International Journal of Engineering & Technology, 7 (4.10) (2018) 863-868

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper



Entropy Generation on MHD Flow and Heat Transfer of Non-Newtonian Fluid Flow Over a Non-Linear Radially Stretching Sheet

S.Sreenadh[1], V. Ramesh Babu[2], G. Gopi Krishna[3], S.R.Mishra[4] and A.N.S.Srinivas[5*]

[1]*Department of Mathematics, Sri Venkateswara University, Tirupati, 517 502, A.P., India.*
[2]*Department of Mathematics, Rashtriya Sanskrit Vidyapeetha, Tirupati, 517 502, A.P., India*
[3]*Department of Mathematics, Marri Laxman Reddy Institute of Technology and Management, Hyderabad - 500043*
[4] *Department of Mathematics, Siksha O Anusandhan, Bhubaneswar - 751030, Odisha, India.*
[5]*Department of Mathematics, School of Advanced Sciences, VIT, Vellore, 632 014, T.N., India.*
**Corresponding author E-mail: anssrinivas@vit.ac.in*

Abstract

An investigation is made for analyzing the behavior of MHD flow phenomena of a non-Newtonian fluid over a non-linear radially stretching sheet by using numerical technique. Magnetic field is considered in normal direction to the stretching sheet. With use of similarity transformations, the pdes are transformed into odes. The solution of theses odes are performed by using fourth order Runge - Kutta method along with shooting technique. The significance of different physical parameters characterizes the flow phenomena are analyzed with the use of graphs. The Jeffrey parameter λ_1 and magnetic parameter M has significant effect on velocity and temperature distribution over a non-linear stretching sheet. It is noticed that, the higher magnetic parameter M results the increase in entropy generation number where the opposite nature is noticed in the case of Bejan number.

Keywords: *Axisymmetric flow; Heat transfer; Magnetic effect; Non-Newtonian fluid; Entropy generation.*

1. Introduction

Many researchers investigated the significance of flow with heat transfer mechanism on a stretching sheet due to its important applications in science and engineering. Some of the applications include manufacturing the glass fibre, cooling, drying of papers and polymer extrusion etc. In this process, the final product of the material depends upon the properties of transfer of heat over the stretched surface. Crane [1] investigated the solution for various fluid flow over stretching sheet. Wang [2] studied stretching sheet problem in the three dimensions. Later, the flow problems over a stretching sheet were considered by the many researchers. Ariel [3] analyzed computation of flow past a stretching sheet with homotopy perturbation method. Dissipative effect on the fluid flow over a nonlinearly stretching sheet is examined by Alinejad and Samarbaksh [4]. The MHD effect on viscous fluid flow over a nonlinear porous sheet is investigated by Nadeem and Hussain [5]. Qasim [6] discussed the effects of flow phenomena on Jeffrey fluid flow. Hassanien et al., [7] presented heat transfer effects by considering Power-law fluid for his discussion. The enhanced boundary condition by taking Newtonian heating over a stretching sheet is examined by Salleh et al., [8]. Khan and Pop [9] studied the nanofluid flow over a stretching sheet. Makinde and Aziz [10] extended the study with enhance boundary conditions. The steady, laminar nanofluid flow over the non-linear stretching of a flat surface is numerically examined by Rana and Bhargava [11]. Mixed convection and MHD effects on Jeffrey fluid flow over an exponentially stretching sheet is discussed by Ahmad et al. [12]. Ahmed et al. [13] observed the influence of MHD and convective heat properties on non-newtonia fluid. Liao [14] discussed the analytic solution of fluid flows with MHD. Ishak [15] considered the linearly stretching sheet to understand the influence of radiation on the thermal boundary layer flow with constant surface temperature.

Abel and Mahesha [16] studied significance of variable thermal conductivity, and radiation on viscoelastic fluid flow. Rashidi et al., [17] investigated the significance of radiation on flow of MHD fluid on a permeable vertical stretching sheet. Pramanik [18] analyzed the thermal radiation and heat transfer effects on Casson fluid flow past an exponentially porous medium. Shahzad et al. [19] presented the exact solution for asymmetric flow in the case of non-linear radially stretched surface. Further Shahzad et al. [20] examined the exact solution flow problem with heat transfer.

In view of above studies, the entropy generation on MHD flow phenomena of fluid over a non-linear radially stretching sheet is examined numerically. The influence of different parameters on flow quantities are analyzed graphically.

2. Mathematical formulation:

Fig. 1 illustrates the magneto hydrodynamic Jeffrey fluid flow over a nonlinear radially stretching surface corresponds to the plane $z = 0$. The flow is produced when the sheet is stretched along the radial direction. The velocity of the sheet is defined as $U(r) = cr^3$, which is nonlinear and c is a dimensional constant.

Copyright © 2018 Authors. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Here the sheet surface and ambient temperatures are T_w and T_∞ respectively, where $T_w > T_\infty$.

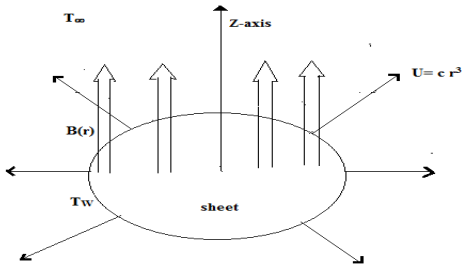


Fig.1: Physical Model

For an incompressible Jeffrey fluid, the constitutive equations are

$$\bar{T} = -\bar{p}\bar{I} + \bar{s}$$

$$\bar{s} = \frac{\mu}{1+\lambda_1}\left(\dot{\bar{\gamma}} + \lambda_2 \ddot{\bar{\gamma}}\right)$$

here $\bar{T}$ is Cauchy's stress tensor, $\bar{p}$ is pressure, $\bar{I}$ is an identity tensor, $\bar{s}$ is extra stress tensor , λ_1 is ratio of relaxation to retardation time, λ_2 is retardation time, $\dot{\gamma}$ is shear rate and dots over the quantities indicate differentiation with respect to time.

Following [20], the governing equations with boundary conditions for the flow of a Jeffrey fluid are

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = \frac{1}{1+\lambda_1}\frac{\partial^2 u}{\partial z^2} - \frac{\sigma B^r(r)}{\rho}u + \rho g\beta\left(T - T_\infty\right) \tag{2}$$

$$u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \alpha\frac{\partial^2 T}{\partial z^2} + \frac{\nu}{1+\lambda_1}\left(\frac{\partial u}{\partial z}\right)^2 \tag{3}$$

$$\left.\begin{aligned} &u = U(r) = cr^3, w = 0, T = T_w = T_\infty + A_0\left(\frac{r}{l}\right)^6 \ at \ z = 0 \\ &u \to 0, T = T_\infty \ as \ z \to \infty \end{aligned}\right\} \tag{4}$$

here u the radial and w the axial velocity respectively, $\nu = \frac{\mu}{\rho}$ is the kinematic viscosity, μ = coefficient of viscosity, ρ = fluid density, T = the fluid temperature, $\alpha = \frac{k}{\rho c_p}$ is the thermal diffusivity and $B(r) = B_0 r$ is the variable magnetic field.
Here ,

$$\begin{aligned} &u = U(r) = cr^3 f^{l}(\eta),\ w = -\sqrt{cv}r[3f(\eta) + \eta f^{l}(\eta)], \\ &\theta = \frac{T - T_\infty}{T_w - T_\infty}, Gr = \frac{\rho g\beta(T_w - T_0)}{c^2 r^5}, \\ &\eta = \sqrt{\frac{c}{\nu}}rz, \Pr = \frac{\rho C_p}{\mu}, \\ &Ec = \frac{U^2}{c_p(T_w - T_\infty)}, M = \frac{\sigma B_0^2}{\rho c}. \end{aligned} \tag{5}$$

The momentum equation along with boundary conditions, equations (4) and (5) reduces to

$$\frac{1}{1+\lambda_1}f''' + 3ff'' - 3\left(f'\right)^2 - Mf' + Gr\theta = 0 \tag{6}$$

$$\theta'' - 6\Pr f'\theta + 3\Pr f\theta' = -\frac{\Pr Ec}{1+\lambda_1}\left(f''\right)^2 \tag{7}$$

$$\left.\begin{aligned} &f(\eta) = 0,\ f'(\eta) = 1, \theta(\eta) = 1 \ \text{at} \ \ \eta = 0 \\ &f'(\eta) = 0, \theta(\eta) = 0 \qquad\quad \text{at} \ \ \eta = \infty \end{aligned}\right\} \tag{8}$$

3. Entropy analysis:

The entropy generation for a conducting fluid through a deformable vertical porous layer is given as

$$E_G = \frac{k}{T_\infty^2}\left(\frac{\partial T}{\partial z}\right)^2 + \frac{\mu}{(1+\lambda_1)T_\infty}\left(\frac{\partial u}{\partial z}\right)^2 + \frac{\sigma B^2(r)}{T_\infty}u^2 \tag{9}$$

The entropy generation number can be determined as

$$Ns = \frac{T_\infty^2 L^2 E_G}{k(T_w - T_\infty)^2} \tag{10}$$

Hence

$$Ns = \mathrm{Re}\,\theta^{'2} + \frac{\mathrm{Re}}{1+\lambda_1}\frac{Br}{\Omega}f^{''2} + M\,\mathrm{Re}\left(\frac{Br}{\Omega}\right)f^{'2} \tag{11}$$

Where $Br = \frac{\mu U^2}{k(T_w - T_\infty)}$ the Brinkman is number, $\mathrm{Re} = \frac{\nu}{cr^2 L^2}$ and $\Omega = \frac{T_w - T_\infty}{T_\infty}$ the temperature difference.

Also N_S can be expressed as a summation of N_1 and N_2. Here N_1 and N_2 represent the entropy due to heat transfer and fluid friction respectively

$$N_1 = \mathrm{Re}\,\theta^{'2}\ ,\ N_2 = \frac{\mathrm{Re}}{1+\lambda_1}\left(\frac{Br}{\Omega}\right)f^{''2} + M\,\mathrm{Re}\left(\frac{Br}{\Omega}\right)f^{'2} \tag{12}$$

The Bejan number (Be) as

$$Be = \frac{N_1}{Ns} = \frac{1}{1+\Phi} \tag{13}$$

Where $\Phi = \frac{N_2}{N_1}$. Hence it is clear to note that for $0 \le \Phi \le 1$ heat transfer dominates and for $\Phi > 1$ fluid friction dominates. Further the contributions of both fluid friction and heat transfer for entropy generation are equal in the case of $\Phi = 1$.

4. Results and discussions:

In this study, the MHD heat transfer analysis for Jeffrey fluid flow over a nonlinear radial stretching sheet is investigated. Numerical solution such as Runge-Kutta forth order with shooting technique is implemented to handle the transformed ODEs. The computations are carried out by using Matlab software to analyze the quantitative effects of the different physical parameters. The results are discussed for parameters such as magnetic (M), Jeffrey parameter (λ_1), Prandtl number (Pr), Eckert number (Ec), and Grashof

number(Gr).To study the numerical computation, we used $Ec = 0.2, Gr = 5, M = 2, \Pr = 5, \lambda_1 = 0.2$.

Figs.2–6 illustrates the significance of the various physical parameters on the non-dimensional fluid velocity. It is known from the Fig. 2 that the velocity enhances with an increasing Eckert number Ec . The increase in $\Pr$ fluid velocity reduces which is given in Fig.3. It is noticed from Fig. 4 that the growing values of Grashof number Gr enhances the fluid velocity. The effect of Jeffrey parameter λ_1 and magnetic parameter M on fluid velocity is presented in Figs. 5 and 6 respectively. It is noticed that in velocity enhances with increasing Jeffrey parameter whereas effect is reversed for higher values of magnetic parameter M .

Figs. 7 – 11 illustrate the influence of aforesaid parameters on temperature. The higher values of magnetic parameter M and Eckert number Ec yields the decrease in temperature which is shown in Figs. 7 and 8 respectively. Figs. 9 and 10 shows the change in temperature for various values of Prandtl number $\Pr$ and Jeffrey parameter λ_1 i. e., the temperature decreases as $\Pr$ and λ_1 increases. Fig. 11 shows the significance of Grashof number Gr on temperature. It is clear that the increasing values of Gr reduces the temperature.

The change in entropy generation number for $\lambda_1, \mathrm{Re}, M$ and Br/Ω are presented in Figs. 12 – 15 respectively. It is clear from Fig. 12 that the temperature lessens as Jeffrey parameter increases. But from Figs. 13 – 15, the opposite nature is observed i.e., with higher values of Reynolds number Re , M and Br/Ω .

The influence of Jeffrey parameter λ_1 , M and group parameter Br/Ω on Bejan number profiles is shown in Figs. 16 – 18 respectively. Fig. 16 shows that the Bejan number profiles increases with increasing Jeffrey parameter λ_1 . The Bejan number profiles decreases with growing M and Br/Ω is illustrated in Figs. 17 – 18 respectively.

5. Conclusions:

The numerical study of entropy generation on MHD flow over a non-linear radially stretched sheet is presented. The nonlinear coupled ODEs are solved by shooting technique along with Runge – Kutta method of order four. The significance of pertinent physical parameters on flow quantities are presented through graphs. The important aspects of aforesaid discussions are as follows,

(i) The velocity of the fluid over a stretching sheet increases with increasing values of Ec, Gr and λ_1 , where fluid velocity decreases as M and $\Pr$ increase.

(ii) The temperature distribution in the fluid increases with increasing values of Ec and M , where the opposite behavior is observed in the case of $\Pr, \lambda_1$ and Gr .

(iii) The entropy generation number increases as M, Re and Br/Ω increases. The decrease in entropy generation number is noticed for increasing values of as λ_1 .

(iv) The Bejan number increases as λ_1 increases whereas it retards with increasing M and Br/Ω .

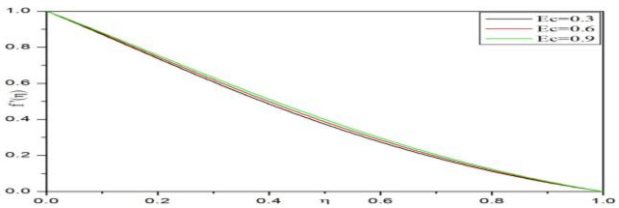


Fig 2: Velocity profiles for various values of Ec

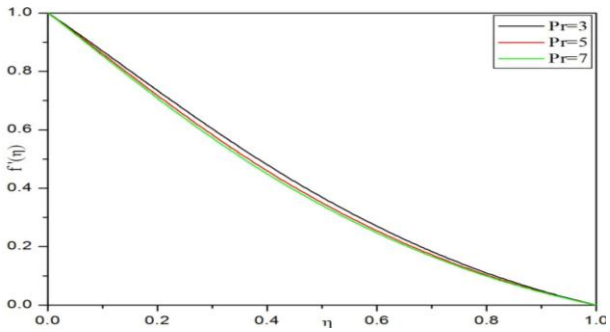


Fig 3: Velocity profiles for different values of $\Pr$

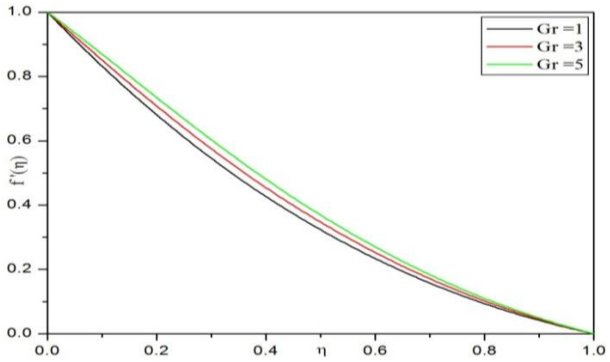


Fig 4: Velocity profiles for different values of Gr

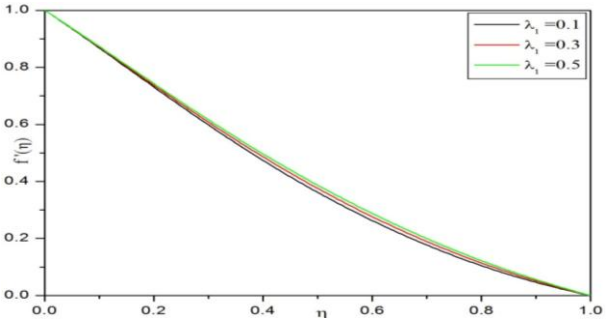


Fig 5: Velocity profiles for different values of λ_1

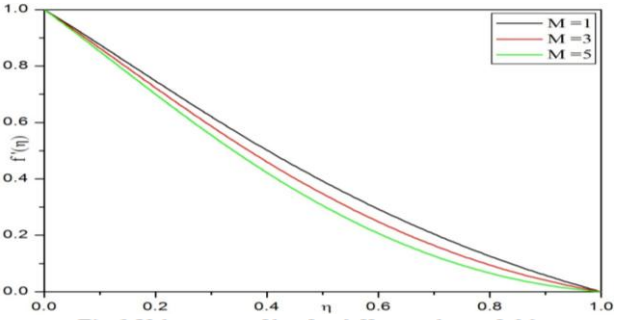


Fig 6: Velocity profiles for diffrent values of M

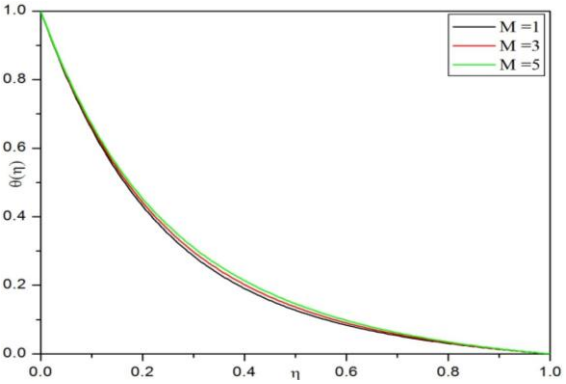


Fig 7: Temputaure profiles for diffrent values of M

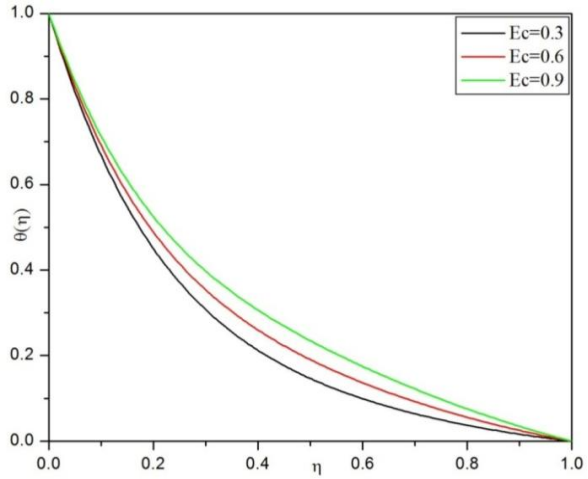


Fig 8: Temperature profiles for diffrent values of Ec

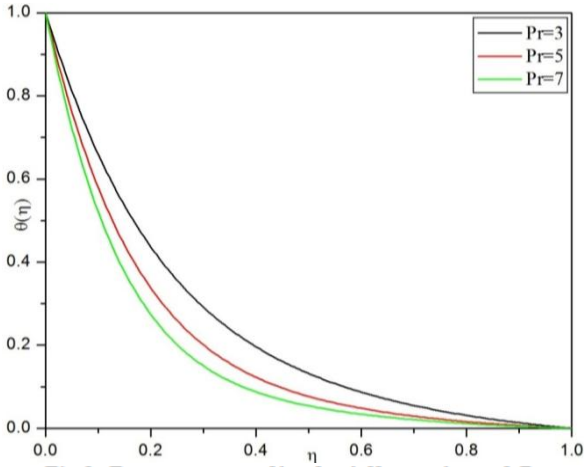


Fig 9: Temperature profiles for diffrent values of Pr

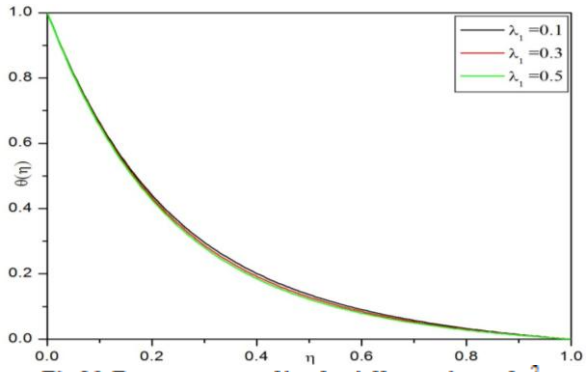


Fig 10: Tempurature profiles for diffrent values of λ_1

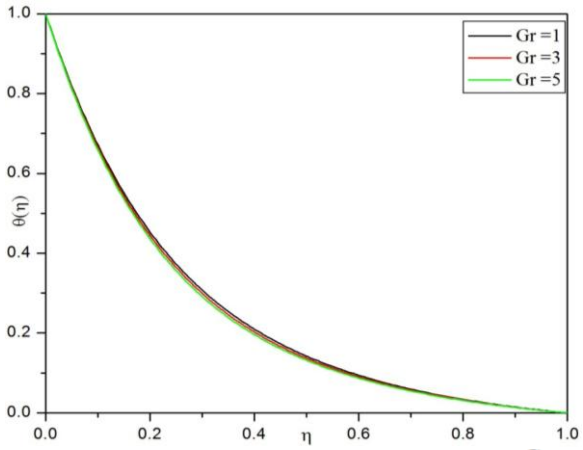


Fig 11: Temperature profiles for diffrent values of Gr

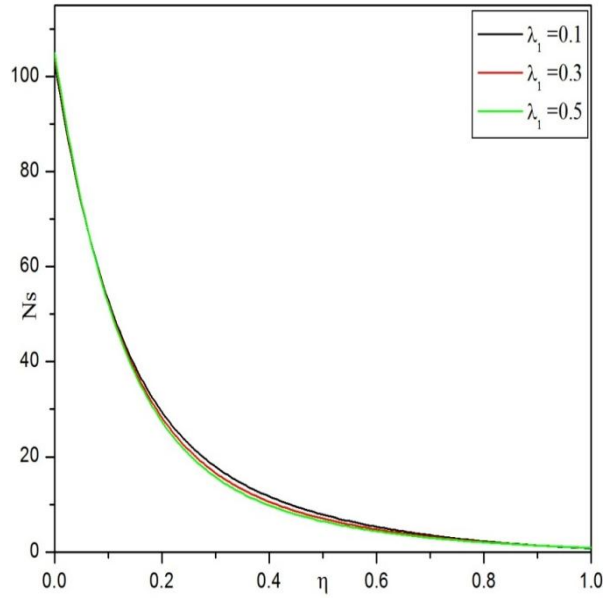


Fig 12: Entropy generation number profiles for diffrent values of λ_1

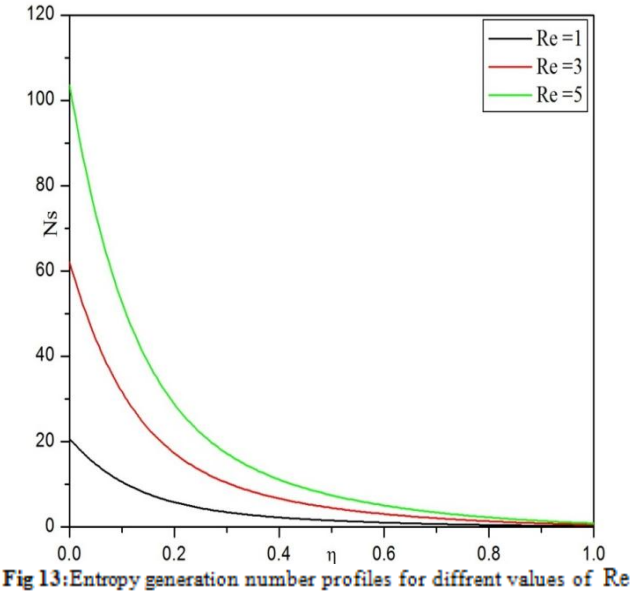


Fig 13: Entropy generation number profiles for diffrent values of Re

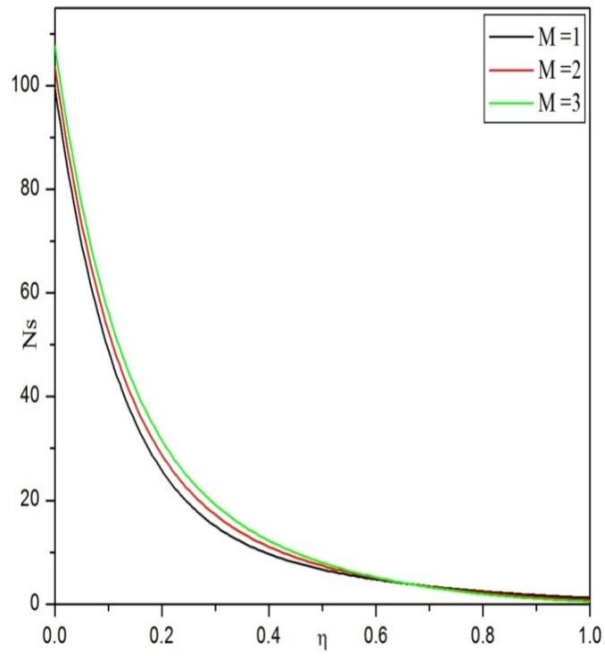


Fig 14: Entropy generation number profiles diffrent values of M

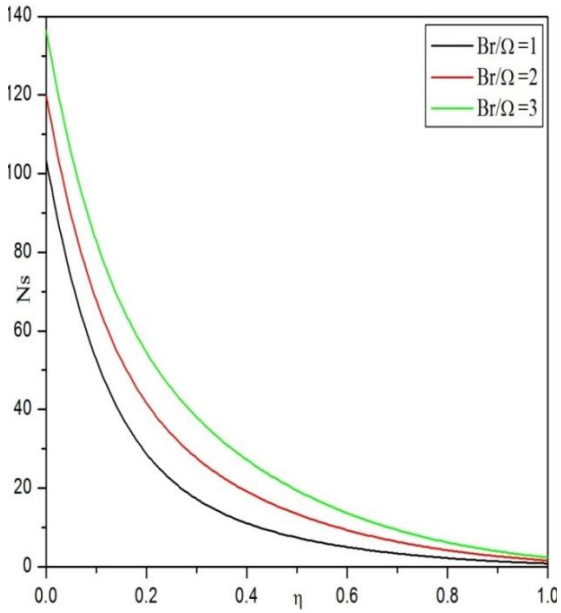


Fig 15: Entropy generation number profiles for diffrent values of Br/Ω

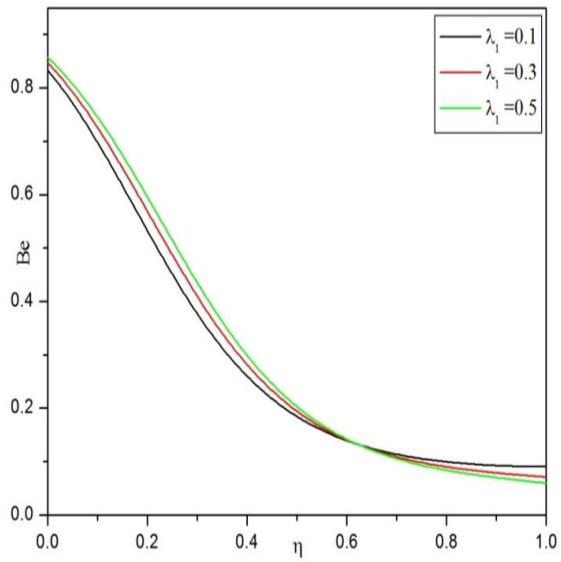


Fig 16: Bejan Number profiles for diffrent values of λ_1

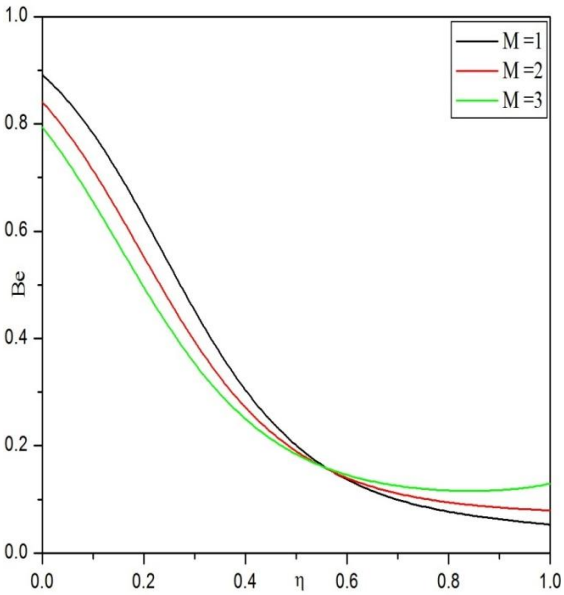


Fig 17: Bejan number profiles for diffrent values of M

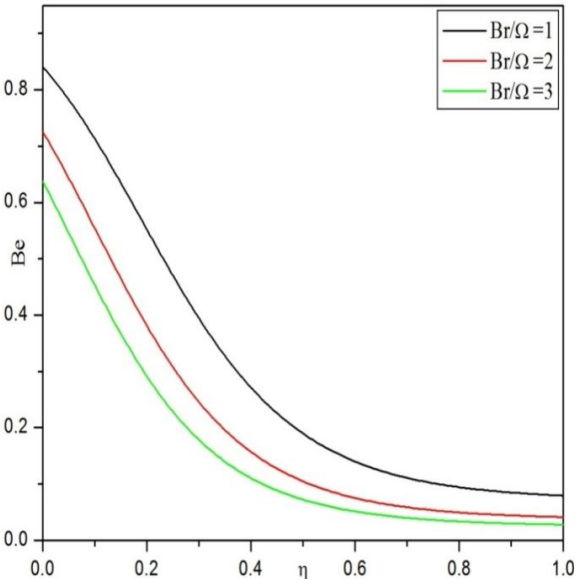


Fig 18: Bejan number profiles for diffrent values of Br/Ω

References

[1] Crane, L.J., "Flow past a stretching plate". *Z. Angew. Math. Phys.*, Vol. 21 (1970), pp. 645–655. https://doi.org/10.1007/BF01587695.

[2] Wang, C.Y., "The three-dimensional flow due to a stretching flat surface". *AIP Physics of Fluids*, Vol. 27(8) (1984), pp. 1915–1917. https://doi.org/10.1063/1.864868.

[3] Ariel, P. D., "Axisymmetric flow due to a stretching sheet with partial slip". *Computers and Mathematics with Applications*, Vol. 54 (7-8) (2007), pp. 1169-1183. https://doi.org/10.1016/j.camwa.2006.12.063.

[4] Alinejad, J. & Samarbakhsh, S. "Viscous flow over nonlinearly stretching sheet with effects of viscous dissipation". *Journal of Applied Mathematics*, Vol.10, Article ID: 587834 (2012). http://dx.doi.org/10.1155/2012/587834.

[5] Nadeem, S. & Hussain, A., "MHD flow of a viscous fluid on a nonlinear porous shrinking sheet with Homotopy analysis method". *.Applied. Mathematics and Mechanics*, Vol. 30 (2009), pp.1569-1578. https://doi.org/10.1007/s10483-009-1208-6.

[6] Qasim, M. "Heat and mass transfer in a Jeffrey fluid over a stretching sheet with heat source/sink". *Alex. Eng. Journal*, Vol. 52(4) (2013), pp. 571-575. https://doi.org/10.1016/j.aej.2013.08.004.

[7] Hassanien, I.A., Abdullah, A. & Gorla, R.S.R.," Flow and heat transfer in a Power-Law fluid over a nonisothermal stretching sheet". *Mathematical Computer Modeling*. Vol. 28(9) (1998), pp.105-116. https://doi.org/10.1016/S0895-7177(98)00148-4.

[8] Salleh, M. Z., Nazar, R. and Pop, I. "Boundary layer flow and heat transfer over a stretching sheet with Newtonian heating". *Journal of the Taiwan Institute of Chemical Engineers*, Vol. 41(6) (2010), pp. 651-655. https://doi.org/10.1016/j.jtice.2010.01.013.

[9] Khan, W.A. and Pop, I. "Boundary layer flow of a nanofluid past a stretching sheet". *International Journal of Heat and Mass Transfer*, Vol. 53 (11-12) (2010), pp.2477–2483. https://doi.org/10.1016/j.ijheatmasstransfer.2010.01.032.

[10] Makinde, O.D. & Azizb, A., "Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition". *Intr. Journal of Thermal Sciences*, Vol. 50(7) (2011), pp.1326-1332. https://doi.org/10.1016/j.ijthermalsci.2011. 02. 019.

[11] Rana, P. & Bhargava, R. "Flow and heat transfer of a nanofluid over a nonlinearly stretching sheet: A numerical study". *Communications in Nonlinear Science and Numerical Simulation*, Vol. 17(1) (2012), pp.212–226. https://doi.org/10.1016/j.cnsns.2011.05.009.

[12] Ahmad, K., Hanouf, Z. and Ishak,A.," Mixed convection Jeffrey fluid flow over an exponentially stretching sheet with magneto hydrodynamic effect.". *AIP Advances*,Vol.6(3)(2016), pp.035024(1-7) https://doi.org/10.1063/1.4945401.

[13] Ahmed, J., Shahzad, A., Khan, M. and Ali, R. "A note on convective heat transfer of an MHD Jeffrey fluid over a stretching sheet". AIP Advances, Vol. 5(11) (2015), pp.117117 (1-11). https://doi.org/10.1063/1.4935571.

[14] Liao, S. J., "On the analytic solution of magnetohydrodynamic flows of non-Newtonian fluids over a stretching sheet". *Journal of Fluid Mechanics*, Vol. 488 (2003) pp. 189-212. https://doi.org/10.1017/S0022112003004865

[15] Ishak, A., "Thermal boundary layer flow over a stretching sheet in a micropolar fluid with radiation effect". *Meccanica*, Vol. 45(3) (2010) pp. 367-373. https://doi.org/10.1007/s11012-009-9257-4.

[16] Abel, M. S. & Mahesha, N., "Heat transfer in MHD viscoelastic fluid flow over a stretching sheet with variable thermal conductivity, non-uniform heat source and radiation". *Applied Mathematical Modeling*, Vol. 32(10) (2008), pp. 1965-1983. https://doi.org/10.1016/j.apm.2007.06.038.

[17] Rashidi,M. M., Rostami, B., Freidoonimehr, N. and Abbasbandy, S. "Free convective heat and mass transfer for MHD fluid flow over a permeable vertical stretching sheet in the presence of the radiation and buoyancy effects". *Ain Shams Engineering Journal*, Vol. 5(3), (2014), pp.901-912. https://doi.org/10.1016/j.asej.2014.02.007.

[18] Pramanik, S. "Casson fluid flow and heat transfer past an exponentially porous stretching surface in presence of thermal radiation". *Ain Shams Engineering Journal*, Vol. 5(1) (2014), pp. 205-212. https://doi.org/10.1016/j.asej.2013.05.003.

[19] Shahzad, A., Ali, R. and Khan, M. "On exact solution for asymmetric flow and heat transfer over a nonlinear radially stretching sheet". *Chin. Phys. Lett*, Vol. 29 (2012), pp. 084705 (1-4). http://doi:10.1088/0246-307x/29/8/084705.

[20] Shahzad, A., Ahmed, J. and Khan, M. (2016). "On heat transfer analysis of axisymmetric flow of viscous fluid over a nonlinear radially stretching sheet". Alexandria En*gineering Journal*, Vol.55 (3) (2016)**,** pp. 2423-2429. https://doi.org/10.1016/j.aej.2016.04.013.