{\mu}_{\acute{\omega}}(x) \geq \text{rmin}\{\overline{\mu}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \overline{\mu}_{\acute{\omega}}(y)\}$, essentially

$\overline{\sigma}_{\acute{\omega}}(x) \leq \text{rmax}\{\overline{\sigma}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \overline{\sigma}_{\acute{\omega}}(y)\}$.

Therefore, B is i-v IF INK-ideal of U.

Theorem 4.8. If $\overline{\mu}_{\acute{\nu}}$ is a i-v IF INK-ideal of INK-algebra U, then $\overline{\mu}_{\square\square}$ is also i-v IF INK-ideal of INK-algebra U.

Proof.

$\overline{\mu}_{\acute{\nu}}(0) \geq \overline{\mu}_{\acute{\nu}}(x)$

$[\overline{\mu}_{\acute{\nu}}(0)] \geq [\overline{\mu}_{\acute{\nu}}(x)]$

$[\overline{\mu}_{\acute{\nu}}(0)]^m \geq [\overline{\mu}_{\acute{\nu}}(x)]^m$

$\overline{\mu}_{\acute{\nu}}(0)^m \geq \overline{\mu}_{\acute{\nu}}(x)^m$

$\overline{\mu}_{\acute{\nu}}{}^m(0) \geq \overline{\mu}_{\acute{\nu}}{}^m(x)$

$\overline{\mu}_{\acute{\nu}}(x) \geq \text{r min}\{\overline{\mu}_{\acute{\nu}}((z\cdot x)\cdot(z\cdot y)), \overline{\mu}_{\acute{\nu}}(y)\}$

$[\overline{\mu}_{\acute{\nu}}(x)]^m \geq [\text{r min}\{\overline{\mu}_{\acute{\nu}}((z\cdot x)\cdot(z\cdot y)), \overline{\mu}_{\acute{\nu}}(y)\}]^m$

$\overline{\mu}_{\acute{\nu}}(x)^m \geq \text{r min}\{\overline{\mu}_{\acute{\nu}}((z\cdot x)\cdot(z\cdot y))^m, \overline{\mu}_{\acute{\nu}}(y)^m\}$

$\overline{\mu}_{\acute{\nu}}{}^n(x) \geq \text{r min}\{\overline{\mu}_{\acute{\nu}}{}^m((z\cdot x)\cdot(z\cdot y)), \overline{\mu}_{\acute{\nu}}{}^m(y)\}$

and

$\overline{\sigma}_{\acute{\nu}}(0) \leq \overline{\sigma}_{\acute{\nu}}(x)$

$[\overline{\sigma}_{\acute{\nu}}(0)] \leq [\overline{\sigma}_{\acute{\nu}}(x)]$

$[\overline{\sigma}_{\acute{\nu}}(0)]^m \leq [\overline{\sigma}_{\acute{\nu}}(x)]^m$

$\overline{\sigma}_{\acute{\nu}}(0)^m \leq \overline{\sigma}_{\acute{\nu}}(x)^m$

$\overline{\sigma}_{\acute{\nu}}{}^m(0) \leq \overline{\sigma}_{\acute{\nu}}{}^m(x)$

$\overline{\sigma}_{\acute{\nu}}(x) \leq \text{r min}\{\overline{\sigma}_{\acute{\nu}}((z\cdot x)\cdot(z\cdot y)), \overline{\sigma}_{\acute{\nu}}(y)\}$

$[\overline{\sigma}_{\acute{\nu}}(x)]^m \leq [\text{r min}\{\overline{\sigma}_{\acute{\nu}}((z\cdot x)\cdot(z\cdot y)), \overline{\sigma}_{\acute{\nu}}(y)\}]^m$

$\overline{\sigma}_{\acute{\nu}}(x)^m \leq \text{r min}\{\overline{\sigma}_{\acute{\nu}}((z\cdot x)\cdot(z\cdot y))^m, \overline{\sigma}_{\acute{\nu}}(y)^m\}$

$\overline{\sigma}_{\acute{\nu}}{}^m(x) \leq \text{r min}\{\overline{\sigma}_{\acute{\nu}}{}^m((z\cdot x)\cdot(z\cdot y)), \overline{\sigma}_{\acute{\nu}}{}^m(y)\}$.

Hence complete the proof.

Theorem 4.10. If $\overline{\mu}_{\acute{\nu}}$ is also a i-v intuitionistic fuzzy INK-ideal of INK-algebra U, then $\overline{\mu}_{\acute{\nu}\cap\acute{\omega}}$ is also a INK-ideal of INK-algebra U.

Proof. For all $x, y, z \in U$

$\overline{\mu}_{\acute{\nu}}(0) \geq \overline{\mu}_{\acute{\nu}}(x)$

$\min\{\overline{\mu}_{\acute{\nu}}(0), \overline{\mu}_{\acute{\omega}}(0)\} \geq \min\{\overline{\mu}_{\acute{\nu}}(x), \overline{\mu}_{\acute{\omega}}(x)\}$

$\overline{\mu}_{\acute{\nu}\cap\acute{\omega}}(0) \geq \overline{\mu}_{\acute{\nu}\cap\acute{\omega}}(x)$

$\overline{\mu}_{\acute{\nu}}(x) \geq \text{r min}\{\overline{\mu}_{\acute{\nu}}((z\cdot x)\cdot(z\cdot y)), \overline{\mu}_{\acute{\nu}}(y)\}$

$\overline{\mu}_{\acute{\omega}}(x) \geq \text{r min}\{\overline{\mu}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \overline{\mu}_{\acute{\omega}}(y)\}$

$\{\overline{\mu}_{\acute{\nu}}(x), \overline{\mu}_{\acute{\omega}}(x)\} \geq \{\text{r min}\{\overline{\mu}_{\acute{\nu}}((z\cdot x)\cdot(z\cdot y)), \overline{\mu}_{\acute{\nu}}(y)\},$

$\text{r min}\{\overline{\mu}_{\acute{\omega}}((z\cdot x)\cdot(z\cdot y)), \overline{\mu}_{\acute{\omega}}(y)\}\}$

$\min\{\overline{\mu}_{\acute{\nu}}(x), \overline{\mu}_{\acute{\omega}}(x)\} \geq \{\text{r min}\{\overline{\mu}_{\acute{\nu}}((z\cdot x)\cdot(z\cdot y)), \overline{\mu}_{\acute{\nu}}(y)\},$

r min{ $\overline{\mu}_{\psi}((z\cdot x)\cdot(z\cdot y)), \overline{\mu}_{\phi}(y)\}$}
$\geq$ min {r min {$\overline{\mu}_{\psi}((z\cdot x)\cdot(z\cdot y)), \overline{\mu}_{\phi}((z\cdot x)\cdot(z\cdot y))\}$,
r min {$\overline{\mu}_{\psi}(y), \overline{\mu}_{\phi}(y)\}$}
$\overline{\mu}_{\psi\cap\phi}(x) = r\min\{\overline{\mu}_{\psi\cap\phi}((z\cdot x)\cdot(z\cdot y)), \overline{\mu}_{\psi\cap\phi}(y)\}$.
and
$\overline{\sigma}_{\psi}(0) \leq \overline{\sigma}_{A}(x)$
max $\{\overline{\sigma}_{\psi}(0), \overline{\sigma}_{\phi}(0)\} \leq \max\{\overline{\sigma}_{\psi}(x), \overline{\sigma}_{\phi}(x)\}$
$\overline{\sigma}_{\psi\cap\phi}(0) \leq \overline{\sigma}_{\psi\cap\phi}(x)$
$\overline{\sigma}_{\psi}(x) \leq r\max\{\overline{\sigma}_{\psi}((z\cdot x)\cdot(z\cdot y)), \overline{\sigma}_{\psi}(y)\}$
$\overline{\sigma}_{\phi}(x) \leq r\max\{\overline{\sigma}_{\phi}((z\cdot x)\cdot(z\cdot y)), \overline{\sigma}_{\phi}(y)\}$
$\{\overline{\sigma}_{\psi}(x), \overline{\sigma}_{\phi}(x)\} \leq \{r\max\{\overline{\sigma}_{\psi}((z\cdot x)\cdot(z\cdot y)), \overline{\sigma}_{\psi}(y)\}$,
r max{ $\overline{\sigma}_{\phi}((z\cdot x)\cdot(z\cdot y)), \overline{\sigma}_{\phi}(y)\}\}$
max $\{\overline{\sigma}_{\psi}(x), \overline{\sigma}_{\phi}(x)\} \leq \{r\max\{\overline{\sigma}_{\psi}((z\cdot x)\cdot(z\cdot y)), \overline{\sigma}_{\psi}(y)\}$,
r max $\{\overline{\sigma}_{\phi}((z\cdot x)\cdot(z\cdot y)), \overline{\sigma}_{\phi}(y)\}\}$
$\leq$ max {r max $\{\overline{\sigma}_{\psi}((z\cdot x)\cdot(z\cdot y)), \overline{\sigma}_{\phi}((z\cdot x)\cdot(z\cdot y))\}$,
r max $\{\overline{\sigma}_{\psi}(y), \overline{\sigma}_{\phi}(y)\}\}$
$\overline{\sigma}_{\psi\cap\phi}(x) = r\max\{\overline{\sigma}_{\psi\cap\phi}((z\cdot x)\cdot(z\cdot y)), \overline{\sigma}_{\psi\cap\phi}(y)\}$.
Complete the proof.

Theorem 4.9. If $\overline{\mu}_{\psi}$ is also a i-v IF INK-ideal of U, then $\overline{\mu}_{\psi\cup\phi}$ is also a INK-ideal of U.

Proof. For all x, y, z $\in$ U.
$\overline{\mu}_{\psi}(0) \geq \overline{\mu}_{\psi}(x)$
min $\{\overline{\mu}_{\psi}(0), \overline{\mu}_{\phi}(0)\} \geq \min\{\overline{\mu}_{\psi}(x), \overline{\mu}_{\phi}(x)\}$
$\overline{\mu}_{\psi\cup\phi}(0) \geq \overline{\mu}_{\psi\cup\phi}(x)$
$\overline{\mu}_{\psi}(x) \geq r\min\{\overline{\mu}_{\psi}((z\cdot x)\cdot(z\cdot y)), \overline{\mu}_{\psi}(y)\}$
$\overline{\mu}_{\phi}(x) \geq r\min\{\overline{\mu}_{\phi}((z\cdot x)\cdot(z\cdot y)), \overline{\mu}_{\phi}(y)\}$
$\{\overline{\mu}_{\psi}(x), \overline{\mu}_{\phi}(x)\} \geq \{r\min\{\overline{\mu}_{\psi}((z\cdot x)\cdot(z\cdot y)), \overline{\mu}_{\psi}(y)\}$,
r min{ $\overline{\kappa}_{\phi}((z\cdot x)\cdot(z\cdot y)), \overline{\kappa}_{\phi}(y)\}\}$
min $\{\overline{\mu}_{\psi}(x), \overline{\mu}_{\phi}(x)\} \geq \{r\min\{\overline{\mu}_{\psi}((z\cdot x)\cdot(z\cdot y)), \overline{\mu}_{A}(y)\}$,
r min{ $\overline{\mu}_{\phi}((z\cdot x)\cdot(z\cdot y)), \overline{\mu}_{\phi}(y)\}\}$
$\geq$ min {r min{$\overline{\mu}_{\psi}((z\cdot x)\cdot(z\cdot y)), \overline{\mu}_{\phi}((z\cdot x)\cdot(z\cdot y))\}$,
r min $\{\overline{\mu}_{\psi}(y), \overline{\mu}_{\phi}(y)\}\}$
$\overline{\mu}_{\psi\cup\phi}(x) = r\min\{\overline{\mu}_{\psi\cup\phi}((z\cdot x)\cdot(z\cdot y)), \overline{\mu}_{\psi\cup\phi}(y)\}$.
and
$\overline{\sigma}_{\psi}(0) \leq \overline{\sigma}_{\psi}(x)$
max $\{\overline{\sigma}_{\psi}(0), \overline{\sigma}_{\phi}(0)\} \leq \max\{\overline{\sigma}_{\psi}(x), \overline{\sigma}_{\phi}(x)\}$
$\overline{\sigma}_{AUB}(0) \leq \overline{\sigma}_{AUB}(x)$
$\overline{\sigma}_{\psi}(x) \leq r\max\{\overline{\sigma}_{\psi}((z\cdot x)\cdot(z\cdot y)), \overline{\sigma}_{\psi}(y)\}$
$\overline{\sigma}_{\phi}(x) \leq r\max\{\overline{\sigma}_{\phi}((z\cdot x)\cdot(z\cdot y)), \overline{\sigma}_{\phi}(y)\}$
$\{\overline{\sigma}_{\psi}(x), \overline{\sigma}_{\phi}(x)\} \leq \{r\max\{\overline{\sigma}_{\psi}((z\cdot x)\cdot(z\cdot y)), \overline{\sigma}_{\psi}(y)\}$,
r max{ $\overline{\sigma}_{\phi}((z\cdot x)\cdot(z\cdot y)), \overline{\sigma}_{\phi}(y)\}\}$
max $\{\overline{\sigma}_{\psi}(x), \overline{\sigma}_{\phi}(x)\} \leq \{r\max\{\overline{\sigma}_{\psi}((z\cdot x)\cdot(z\cdot y)), \overline{\sigma}_{\psi}(y)\}$,
r max $\{\overline{\sigma}_{\phi}((z\cdot x)\cdot(z\cdot y)), \overline{\sigma}_{\phi}(y)\}\}$
$\leq$ max {r max $\{\overline{\sigma}_{\psi}((z\cdot x)\cdot(z\cdot y)), \overline{\sigma}_{\phi}((z\cdot x)\cdot(z\cdot y))\}$,
r max $\{\overline{\sigma}_{\psi}(y), \overline{\sigma}_{\phi}(y)\}\}$
$\overline{\sigma}_{\psi\cup\phi}(x) \leq r\max\{\overline{\sigma}_{\psi\cup\phi}((z\cdot x)\cdot(z\cdot y)), \overline{\sigma}_{\psi\cup\phi}(y)\}$.
Hence complete the proof.

5. Homomorphism of i-v Intuitionistic fuzzy INK-Algebra

Definition 5.1. *Let (U, ·, 0) and (V, $\cdot'$, $0'$) be INK-algebras. A function φ: U→V is called a homomorphism if*
$\varphi(x\cdot y) = \varphi(x)\cdot'\varphi(y), \forall x, y \in U.$
For any interval-valued IFS χ = (V, $\overline{\mu}_{\chi}, \overline{\sigma}_{\chi}$) in V we define a new i-v φ B = (U, φ $\overline{\mu}_{\chi}$, φ $\overline{\sigma}_{\chi}$) *in U, by*
φ $\overline{\mu}_{\chi} = \overline{\mu}_{\chi}(\varphi(x))$ and φ $\overline{\sigma}_{\chi} = \overline{\sigma}_{\chi}(\varphi(x))$.

Theorem 5.2. Let (U, ·, 0) and (V, $\cdot'$, $0'$) be INK-algebras. An onto homomorphic image of an i-v IF INK-ideal of U is also an i-v IF INK-ideal of V.

Proof. Let φ: U →V be an onto homomorphism of INK-algebras. Suppose χ = (V, $\overline{\mu}_{\chi}, \overline{\sigma}_{\chi}$) is the image of an i-v IF INK-ideal φ $_{\chi}$ = (U, φ $\overline{\mu}_{\chi}$, φ$\overline{\sigma}_{\chi}$) of U. We have to prove that A = (V, $\overline{\mu}_{\chi}, \overline{\sigma}_{\chi}$) is an i-v IF INK-ideal of V. Since h: U →V is onto, then x, y, z$\in$ V there exist x, y, z ∈ U such that φ (x) = x and
φ (y) = y^1 also we study φ (0) = 0^1.
Then
$\overline{\mu}_{\chi}(0') = \overline{\mu}_{\chi}(\varphi(0))$
$= \varphi\overline{\mu}_{\chi}(0)$
$\geq \varphi\overline{\mu}_{\chi}(x)$
$= \overline{\mu}_{\chi}(\varphi(x))$
$\overline{\mu}_{\chi}(0') = \overline{\mu}_{\chi}(x')$
and
$\overline{\sigma}_{\chi}(0') = \overline{\sigma}_{\chi}(\varphi(0))$
$= \varphi\overline{\sigma}_{\chi}(0)$
$\leq \varphi\overline{\sigma}_{\chi}(x)$
$= \overline{\sigma}_{A}(\varphi(x))$
$\overline{\sigma}_{\chi}(0') = \overline{\sigma}_{\chi}(x')$.
Also
$\overline{\mu}_{\chi}(x^1) = \overline{\mu}_{\chi}(\varphi(x))$
$= \varphi\overline{\mu}_{\chi}(x)$
$\geq \min\{\varphi\overline{\mu}_{\chi}((z\cdot x)\cdot(z\cdot y)), \varphi\overline{\mu}_{\chi}(y)\}$
$= \min\{\overline{\mu}_{\chi}(\varphi((z\cdot x)\cdot(z\cdot y)), \overline{\mu}_{\chi}(\varphi(y))\}$
$= \min\{\overline{\mu}_{\chi}(\varphi(z\cdot x)\cdot\varphi(z\cdot y)), \overline{\mu}_{\chi}(\varphi(y))\}$
$= \min\{\overline{\mu}_{\chi}((\varphi(z)\cdot\varphi(x))\cdot'(\varphi(z)\cdot'\varphi(y)), \overline{\mu}_{\chi}(\varphi(y))\}$
$\overline{\mu}_{\chi}(x') = \min\{\overline{\mu}_{\chi}((z\cdot x)\cdot(z\cdot y)), \overline{\mu}_{\chi}(y)\}$
and
$\overline{\sigma}_{\chi}(x') = \overline{\sigma}_{\chi}(\varphi(x))$
$= \varphi\overline{\sigma}_{\chi}(x)$
$\leq \max\{\varphi\overline{\sigma}_{\chi}((z\cdot x)\cdot(z\cdot y)), \varphi\overline{\sigma}_{\chi}(y)\}$
$= \max\{\overline{\sigma}_{\chi}(\varphi((z\cdot x)\cdot(z\cdot y)), \overline{\sigma}_{\chi}(\varphi(y))\}$
$= \max\{\overline{\sigma}_{\chi}(\varphi(z\cdot x)\cdot'\varphi(z\cdot y)), \overline{\sigma}_{\chi}(\varphi(y))\}$
$= \max\{\overline{\sigma}_{\chi}((\varphi(z)\cdot\varphi(x))\cdot'(\varphi(z)\cdot'\varphi(y)), \overline{\sigma}_{\chi}(\varphi(y))\}$
$\overline{\sigma}_{\chi}(x^1) = \max\{\overline{\sigma}_{\chi}((z\cdot x)\cdot(z\cdot y)), \overline{\sigma}_{\chi}(y)\}$
Hence A = (V, $\overline{\mu}_{\chi}, \overline{\sigma}_{\chi}$) is an i-v IF INK-ideal of U.

Theorem 5.3. Let (X, ·,0) and (Y, $\cdot'$, $0'$) be INK-algebras. An onto homomorphic inverse image of an i-v IF INK-ideal of V is also an i-v IF INK-ideal of U.

Proof. Let ψ: U → V be an onto homomorphism of INK-algebras. Suppose ψ_{χ} = (X, $\psi\overline{\mu}_{\chi}, \psi\overline{\sigma}_{\chi}$) is the inverse image of an i-v IF INK-ideal χ = (Y, $\overline{\mu}_{\chi}, \overline{\sigma}_{\chi}$) of V.
ψ_{χ} = (X, $\psi\overline{\mu}_{\chi}, \psi\overline{\sigma}_{\chi}$) is an i-v INK-ideal of U.
For any x, y, z$\in$ V there exist x, y, z ∈ U.
such that ψ (x) = x and ψ (y) = y and ψ (0) = 0.
$\psi\overline{\mu}_{\chi}(0) = \overline{\mu}_{\chi}(\psi(0))$
$= \overline{\mu}_{\chi}(0^1)$
$\overline{\mu}_{\chi}(x')$
$= \overline{\mu}_{\chi}(\psi(x))$
implies that $\psi\overline{\mu}_{\chi}(0) = \psi\overline{\mu}_{\chi}(x)$ and
$\psi\overline{\sigma}_{\chi}(0) = \overline{\sigma}_{\chi}(\psi(0))$
$= \overline{\sigma}_{\chi}(0')$

$\leq \overline{\sigma}_{\chi}(x')$
$= \overline{\sigma}_{\chi}(\psi(x))$
Implies that $\psi\overline{\sigma}_{\chi}(0) = \psi\overline{\sigma}_{\chi}(x)$.
$\psi\overline{\aleph}_{\chi}(x) = \overline{\aleph}_{\chi}(\psi(x))$
$=\overline{\aleph}_{\chi}(x^1)$
$\psi\overline{\aleph}_{\chi}(x) = \min\{\overline{\aleph}_{\chi}((z' \cdot x') \cdot (z' \cdot y')), \overline{\aleph}_{\chi}(y')\}$
$= \min\{\overline{\aleph}_{\chi}((\psi(z) \cdot \psi(x)) \cdot (\psi(z) \cdot \psi(y)), \overline{\aleph}_{\chi}(\psi(y))\}$
$= \min\{\overline{\aleph}_{\chi}(\psi(z \cdot x) \cdot \psi(z \cdot y)), \overline{\aleph}_{\chi}(\psi(y))\}$
$= \min\{\overline{\aleph}_{\chi}(\psi((z \cdot x) \cdot (z \cdot y)), \overline{\aleph}_{\chi}(\psi(y))\}$
$\geq \min\{\psi\overline{\aleph}_{\chi}((z \cdot x) \cdot (z \cdot y)), \psi\overline{\aleph}_{\chi}(y)\}$
and
$\psi\overline{\sigma}_{\chi}(x) = \overline{\sigma}_{\chi}(\psi(x))$
$= \overline{\sigma}_{\chi}(x')$
$\psi\overline{\sigma}_{\chi}(x) = \max\{\overline{\sigma}_{\chi}((z' \cdot x') \cdot (z' \cdot y')), \overline{\sigma}_{\chi}(y')\}$
$= \max\{\overline{\sigma}_{\chi}((\psi(z) \cdot \psi(x)) \cdot (\psi(z) \cdot \psi(y)), \overline{\sigma}_{\chi}(\psi(y))\}$
$= \max\{\overline{\sigma}_{\chi}(\psi(z \cdot x) \cdot \psi(z \cdot y)), \overline{\sigma}_{\chi}(\psi(y))\}$
$= \max\{\overline{\sigma}_{\chi}(\psi((z \cdot x) \cdot (z \cdot y)), \overline{\sigma}_{\chi}(\psi(y))\}$
$\psi\overline{\sigma}_{\chi}(x) \leq \max\{\psi\overline{\sigma}_{\chi}((z \cdot x) \cdot (z \cdot y)), \psi\overline{\sigma}_{\chi}(y)\}$

Therefore, complete the proof.

6. Conclusions

In this paper, we INK-algebra between the current value of the member and non-member functions of IF-INK classical with introducing the concept, and to study their properties.

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