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Research paper

On Maximal Semi Continuous Functions in Topological Spaces

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Abstract

Study of functions called maximal semi continuous, strongly maximal semi continuous, maximal semi irresolute, maximal semi s-continuous and super maximal semi continuous functions are introduce and investigate. During this process, some of their properties are obtained.

Keywords: *Minimal, maximal and semi maximal open sets.*

1. Introduction

In 1963, N. Levine [3] studied semi open sets. Family of semi open sets is represented by SO(X) and its complement is said to be semi-closed set of X. Also, family of semi closed sets is denoted by SC(X). Later, F. Nakaoka and N. Oda [4], [5], [6], investigated minimal open and closed sets, maximal open and closed sets, Family of minimal open (minimal closed) sets and maximal open (maximal closed) sets are denoted by $M_iO(X)$ ($M_iC(X)$) and $M_aO(X)$ ($M_aC(X)$) respectively. Also in 2009 and 2011, S. S. Benchalli et. al., [1] and [2], studied semi maximal open and semi minimal closed sets, also minimal open sets and maps in topological spaces. Family of semi maximal open sets and semi minimal closed is denoted by $SM_aO(X)$ and $SM_iO(X)$ respectively. Throughout the present paper the sets X or Y always mean topological spaces and A or B denote subsets where cl(B) is its closure, int(B) is its interior and g denote a map from X to Y.

2. Preliminaries

2.1 Definition [3]: B of X is known as semiopen if ∃ some open set U ∋ $U \subset A \subset cl(U)$.

2.2 Definition [4]: An open set W is known as minimal open if open set present in W is ϕ or W where W is proper nonempty.

2.3 Definition [5]: An open set W is known as maximal open if open set contains W is X or W where W is proper nonempty.

2.4 Definition [6]: A closed set Z is known as minimal closed if closed set present in Z is ϕ or Z., where Z is proper nonempty.

2.5 Definition [6]: A closed set Z is known as maximal closed if closed set contains Z is X or Z. where Z is proper nonempty.

2.6 Definition [2]: A map g is

i) minimal continuous if $g^{-1}(M)$ is open set, ∀ $M \in M_iO(Y)$
ii) maximal continuous if $g^{-1}(M)$ is an open, ∀ $M \in M_aO(Y)$
iii) minimal irresolute if $g^{-1}(M) \in M_iO(X)$, ∀ $M \in M_iO(Y)$
iv) maximal irresolute if $g^{-1}(M) \in M_iO(X)$, ∀ $M \in M_iC(Y)$
v) minimal-maximal continuous if $g^{-1}(M) \in M_aO(X)$, ∀ $M \in M_aO(Y)$
vi) maximal-minimal continuous if $g^{-1}(M) \in M_iO(X)$, ∀ $M \in M_aO(Y)$

3. Maximal semi continuous functions.

3.1 Definition: A map g is

i) maximal semi continuous if ∀ $M \in M_aO(Y)$, $g^{-1}(M) \in SO(X)$.
ii) strongly maximal semi continuous if ∀ $M \in M_aO(Y)$, $g^{-1}(M) \in M_aSO(X)$.
iii) maximal semi irresolute if ∀ $M \in M_aSO(Y)$, $g^{-1}(M) \in M_aSO(X)$.
iv) maximal semi s-continuous if ∀ $M \in M_aSO(Y)$, $g^{-1}(M) \in SO(X)$.
v) super maximal semi continuous if ∀ $M \in M_aSO(Y)$, $g^{-1}(M) \in SM_aO(X)$.

3.2 Theorem: If g is continuous map, then g is maximal semi continuous and not conversely.

Proof: Assume that g is a continuous map. Take $M \in M_aO(Y)$. As maximal open imply open set, M is open in Y. Then X contains $g^{-1}(M)$ as open set. Since every open imply semi open set. Then $g^{-1}(M) \in SO(X)$, ∀ $M \in M_aO(Y)$. Hence g is maximal semi continuous.

3.3 Example: Let X = Y = {1, 2, 3} with topologies $\tau = \{\phi, \{1\}, \{1, 2\}, X\}$ and $\mu = \{\phi, \{2\}, \{1, 2\}, Y\}$. Let g be an identity function which is maximal semi continuous but not continuous as open set {2} in Y, $g^{-1}(\{2\}) = \{2\}$ is not open set.

3.4 Theorem: If g is semi continuous, then g is maximal semi continuous and not conversely.

Proof: Take $M \in M_aO(Y)$. Then M is an open in Y and $g^{-1}(M) \in SO(X)$. Thus g is maximal semi continuous.

3.5 Example: By Example 3.3, g is maximal semi continuous but not semi continuous as open set {2} in Y, $g^{-1}(\{2\}) = \{2\} \notin SO(X)$. Therefore g is not semi continuous.

3.6 Theorem: If g is strongly maximal semi continuous, then g is maximal semi continuous and converse is not true.

Proof: Take $M \in M_aO(Y)$. As g is strongly maximal semi continuous, $g^{-1}(M) \in M_aSO(X)$. Since maximal semi open set implies semi open set, then $g^{-1}(M) \in SO(X)$, $\forall\, M \in M_aO(Y)$. Hence g is maximal semi continuous.

3.7 Example: Let X = Y = {1, 2, 3, 4} with topologies $\tau = \{\phi, \{1\}, \{1, 2\}, \{1, 2, 3\}, X\}$ and $\mu = \{\phi, \{1\}, \{2\}, \{1, 2\}, Y\}$. Let g be an identity function which is maximal semi continuous but not strongly maximal semi continuous as $\{1, 2\} \in M_aO(Y)$, then $g^{-1}(\{1, 2\}) = \{1, 2\} \notin M_aSO(X)$.

3.8 Theorem: Every maximal semi irresolute is strongly maximal semi continuous, but not converse is true.

Proof: Assume that g is maximal semi irresolute map. Let $M \in M_aO(Y)$, then $M \in M_aSO(Y)$. By hypothesis, we have $g^{-1}(M) \in M_aSO(X)$. Hence g is strongly maximal semi continuous.

3.9 Example: By Example 3.3, g is strongly maximal semi continuous but not maximal semi irresolute as $\{2, 3\} \in M_aSO(Y)$, then $g^{-1}(\{2, 3\}) = \{2, 3\} \notin M_aSO(X)$.

3.10 Theorem: If g is maximal semi irresolute then g is maximal semi continuous and not conversely.

Proof: Consider a maximal semi irresolute map g. Take $M \in M_aO(Y)$ Then $M \in M_aSO(Y)$. As g is maximal semi irresolute, $g^{-1}(M) \in M_aSO(X)$. This implies $g^{-1}(M) \in SO(X)$. Thus g is maximal semi continuous.

3.11 Example: By Example 3.3, g is maximal semi continuous but not maximal semi irresolute as $\{2, 3\} \in M_aSO(Y)$, then $g^{-1}(\{2, 3\}) = \{2, 3\} \notin M_aSO(X)$.

3.12 Theorem If g is maximal semi s – continuous, then it is maximal semi continuous and converse is not true.

Proof: Assume g is maximal semi s - continuous. Let $M \in M_aO(Y)$. Then $M \in M_aSO(Y)$. This implies $g^{-1}(M) \in SO(X)$. Hence g is maximal semi continuous.

3.13 Example: By Example 3.3, g is maximal semi continuous but not maximal semi s-continuous as $\{2, 3\} \in M_aSO(Y)$, then $g^{-1}(\{2, 3\}) = \{2, 3\} \notin SO(X)$.

3.14 Remark: Maximal semi s-continuous and strongly maximal semi continuous functions are independent.

3.15 Example : Let X = Y = {1, 2, 3}, $\tau_1 = \{\phi, \{1\}, \{2\}, \{1, 2\}, X\}$, $\tau_2 = \{\phi, \{1\}, \{1, 2\}, X\}$, $\mu_1 = \{\phi, \{1\}, \{1, 2\}, Y\}$ and $\mu_2 = \{\phi, \{2\}, \{1, 2\}, Y\}$. Let $f_1 : (X, \tau_1) \to (Y, \mu_1)$ and $f_2 : (X, \tau_2) \to (Y, \mu_2)$ are an identity functions. Then f_1 is maximal semi s-continuous but not strongly maximal semi continuous. Now $\{1\} \in M_aO(Y)$, then $f_1^{-1}(\{1\}) = \{1\} \notin M_aSO(X)$. Also f_2 is strongly maximal semi continuous but not maximal semi s-continuous. Now $\{2, 3\} \in M_aSO(Y)$, then $f_2^{-1}(\{2, 3\}) = \{2, 3\} \notin SO(X)$.

3.16 Theorem: Every super maximal semi continuous function is maximal semi continuous.

Proof: Consider a super maximal semi continuous function g. Let $M \in M_aO(Y)$. Then $M \in M_aSO(Y)$. By hypothesis $g^{-1}(M) \in SM_aO(X)$. As every semi maximal open is semi open, we have $g^{-1}(M) \in SO(X)$. Hence g is maximal semi continuous

3.17 Example: Let X = Y = {1, 2, 3} with $\tau = \{\phi, \{1\}, \{1, 2\}, X\}$ and $\mu = \{\phi, \{1\}, \{1, 3\}, Y\}$. Let g is identity map. Then g is maximal semi continuous and not super maximal semi continuous as $\{1, 3\} \in M_aSO(Y)$, then $g^{-1}(\{1, 3) = \{1, 3\} \notin S_aMO(X)$.

3.18 Theorem: Every super maximal semi continuous map is maximal semi s – continuous, but not converse is true.

Proof: Assume g is super maximal semi continuous map. Take $M \in M_aSO(Y)$. By hypothesis, $g^{-1}(M) \in SM_aO(X)$. This implies $g^{-1}(M) \in SO(X)$. Hence g is maximal semi s - continuous.

3.19 Example: By Example 3.17, g is maximal semi s-continuous but not super maximal semi continuous as $\{1, 3\} \in M_aSO(Y)$, then $g^{-1}(\{1, 3) = \{1, 3\} \notin SM_aO(X)$.

3.20 Theorem: If g is super maximal semi continuous map, then g is semi maximal continuous and not conversely.

Proof: Given g is super maximal semi continuous map. Let $M \in M_aO(Y)$. Then $M \in M_aSO(Y)$. By hypothesis, $g^{-1}(M) \in SM_aO(X)$. Hence g is semi maximal continuous.

3.21 Example: Let X = Y = {1, 2, 3} with $\tau = \{\phi, \{1\}, \{1, 2\}, X\}$ and $\mu = \{\phi, \{1\}, \{1, 3\}, Y\}$. Here, g is identity map. Then g is semi maximal continuous and not super maximal semi continuous. Now $\{1, 3\} \in M_aSO(Y)$, then $g^{-1}(\{1, 3) = \{1, 3\} \notin SM_aO(X)$.

3.22 Theorem: A mapping g is maximal semi continuous iff $g^{-1}(F) \in SC(X)$, $\forall\, F \in M_iC(Y)$.

Proof: Assume that g is a maximal semi continuous map and $F \in M_iC(Y)$ implies $(Y - F) \in M_aO(Y)$. As g is maximal semi continuous, we have $g^{-1}(Y - F) \in SO(X)$. But $g^{-1}(Y - F) = X - g^{-1}(F)$ belongs to SO(X). Therefore $g^{-1}(F) \in SC(X)$.

Conversely, let $g^{-1}(F) \in SC(X)$, $\forall\, F \in M_iC(Y)$. Let $M \in M_aO(Y)$. Then $(Y - M) \in M_iC(Y)$ and by hypothesis $g^{-1}(Y - M) = X - g^{-1}(M)$ belongs to SC(X) implies $g^{-1}(M) \in SO(X)$. Hence g is maximal semi continuous.

3.23 Theorem: If g is maximal semi continuous then restriction function $g_A : A \to Y$ is maximal semi continuous.

Proof: Consider a maximal semi continuous function g and non-empty subset A of X. Let $M \in M_aO(Y)$. By hypothesis, $g^{-1}(M) \in SO(X)$. Therefore by definition of $f_A : A \to Y$ it is evident that $g_A^{-1}(M) = A \cap g^{-1}(M)$. Therefore $A \cap g^{-1}(M)$ is semi open set in A. Therefore by definition $g_A : A \to Y$ is maximal semi continuous.

3.24 Theorem: A mapping g is maximal semi continuous iff for any $p \in X$ and $M \in M_aO(Y)$ containing $g(p)$, $\exists\, N \in SO(X) \ni p \in N$ and $f(N) \subset M$.

Proof: Let $M \in M_aO(Y)$ containing g (p) for $p \in N$, where $N \in SO(X)$. As g is maximal semi continuous, we have $g^{-1}(M) \in SO(X)$. Take $N = g^{-1}(M)$ which implies $g(N) = g(g^{-1}(M)) \subset M$. Therefore $g(N) \subset M$.

Conversely, let $M \in M_aO(Y)$. By hypothesis, $\exists N \in SO(X)$, $\ni p \in N$ which implies $g(p) \in g(N) \subset M$ which implies $p \in g^{-1}(g(N)) \subset g^{-1}(M)$. Thus $g^{-1}(M) \in SO(X)$, $\forall M \in M_aO(Y)$. Therefore g is maximal semi continuous.

3.25 Theorem: A mapping g is strongly maximal semi continuous iff $g^{-1}(F) \in M_iSC(X)$, $\forall F \in M_iC(Y)$.

Proof : Assume that g is a strongly maximal semi continuous map and $F \in M_iC(Y)$, $(Y - F) \in M_aO(Y)$. By definition, $g^{-1}(Y - F) \in M_aSO(X)$. But $g^{-1}(Y - F)$ belongs to $M_aSO(X)$. Therefore $g^{-1}(F) \in M_iSC(X)$.

Conversely, let $g^{-1}(F) \in M_iSC(X)$ $\forall F \in M_iC(Y)$. Let $M \in M_aO(Y)$. Then $(Y - M) \in M_iC(Y)$ and by hypothesis $g^{-1}(Y - M) \in M_iSC(X)$, that is $g^{-1}(M) \in M_aSO(X)$. Hence g is strongly maximal semi continuous.

3.26 Theorem: If g is strongly maximal semi continuous then $g_A : A \to Y$ is strongly maximal semi continuous, where A has the relatived topology.

Proof: The following the Theorem 3.23.

3.27 Theorem: Mapping g is strongly maximal semi continuous iff for $p \in X$ and $M \in M_aO(Y)$ containing g(p), $\exists N \in M_aSO(X) \ni p \in N$ and $g(N) \subset M$.

Proof: The following the Theorem 3.24.

3.28 Theorem: A mapping g is maximal semi irresolute iff $g^{-1}(F) \in M_iSC(X)$, $\forall F \in M_iSC(Y)$.

Proof : Assume that g is maximal semi irresolute and $F \in M_iSC(Y)$, $(Y - F) \in M_aSO(Y)$. As g is maximal semi irresolute, we have $g^{-1}(Y - F) \in M_aSO(X)$. Therefore $g^{-1}(F) \in M_iSC(X)$.

Conversely, let $g^{-1}(F) \in M_iSC(X)$ $\forall F \in M_iSC(Y)$. Let $M \in M_aSO(Y)$. Then $(Y - M) \in M_aSC(Y)$ and by hypothesis $g^{-1}(Y - M) = X - g^{-1}(M)$ belongs to $M_iSC(X)$, that is
$g^{-1}(M) \in M_aSO(X)$. Hence g is maximal semi irresolute.

3.29 Theorem: If g is maximal semi irresolute then $g_A : A \to Y$ is maximal semi irresolute, where A has the relatived topology.

Proof: The following the Theorem 3.23.

3.30 Theorem: If g is maximal semi irresolute iff for $p \in X$ and $M \in M_aSO(Y)$ containing g (p), $\exists N \in M_aSO(X) \ni p \in N$ and $g(N) \subset M$.

Proof: The following the Theorem 3.24.

3.31 Theorem: If g is maximal semi s - continuous then $g_A : A \to Y$ is maximal semi s - continuous, where A has the relatived topology.

Proof: The following the Theorem 3.23.

3.32 Theorem: A mapping g is maximal semi s - continuous iff for $p \in X$ and $M \in M_aSO(Y)$ containing g (p), $\exists N \in SO(X) \ni p \in N$ and $g(N) \subset M$.

Proof: The following the Theorem 3.24.

3.33 Theorem: A mapping g is super maximal semicontinuous iff $g^{-1}(F) \in SM_iC(X)$, $\forall F \in M_iSC(Y)$.

Proof: Assume that g is super maximal semi continuous and F $F \in M_iSC(Y)$, $(Y - F) \in M_aSO(Y)$. By definition, $g^{-1}(Y - F) \in SM_aO(X)$. But $g^{-1}(Y - F) = X - g^{-1}(F)$ belongs to $SM_aO(X)$. Therefore $g^{-1}(F) \in SM_aC(X)$.

Conversely, let $g^{-1}(F) \in SM_iC(X)$, $\forall F \in M_iSC(Y)$. Let $M \in M_aSO(Y)$. Then $(Y - M) \in M_iSC(Y)$ and by hypothesis $g^{-1}(Y - M) = X - g^{-1}(M)$ is in $SM_iC(X)$, that is $g^{-1}(M) \in SM_aO(X)$. Hence g is super maximal semi continuous.

3.34 Theorem: If g is super maximal semi continuous then $g_A : A \to Y$ is super maximal semi continuous, where A has the relatived topology.

Proof: The following the Theorem 3.23.

3.35 Theorem: If g is super maximal semi continuous iff for $p \in X$ and $M \in M_aSO(Y)$ containing g(p), $\exists N \in SM_aO(X) \ni p \in N$ and $g(N) \subset M$.

Proof: The following the Theorem 3.24.

3.36 Theorem: If g is super maximal semicontinuous and $h : Y \to Z$ is maximal semi irresolute. Then $h \circ g : X \to Z$ is super maximal semicontinuous.

3.37 Remark: Composition of maximal semi continuous is not maximal semi continuous, which is shown below.

3.38 Example: Let X = Y = Z = {1, 2, 3} with $\tau = \{\phi, \{2\}, \{2, 3\}, X\}$, $\mu = \{\phi, \{1\}, \{1, 3\}, Y\}$, and $\eta = \{\phi, \{3\}, \{1, 3\}, Z\}$. Let g and h be identity functions. Then g and h are maximal semi continuous functions but not h o g is maximal semi continuous. Now $\{1, 3\} \in M_aO(Z)$, then $(h \circ g)^{-1}(\{1, 3\}) = \{1, 3\} \notin SO(X)$.

3.39 Theorem : If g is continuous (resp. semi continuous) and h is continuous (resp. maximal continuous, strongly maximal continuous, maximal - minimal continuous) functions. Then h o g is maximal semi continuous.

3.40 Theorem : If g is strongly maximal continuous (resp. maximal semi continuous) and h is strongly maximal continuous functions. Then hog is maximal semi continuous.

3.41 Remark: Composition of strongly maximal semi continuous is not strongly maximal semi continuous, which is shown below.

3.42 Example: By Example 3.38, we have g and h are strongly maximal semi continuous functions but not h o g is strongly maximal semi continuous. Now $\{1, 3\} \in M_aO(Z)$, then $(h \circ g)^{-1}\{1, 3\} = \{1, 3\} \notin M_aSO(X)$.

3.43 Theorem : If g is strongly maximal continuous (resp. strongly maximal semi continuous) and h is strongly maximal continuous functions. Then hog is strongly maximal semi continuous.

3.44 Theorem : If g and h are maximal semi irresolute. Then hog is maximal semi irresolute

3.45 Remark: The composition of maximal semi s - continuous need not be maximal semi s - continuous, which is shown below.

3.46 Example: Let X = Y = Z = {1, 2, 3} with $\tau = \{\phi, \{2\}, \{3\}, \{2, 3\}, X\}$, $\mu = \{\phi, \{1\}, \{2\}, \{1, 2\}, Y\}$, and $\eta = \{\phi, \{1\}, \{2, 3\}, Z\}$. Let g and h be an identity functions. Then g and h are maximal semi s-continuous functions but not hog is maximal semi s-continuous. Now $\{1\} \in M_aSO(Z)$, then$(h \circ g)^{-1}(\{1\}) = \{1\} \notin SO(X)$.

3.47 Theorem : If g is maximal semi irresolute (resp. maximal semi s - continuous) and h is maximal semi irresolute. Then hog is maximal semi s - continuous.

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