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Research paper

On Sαrw-Homeomorphism in Topological Spaces

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Abstract

New class of homeomorphisms named as sαrw-homeomorphism and sαrw*-homeomorphism are explored & elaborated. Few basic properties are inspected. Their relations with some existing homeomorphisms in topological spaces are studied.

Keywords: *homeomorphism; sαrw-homeomorphism and sαrw*-homeomorphism.*

1. Introduction

The concept of generalized homeomorphism was introduced and studied in the year 1991 by Balachandran et all [2]. N. Nagaveni [7] introduced and studied rwg homeomorphism in topological spaces. In the year 2002, M Sheik John [10] introduced and studied w-homeomorphism in topological space and rgα-homeomorphisms, rps-homeomorphisms and gs and sg homeomorphisms have been introduced and studied by A. Vadivel [11], T. Shyla Isac Mary [8], H. Maki [6] respectively. This paper aims to explore and elaborate sαrw-homeomorphism, sαrw*-homeomorphism and its relation with some existing homeomorphisms in topological spaces. Few of its properties are investigated.

2. Preliminaries

Without considering any separation axioms, a topological space Y or (Y,τ) and Z or (Z,σ) is assumed. For $B \subseteq Y$, where Y is a topological space. Closure of B- cl (B), interior of B- int (B), and complement of B - Y-B or B^C

Definition 2.2: g: (Y, τ) → (Z, σ) is a map.
i) If g^{-1}(F) is r-closed in Y ∀ closed F of Z then it is called as regular-continuous (in brief r- continuous) [1].
ii) If g^{-1}(F) is r- closed in Y ∀ closed F of Z, then it is called as completely–continuous [1].
iii) If g^{-1}(F) is g–closed in Y ∀ closed F of Z, then it is called as g– Continuous [2].
iv) If g^{-1}(F) is sαrw–closed in Y ∀ closed F of Z, then it is called as sαrw–continuous [4].
v) If g^{-1}(F) is closed set in Y ∀ sαrw–closed F in Z, then it is called as strongly sαrw–continuous [4].
vi) If g^{-1}(F) is αrw–closed in Y ∀ closed F of Z, then it is called as αrw– continuous [12].

Definition 2.5: g: (Y, τ) → (Z, σ) is map
i) If g (F) is g-open in (Z, σ) ∀ open F of (Y, τ), then it is called as g-open map [2].
ii) If g (F) is gpr-open in (Z, σ) ∀ open set F of (Y, τ), then it is called as gpr-open map [5].
iii) If g (F) is open in (Z, σ) ∀ regular open set F of (Y, τ), then it is called as regular open map [9].
iv) If g (F) is rwg-open in (Z, σ) ∀ open set F of (Y, τ), then it is called as rwg-open map [7].
v) If g (F) is wg-open in (Z, σ) ∀ open set F of (Y, τ), then it is called as wg-open map [7].
vi) If g (F) is w-open in (Z, σ) ∀ open set F of (Y, τ), then it is called as w-open map [10].

Definition 2.6: g: (Y, τ) → (Z, σ) is a map.
i) If g is open and continuous then it is called as homeomorphism
ii) If g is g-continuous and g-open then it is called as g- homeomorphism [2].
iii) If g is rgα-continuous and rgα-open then it is called as rgα-homeomorphism [11].
iv) If g is rwg-continuous and rwg-open then it is called as rwg-homeomorphism [7].
v) If g is sg-continuous and sg-open then it is called as sg-homeomorphism [6].
vi) If g is gs-continuous and gs-open then it is called as gs- homeomorphism [6].
vii) If g is rps-open and rps-continuous then it is called as rps-homeomorphism [11].

Results 2.7 [3]:
i. In Y, each closed set (resp regular-closed, α-closed)is sαrw closed .
ii. In Y, each sαrw-closed set is sg-closed.
iii. In Y, each sαrw-closed set is gsp-closed (resp rps-closed, gs closed , gspr-closed) .

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3. sαrw-homeomorphism in topological Space

Definition 3.1: g: (Y, τ) → (Z, σ) is a bijective map and if g is sαrw-continuous as well as sαrw-open then we call it sαrw-homeomorphism.

Theorem 3.2: Each homeomorphism is sαrw-homeomorphism.
Proof: g: (Y, τ) → (Z, σ) is a homeomorphism. g is bijective, continuous and open because each continuous map is sαrw-continuous as well as each open map is sαrw-open. Thus f is sαrw-homeomorphism.

Remark 3.3: Each sαrw-homeomorphism need not homeomorphism.

Example 3.4: Let Y=Z={1, 2 ,3}, τ ={ϕ,{1},{2},{1, 2},Y} and σ ={ϕ,{1},{2},{1, 2},{1, 3}, Z }.g: Y→Z is defined as g (1)=2 , g(2)=3,g(3)=3,then g is sαrw-homeomorphism but not homeomorphism as closed set F= {2} in Z, g^{-1}(F) ={1} in Y which isn't closed in Y.

Theorem 3.5: Each α-homeomorphism is sαrw-homeomorphism.

Proof: Let g: (Y, τ) → (Z, σ) be homeomorphism. g is bijective, continuous and open map. Because each α-continuous map is sαrw-continuous and each α-open map is sαrw-open, Thus g is sαrw-homeomorphism.

Remark 3.6: Each sαrw- homeomorphism need not be α- homeomorphism.

Example 3.7: From example 3.4 g is sαrw-homeomorphism however isn't α-homeomorphism as closed set F= {2} in Z, then g^{-1}(F) = {1} in X which is n't α-closed in Y.

Theorem 3.8: Each sαrw–homeomorphism is gs-homeomorphism.
Proof: Let g: (Y, τ) → (Z, σ) be sαrw-homeomorphism. g is sαrw-continuous and sαrw-open map, which implies g is gs continuous and gs open map. Thus g is gs-homeomorphism.

Remark 3.9: Each gs –homeomorphism need not be sαrw - homeomorphism

Example 3.10: Let Y={1,2,3}, Y={1,2,3,4}, τ ={ϕ,{1},{2,3}, {1,2,3},Y} and σ ={ϕ,{1},{2},{1, 2},{1, 3 },Z}. g: X→Y be defined as g(1)=2 , g(2)= 3 , g(3)=1, then g is gs–homeomorphism but it isn't sαrw-homeomorphism, as a closed set F= {3} in Z and g^{-1}(F) = {2} in Y which are not sαrw-closed in Y.

Theorem 3.11: Each sαrw–homeomorphism is sg-homeomorphism.

Proof: Let g: (Y, τ) → (Z, σ) be sαrw-homeomorphism. Then g is sαrw-continuous and sαrw-open map. This implies that g is sg continuous and sg open map. Thus g is sg-homeomorphism.

Remark 3.12: Each sg–homeomorphism need not be sαrw - homeomorphism.

Example 3.13: From example 3.10, g is sg–homeomorphism. But not sαrw-homeomorphism as closed set F= {3} in Z, then g^{-1}(F) = {2} in Y which isn't sαrw-closed in Y.

Theorem 3.14: Each sαrw–homeomorphism is rps-homeomorphism.

Proof: Let g: (Y, τ) → (Z, σ) be sαrw-homeomorphism. Then g is sαrw-continuous and sαrw-open map. This implies that g is rps continuous and rps open map. Thus g is rps-homeomorphism.

Remark 3.15: Each rps–homeomorphism need not be sαrw - homeomorphism.

Example 3.16: From example 3.10, g is rps–homeomorphism, but not sαrw-homeomorphism since closed set F= {3} in Z, then g^{-1}(F) = {2} in Y that is not sαrw-closed in Y.

Theorem 3.17: Each sαrw–homeomorphism is gsp-homeomorphism.

Proof: Assume that g: (Y, τ) → (Z, σ) is sαrw-homeomorphism, then g is sαrw-continuous and sαrw-open map. It shows that g is gsp continuous and gsp open map. Thus g is gsp-homeomorphism.

Remark 3.18: Each gsp–homeomorphism need not be sαrw-homeomorphism.

Example 3.19: From example 3.10, g is gsp–homeomorphism, but not sαrw homeomorphism as closed set F= {3} in Z, then g^{-1}(F) = {2} in Y that isn't sαrw-closed in Y.

Theorem 3.20: Each sαrw–homeomorphism is gspr-homeomorphism.

Proof: Assume g: (Y, τ) → (Z, σ) be sαrw-homeomorphism, then g is sαrw-continuous and sαrw-open map. This implies that g is gspr continuous and gspr open map. Thus g is gspr-homeomorphism.

Remark 3.21: Each gspr–homeomorphism need not sαrw - homeomorphism.

Example 3.22: From example 3.10, g is gspr–homeomorphism. But not sαrw-homeomorphism as closed set F= {3} in Z, then g^{-1}(F) = {2} in Y which isn't sαrw-closed in Y.

Remark 3.23: The example below illustates sαrw-homeomorphisms is independent of pre–homeomorphism, β–homeomorphism, gp–homeomorphism, gpr–homeomorphism, swg–homeomorphism, rwg–homeomorphism, wg–homeomorphism, gprw–homeomorphism, rgw–homeomorphism, pgpr–homeomorphism.

Example 3.24: Assume Y={1, 2 ,3, 4}, Z ={1, 2, 3}, τ = { ϕ,{1},{2, 3},{1, 2, 3},Y} and σ ={ϕ,{1},{2},{1 ,2},{1, 3},Z}. g: X→Y be defined as g(1)=2 , g(2)= 3 , g(3)=1, then g is pre–homeomorphism, β–homeomorphism, gp–homeomorphism, gpr–homeomorphism, swg–homeomorphism, rwg–homeomorphism , wg–homeomorphism, gprw–homeomorphism, rgw-homeomorphism, pgpr– homeomorphism, but not sαrw-homeomorphism as closed set F= {3} in Z, then g^{-1}(3) = {2} in Y that isn't sαrw-closed in Y.

Theorem 3.25: Assume that g: Y→Z be bijective and sαrw continuous . Then the below facts are true.

a. g is sαrw-open.
b. g is sαrw-homeomorphism
c. g is sαrw-closed.

Proof:
1. g is sαrw-open.⇒ g is sαrw-homeomorphism

g: Y→Z is bijective, sαrw-continuous map and sαrw-open map. Thus g is a sαrw-homeomorphism.

2. g is sαrw-homeomorphism ⇒ g is sαrw-closed.

g: Y→Z is sαrw-homeomorphism. Let G be any closed in Y and hence G^c is closed in Y. Then g (G^c) is open in Z. But $g(G^c)=g(G)^c$. Thus g is a sαrw-closed.

3. g is sαrw-closed.⇒ g is sαrw-open.

g: Y→Z is bijective, sαrw-continuous and sαrw-closed map. Let G be any open in Y and hence G^c is closed set in Y. Then $g(G^c)$ is closed in Z. But $g(G^c)=g(G)^c$. Thus g is sαrw-open.

Remark 3.26: Ccomposition of 2 sαrw-homeomorphisms is need not be a sαrw-homeomorphism .it is illustrated below

Example 3.27: Assume Y=Z=M= {1, 2, 3}, τ = {ϕ, {1}, {2},{1, 2}, {1, 3},Y} , σ = {ϕ, {1}, {2}, {1, 2},Z} and η = {ϕ, {1}, {2, 3},M}.We define g: (Y, τ) → (Z, σ) by g (1) =1, g (2) =3, g(3) =2 and h: (Y, σ) →(M, η) be the identity mapping. Both g and h are sαrw-homeomorphism but their composition (hog): (Y, τ)→(M, η) isn't a sαrw-homeomorphism as the closed set F={1} in (M, η), but $(hog)^{-1}(F)$={1} isn't sαrw-closed in Y.

Definition 3.28: g: Y→Z is bijective map. Then it is sαrw*-homeomorphism if g and g^{-1} are sαrw-irresolute. Where sαrw*-h(Y,τ) represents the set of all sαrw-homeomorphisms of Y on to itself.

Theorem 3.29: Each sαrw*-homeomorphism is a sαrw-homeomorphism.

Proof: Let g: Y→Z be a sαrw*-homeomorphism. Then g is bijective, g and $g^{-1}$are sαrw-irresolute. Because each sαrw-irresolute map is sαrw-continuous then g and g^{-1} are sαrw-continuous. Also g^{-1}: Z→Y is a sαrw-continuous map is equivalent to sαrw-open map. Thus g is bijective, sαrw-continuous and sαrw-open map. Hence g is a sαrw-homeomorphism.

Each sαrw-homeomorphism need not be sαrw*-homeomorphism.

Example 3.30: Assume Y=Z={1 ,2 ,3}, τ ={ϕ,{1},{2},{1,2},Y} and σ ={ϕ,{1},{2},{1, 2},{1,3},Z}.Let g: Y→Z defined by g(1)=2, g(2)= 3 , g(3)=3, then g is sαrw–homeomorphism but not sαrw*-homeomorphism, because g isn't sαrw-irresolute.

Theorem 3.31: Each sαrw*-homeomorphism is a sg-homeomorphism.

Proof: The proof follows from the facts that each sαrw*-homeomorphism is a sαrw-homeomorphism and each sαrw-homeomorphism is a sg-homeomorphism.
Each sg-homeomorphism need not be sαrw*-homeomorphism.

Example 3.32: Assume Y= {1, 2, 3}, Z= {1, 2, 3, 4}, τ = {ϕ, {1}, {2,3}, Y} and σ = {ϕ, {1}, {2}, {1, 2}, {1, 3}, {1, 2, 3}, Z}. Let g: Y→Z defined by g(1)=2 , g(2)= 3 , g(3)=1, then g is sg–homeomorphism. But not sαrw-homeomorphism as closed set F= {3} in Z, then $g^{-1}(F)$ = {2} in Y which is not sαrw-closed set in Y.

Theorem3.33: Each sαrw*-homeomorphism is a gs-homeomorphism (resp gsp-homeomorphism, rps-homeomorphism, gspr-homeomorphism).
Proof: we know that each sαrw*-homeomorphism is a sαrw-homeomorphism and each sαrw-homeomorphism is a gs-homeomorphism (resp gsp-homeomorphism, rps-homeomorphism, gspr-homeomorphism)
Each gs-homeomorphism is a sαrw*-homeomorphism

Example 3.34: From example 3.27, g is gs-homeomorphism (resp) gsp-homeomorphism, rps-homeomorphism, gspr homeomorphism), but not sαrw-homeomorphism, as closed set F= {3} in Z, then, $f^{-1}(F)$ = {2} in Y which isn't sαrw-closed set in Y.

Remark 3.35: From discussions and facts already known ,relation between sαrw-homeomorphism with some existing homeomorphisms in topological space is shown in following figure.

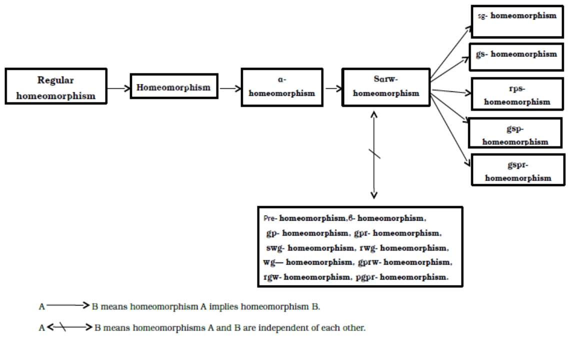

Figure-1

Theorem 3.36: The composition of two sαrw*-homeomorphisms is also a sαrw*-homeomorphism.

Proof: Assume g: X→Y and h: Y→Z is two sαrw*-homeomorphisms. To prove hog: X→Z is sαrw*-homeomorphism. Let V be any closed set in Z. Because h is sαrw-irresolute, h(V) is sαrw-closed in Y. Because g is sαrw-irresolute, $g^{-1}(h^{-1}(V))$ is sαrw-closed in X. But $g^{-1}(h^{-1}(V)) = (hog)^{-1}(V)$. Thus hog is sαrw-irresolute. Let W be any closed in X. Because g is sαrw-irresolute, g (W) is sαrw-closed in Y and also h (g (W)) is sαrw-closed in Z. But (hog) (W) =h(g(W)). Thus (hog) is sαrw-irresolute. Hence (hog) is sαrw*-homeomorphism.

Theorem 3.37: Under the composition of maps, sαrw*-h (Y, τ) is group.

Proof:* be binary operation,*:sαrw*h (Y, τ) (x) sαrw*h (Y, τ) → sαrw*h (Y, τ) defined by f * g = g o f ∀ f, g ∈ sαrw*h (Y, τ). We have (gof) ∈ sαrw*h(X, τ). Also the associative property holds in composition of maps. The identity mapping I: (Y, τ) → (Y, τ) which ∈ sαrw*h (Y, τ), acts as the identity element. For the inverse element, if f ∈ sαrw*h (Y, τ) then f^{-1} ∈ sαrw*h(Y, τ) such that $fof^{-1}= f^{-1}of$=I. Thus, under the composition of maps, the set sαrw*-h(Y, τ) is a group.

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