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Research paper



Quadrilateral Fuzzy Number

Pathinathan T[1]*, Santhoshkumar S[2]

[1]*P. G. and Research Department of Mathematics, Loyola College, Chennai-34, India.*
[2]*P. G. and Research Department of Mathematics, Loyola College, Chennai-34, India.*
**Corresponding author E-mail: pathinathan@gmail.com*

Abstract

Fuzzy numbers are used to represent uncertainty. Various types of fuzzy numbers are used in practical applications. In this paper we define Perfect Pentagonal Fuzzy Number (PPFN), Quadrilateral Fuzzy Number (QNF) and Left skewed Quadrilateral Fuzzy Number and Right skewed Quadrilateral Fuzzy Number. We study their algebraic properties with numerical examples.

Keywords: *fuzzy number, fuzzy operations, Perfect pentagonal fuzzy number (PPFN), quadrilateral fuzzy number (QFN), skewed fuzzy number*.

1. Introduction

Lotfi A. Zadeh [6] introduced fuzzy sets in 1965 to quantify the uncertainty and vagueness of the qualitative words. The concept of fuzzy sets led researchers to define fuzzy numbers. Dubois and Prade [2], defined the fuzzy number as fuzzy subset of the real line [3]. The basic arithmetic structure of fuzzy numbers was developed by Dubois and Prade [1],[2], Mizumoto and Tanaka [8],[9], and Ma et al. [7]. The Triangular fuzzy number and Trapezoidal fuzzy numbers are most popular numbers used in the field of decision making [2],[4],[5]. T. Pathinathan and K. Ponnivalavan introduced Pentagonal Fuzzy Numbers [11] and Diamond Fuzzy numbers [10] in 2014. Also, T. Pathinathan and K. Ponnivalavan studied the Reverse Order Triangular, Trapezoidal and Pentagonal Fuzzy Numbers in 2014[12].

We define Perfect Pentagonal Fuzzy Number based on Pentagonal Fuzzy Number introduced by T. Pathinathan and K. Ponnivalavan [11]. We define Quadrilateral Fuzzy Number (QFN) as Left skewed Quadrilateral Fuzzy Number and Right skewed Quadrilateral Fuzzy Number followed by their Properties and arithmetic operations with numerical examples. The paper is organized as follows: Section two comprises the required preliminaries. In section three we define Perfect Pentagonal Fuzzy Number (PPFN). Also we define Quadrilateral Fuzzy Number (QFN) and explain with arithmetical illustrations. Section four describes conclusion.

2. Preliminaries

2.1. Definition (Fuzzy Set) [6]

A fuzzy set is characterized by a membership function mapping the element of a domain or space or universe of discourse X to the unit interval, $[0,1]$. A fuzzy set $\underset{\sim}{A}$ in a universe discourse X is defined as the following set pairs: $\underset{\sim}{A} = \{x, \mu_{\underset{\sim}{A}}; x \in X\}$. $\mu_{\underset{\sim}{A}} : X \to [0,1]$ is a mapping called the degree of membership function of the fuzzy set $\mu_{\underset{\sim}{A}}(x)$ is called the membership value of $x \in X$ in the fuzzy set $\underset{\sim}{A}$. These membership grades are often represented by real numbers ranging from $[0,1]$.

2.2. Definition (Fuzzy Number) [2],[3]

A fuzzy set set $\underset{\sim}{A}$, defined on the set of real numbers R is said to be fuzzy number, if it satisfies the following characteristics.

a) $\underset{\sim}{A}$ is normal
b) $\underset{\sim}{A}$ is convex set and
c) The support of $\underset{\sim}{A}$ is closed and bounded.

2.3. Definition (Triangular Fuzzy Number) [5]

A fuzzy set $\underset{\sim}{A} = (a,b,c)$, defined on the set of real numbers R is said to be Triangular Fuzzy Numbers, when its membership function is given as below.

$$\mu_{\underset{\sim}{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)} & \text{for } a \le x \le b \\ 1 & \text{for } b = 1 \\ \frac{(c-x)}{(c-b)} & \text{for } b \le x \le c \\ 0 & \text{otherwise} \end{cases}$$

where $a \le b \le c$.

2.4. Definition (Trapezoidal Fuzzy Number) [4]

Trapezoidal Fuzzy Number is defined as $\underset{\sim}{A} = (a,b,c,d)$, where all a,b,c and d are real numbers and its membership function is given below.

$$\mu_{\underset{\sim}{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)} & \text{for } a \le x \le b \\ 1 & \text{for } b \le x \le c \\ \frac{(d-x)}{(d-c)} & \text{for } c \le x \le d \\ 0 & \text{otherwise} \end{cases}$$

where $a \le b \le c \le d$.

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2.5. Definition (Pentagonal Fuzzy Number) [11]

A Pentagonal Fuzzy Number (PNF) of a fuzzy set A is defined as $A_P = (a,b,c,d,e)$ and its membership function is given below,

$$\mu_{A_P}(x) = \begin{cases} \dfrac{(x-a)}{(b-a)} & \text{for } a \le x \le b \\ \dfrac{(x-b)}{(c-b)} & \text{for } b \le x \le c \\ 1 & x = c \\ \dfrac{(d-x)}{(d-c)} & \text{for } c \le x \le d \\ \dfrac{(e-x)}{(e-d)} & \text{for } d \le x \le e \\ 0 & \text{otherwise} \end{cases}$$

and A_P should satisfy the following conditions:

1) $\mu_{A_P}(x)$ is a continuous functions in the interval $[0,1]$.
2) $\mu_{A_P}(x)$ is strictly increasing and continuous function on $[a,b]$and $[c,d]$
3) $\mu_{A_P}(x)$ is strictly decreasing and continuous on $[c,d]$ and $[d,e]$.

3. Quadrilateral Fuzzy Number (QFN)

A Quadrilateral Fuzzy Number is either Left skewed Quadrilateral Fuzzy Number (LsQFN) or Right skewed Quadrilateral Fuzzy Number (RsQFN).

3.1. Perfect Pentagonal Fuzzy Number

3.1.1. Definition

A Pentagonal Fuzzy number is defined as Perfect Pentagonal Fuzzy Number if the following conditions are satisfied along with the existing conditions. We find the slope of line segments corresponding to the values between [a,b], [b,c], [c,d] and [d,e]; say m_1, m_2, m_3 and m_4. i.e., Slope of the line segments are $\frac{y_2 - y_1}{x_2 - x_1} = m_1$, $\frac{y_3 - y_2}{x_3 - x_2} = m_2$, $\frac{y_4 - y_3}{x_4 - x_3} = m_3$ and $\frac{y_5 - y_4}{x_5 - x_4} = m_4$.

1) If $m_1 = m_2$ and $m_3 = m_4$, then this fuzzy number becomes a Triangular Fuzzy Number.
2) If $m_1 \neq m_2$ and $m_3 \neq m_4$, then this fuzzy number is called a Perfect Pentagonal Fuzzy Number (PPFN).

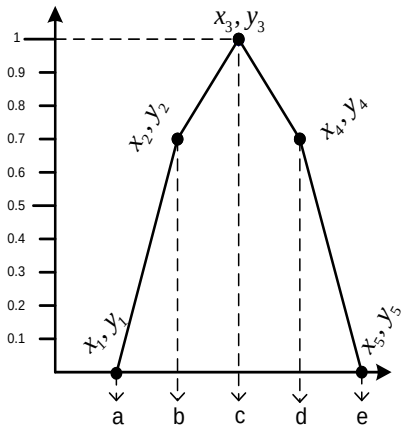


Fig. 1: Perfect Pentagonal Fuzzy Number.

3.2. Right skewed Quadrilateral Fuzzy Number

In Perfect Pentagonal Fuzzy Number, if $m_1 = m_2$ and $m_3 \neq m_4$, then it becomes Right skewed Quadrilateral Fuzzy Number (RsQFN). It defines as follow:

3.2.1. Definition

Right skewed Quadrilateral Fuzzy Number $A_{QR} = (a,b,c,d)$ on the set of real numbers R is defined as follows:

$$\mu_{A_{QR}}(x) = \begin{cases} \dfrac{(x-a)}{(b-a)} & \text{for } a \le x \le b \\ 1 & \text{for } b = 1 \\ \dfrac{(b-x)}{(c-b)} & \text{for } b \le x \le c \\ \dfrac{(c-x)}{(d-c)} & \text{for } c \le x \le d \\ 0 & \text{otherwise} \end{cases}$$

and A_{QR} should satisfy the following conditions:

1) $\mu_{A_{QR}}(x)$ is a continuous functions in the interval $[0,1]$.
2) $\mu_{A_{QR}}(x)$ is strictly increasing and continuous function on $[a,b]$.
3) $\mu_{A_{QR}}(x)$ is strictly decreasing and continuous function on $[b,c]$ and $[c,d]$.

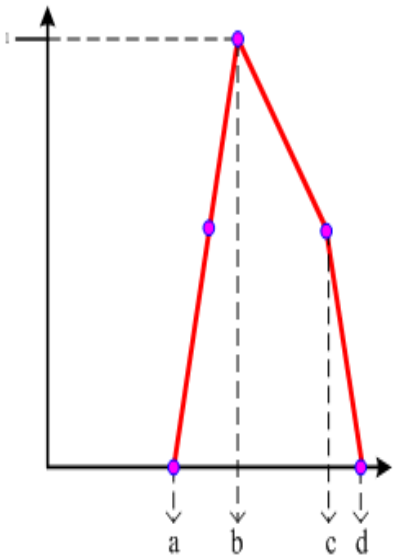


Fig. 2: Right skewed Quadrilateral Fuzzy Number.

3.3. Left skewed Quadrilateral Fuzzy Number

In Perfect Pentagonal Fuzzy Number, $m_1 \neq m_2$ and $m_3 = m_4$, then it becomes Left skewed Quadrilateral Fuzzy Number (LsQFN). It defines as follow:

3.3.1. Definition

Left skewed Quadrilateral Fuzzy Number $A_{QL} = (a,b,c,d)$ on the set of real numbers R is defined as follows:

$$m_{\tilde{A}}(x) = \begin{cases} \dfrac{(x-a)}{(b-a)} & \text{for } a \le x \le b \\ \dfrac{(x-b)}{(c-b)} & \text{for } b \le x \le c \\ 1 & \text{for } c = 1 \\ \dfrac{(c-x)}{(d-c)} & \text{for } c \le x \le d \\ 0 & \text{otherwise} \end{cases}$$

and $\tilde{A}$ should satisfy the following conditions:

1) $m_{\tilde{A}}(x)$ is a continuous functions in the interval $[0,1]$.
2) $m_{\tilde{A}}(x)$ is strictly increasing and continuous function on $[a,b]$ and $[b,c]$.
3) $m_{\tilde{A}}(x)$ is strictly decreasing and continuous function on $[c,d]$.

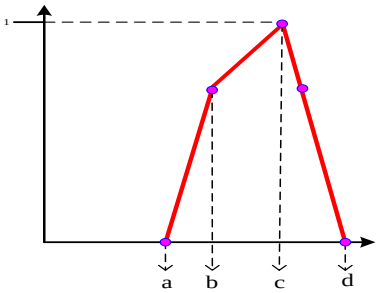


Fig. 3: Left skewed Quadrilateral Fuzzy Number.

3.4. Properties on Left skewed Quadrilateral Fuzzy Number (LsQFN)Left skewed Quadrilateral Fuzzy Number

3.4.1. Union

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two Left skewed Quadrilateral Fuzzy Numbers, then the union of the LsQFNs is, $m_{\tilde{A} \cup \tilde{B}} = \{(a_1 \cup b_1), (a_2 \cup b_2), (a_3 \cup b_3), (a_4 \cup b_4)\}$. The union for n number of LsQFNs is $\bigcup_{k=1}^{n} [m_{\tilde{A}}(x_i)]$ $i = 1,2,3,4$.

3.4.2. Intersection

Similarly the intersection for n LsQFNs is $\bigcap_{k=1}^{n} [m_{\tilde{A}}(x_i)]$ $i = 1,2,3,4$.

3.4.2. Complement

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ be a LsQFN, then the complement of $\tilde{A}$ is, $\bar{\tilde{A}} = 1 - \tilde{A}$. i.e. $\bar{\tilde{A}} = (1-a_1, 1-a_2, 1-a_3, 1-a_4)$.

Note: Properties on Right skewed Quadrilateral Fuzzy Number (RsQFN) are same as LsQFN.

3.5. Arithmetic operations on Left skewed Quadrilateral Fuzzy Number (LsQFN)

3.5.1. Addition

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two Left skewed Quadrilateral Fuzzy Numbers (LsQFN), then the addition of two LsQFN is, $m_{\tilde{A}+\tilde{B}} = ((a_1 + b_1), (a_2 + b_2), (a_3 + b_3), (a_4 + b_4))$.

3.5.2. Subtraction

$$m_{\tilde{A}-\tilde{B}} = ((a_1 - b_4), (a_2 - b_3), (a_3 - b_2), (a_4 - b_1))$$

3.5.3. Scalar Multiplication

$$l \times \tilde{A} = l \times (a_1, a_2, a_3, a_4) = (l \times a_1, l \times a_2, l \times a_3, l \times a_4)$$

3.5.4. Multiplication

$$m_{\tilde{A} \cdot \tilde{B}} = \left[\frac{s a_1}{4}, \frac{s a_2}{4}, \frac{s a_3}{4}, \frac{s a_4}{4}\right], \text{ where } s = b_1 + b_2 + b_3 + b_4.$$

3.5.5. Division

$$\frac{m_{\tilde{A}}}{m_{\tilde{B}}} = \left[\frac{4a_1}{s}, \frac{4a_2}{s}, \frac{4a_3}{s}, \frac{4a_4}{s}\right]$$

Note: Arithmetic operations on Right skewed Quadrilateral Fuzzy Number (RsQFN) are same as LsQFN.

3.6. Numerical example of Arithmetic operations on LsQFN

Let $m_{\tilde{A}} = (0.3, 0.4, 0.7, 0.8)$ and $m_{\tilde{B}} = (0.5, 0.7, 0.8, 0.9)$ are two Left skewed Quadrilateral Fuzzy Numbers (LsQFNs).

3.6.1. Addition

$$m_{\tilde{A}+\tilde{B}} = ((0.3+0.5), (0.4+0.7), (0.7+0.8), (0.8+0.9))$$
$$= (0.8,\ 1.1,\ 1.5,\ 1.7)$$

3.6.2. Subtraction

$$m_{\tilde{A}-\tilde{B}} = ((0.3-0.9), (0.4-0.8), (0.7-0.7), (0.8-0.5))$$
$$= (-0.6,\ -0.4,\ 0,\ 0.3)$$

3.6.3. Scalar Multiplication

$$2 \times \tilde{A} = 2 \times (0.3,\ 0.4,\ 0.7,\ 0.8)$$
$$= (2' 0.3,\ 2' 0.4,\ 2' 0.7,\ 2' 0.8)$$
$$= (0.6,\ 0.8,\ 1.4,\ 1.6)$$

3.6.4. Multiplication

$$s = 0.5 + 0.7 + 0.8 + 0.9 = 2.9;$$
$$m_{\tilde{A} \cdot \tilde{B}} = \left[\frac{2.9' 0.3}{4}, \frac{2.9' 0.4}{4}, \frac{2.9' 0.7}{4}, \frac{2.9' 0.8}{4}\right]$$
$$= (0.2175,\ 0.29,\ 0.5075,\ 0.58)$$

3.6.5. Division

$$\frac{m_{A_{PP}}}{m_{B_{PP}}} = \left(\frac{4' 0.3}{2.9}, \frac{4' 0.4}{2.9}, \frac{4' 0.7}{2.9}, \frac{4' 0.8}{2.9}\right)$$

$$= (0.4137,\ 0.5517,\ 0.9655,\ 1.1034)$$

4. Conclusion

We defined Perfect Pentagonal Fuzzy Number (PPFN). We have introduced and defined Left and Right Skewed Quadrilateral Fuzzy Number (QNF). We propose its algebraic Properties with numerical examples. Quadrilateral Fuzzy Number can be applied to any problem which has four points in representation with one value as normality. This number might be representing various research problems in future.

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