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Research paper

K^{th} Root Transformation for a subclass of Log-Sigmoid Analytic Function based on Quasi-Subordination

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Abstract

In the present investigation, using the concept of quasi-subordination, two subclasses of analytic functions have been introduced. The coefficient inequalities, the Fekete-Szego inequality, upper bounds for k^{th} root transformation were studied. This study is extended to function f^{-1} and for $\frac{z}{f(z)}$.

Keywords: *Analytic function, Starlike function, Convex function, Quasi-subordination, Log-Sigmoid function, k^{th} root transformation*

1. Introduction

The study of Sigmoid functions plays a vital role in geometric function theory. These functions are usual in function analysis related to univalent function theory and sigmoid function is differential every where which is represented by truncated power series and is usually written as

$$h(z)=\frac{1}{1+e^{-z}} \quad (1)$$

This function is monotonically increasing and one-to-one. Let the family of analytic functions in the open disc Δ by A which takes the form

$$f(z)=z+\sum_{n=2}^{\infty}a_nz^n \quad (2)$$

are normalized by $f(0)=0$ and $f'(0)=1$. Thus without loss of generality an univalent analytic function can be written in the form of equation (2) denoted by S . Let f and g be two analytic functions in Δ. The function f is said to be subordinate to g , if there exists a Schwarz function w such that $f(z)=g(w(z))$. It is denoted by $f \prec g$. Furthermore, if g is univalent in Δ then the following equivalence satisfies as $f(0)=g(0)$ and $f(\Delta)\subset g(\Delta)$.

The concept of quasi subordination was introduced by Robertson [17]. Let f and g be two analytic functions in Δ. The function f is said to be quasi-subordinate to g , if there exists φ and w are two analytic functions which satisfies as $|w(z)|<1$ such that $f(z)=\varphi(z)g(w(z))$. It is written as Δ . If $\varphi(z)=1$, then $f(z)=g(w(z))$ so that $f \prec g$ in Δ . If $\varphi(z)=z$, then $f(z)=zg(w(z))$ and it is said that f is majorized by g and written by $f \ll g$ in Δ. Both subordination and majorization are generalized by quasi subordination.

The problem of finding the maximum value of the coefficient functional $|a_3-\mu a_2^2|$, where μ is a real or complex parameter for the class of univalent analytic functions. This work is initiated by Fekete-Szego coefficient functional [6]. The k^{th} root transformation for a univalent analytic function f of the form in a equation (1.2) is denoted by $F(z)$ and is given by

$$F(z)=\left[f\left(z^k\right)\right]^{\frac{1}{k}}=z+\sum_{n=1}^{\infty}b_{nk+1}z^{nk+1}(\forall\, k\in N) \quad (3)$$

This transformation was studied by Ali et al.[1]. Several authors ([2], [10], [11], [21], [23], [24]), have studied the coefficient inequalities corresponding to the k^{th} root transformation for the function f in some subclasses of univalent and multivalent analytic functions. Olatunji et al. [15] introduced and studied the coefficient inequalities for the function f in the class $G_q^{\lambda}(\Phi,s,b)$, $G^{\lambda}(\Phi,s,b)$ and the class involving both quasi subordination and majorization. Murugusundaramoorthy and Janani [9] have studied the Fekete-Szego inequality for the univalent λ-pseudo starlike function in the space of sigmoid functions denoted by $L_{\beta}^{\lambda}(\Phi)$.

$$\mathrm{Re}\left(\frac{z(f'(z))^{\lambda}}{f(z)}\right)>\beta$$

Hamzat Jamiu Olusegun [7] have studied the coefficient inequalities for the function f in the Bazilevic subclass $B(\alpha,n,\gamma,\Phi)$. Several authors ([5], [7], [9], [13], [14], [22]), have introduced and studied the coefficient inequalities in the space of modified sigmoid and sigmoid functions.
Motivated by the above work, using the concept of quasi subordination two new subclasses of analytic functions denoted by $S_q^*(f,f')(\Phi)$ and $S_q^*(f,f',g)(\Phi)$ in the space of modified sigmoid function has been introduced. The coefficient inequalities, Fekete-Szegö inequality, upper bounds for k^{th} root transformation for the function f in these classes are obtained. This study have been extended for f^{-1} and for $\frac{z}{f(z)}$.

2. Preliminaries

Definition 1: The class of functions f analytic in the unit disc Δ, normalized by $f(0)=f'(0)-1=0$ and satisfies the condition

$$(1-\alpha)\left(\frac{zf'(z)}{f(z)}\right)^{\beta}+\alpha\left(\frac{(zf'(z))'}{f(z)}\right)^{1-\beta}\prec_q \Phi(z)$$

This class is denoted by $S_q^*(f,f')(\Phi)$.

Definition 2: The class of functions f analytic in the unit disc Δ, normalized by $f(0)=f'(0)-1=0$ and satisfies the condition

$$(1-\alpha)\left(\frac{z(f*g)'(z)}{(f*g)f(z)}\right)^{\beta}+\alpha\left(\frac{(z(f*g)'(z))'}{(f*g)(z)}\right)^{1-\beta}\prec_q \Phi(z)$$

This class is denoted by $S_q^*(f,f',g)(\Phi)$. For $g(z)=\frac{z}{1-z}$, we have $(f*g)=f$.

Now we recall the following Lemmas:

Lemma 3. [4] Let $p(z)=1+c_1z+c_2z^2+c_3z^3+........$ be an analytic function in Δ satisfying $p(0)=1$ and $\operatorname{Re}\{p(z)\}>0$, for all $z\in\Delta$, then $|c_n|\le 2\ \forall n\ge 1$. The class of all such functions with positive real part is denoted by P.

Lemma 4. [8] Let $w(z)=w_1z+w_2z^2+w_3z^3+.........$, then $|w_1|\le 1$, $\left|w_2-tw_1^2\right|\le 1+(|t|-1)\left|w_1^2\right|\le \max\ \{1,|t|\}$, where $t\in C$

Lemma 5. Let h be a sigmoid function and

$$\Phi(z)=2h(z)=1+\sum_{m=1}^{\infty}\frac{(-1)^m}{2^m}\left(\sum_{n=1}^{\infty}\frac{(-1)^n}{n!}z^n\right)^n$$ then $|\Phi_{n,m}(z)|<2$ and $\Phi(z)\in P,\ |z|<1$ where $\Phi(z)$ is a modified sigmoid function.

Lemma 6. If $\Phi(z)\in P,\ |z|<1$ and it is starlike then f is a normalized univalent function of the form (1.2). Taking $m=1$, Joseph et al.[5] remarked the following:

Remark : Let $\Phi(z)=1+\sum_{n=1}^{\infty}C_nz^n$ where $C_n=\frac{-1(-1)^n}{2n!}$ then $|C_n|\le 1$, n=1,2,3,….. this result is sharp for each n

3. Coefficient estimates for the transforms of the function $f\in S_q^*(f,f',g)(\Phi)$

Theorem 7. If $f\in S_q^*(f,f',g)(\Phi)$, then

$$|b_{k+1}|\le\frac{1}{2kg_2(\beta-3\alpha\beta+2\alpha)} \tag{4}$$

$$|b_{2k+1}|\le\frac{1}{2kg_3(\beta+3\alpha-4\alpha\beta)}\left\{\frac{1}{2}+\max\left[1,|t_1|\right]\right\} \tag{5}$$

Where

$$t_1=\frac{1}{2}+\frac{1}{4(\beta+2\alpha-3\alpha\beta)^2}[g_2^2(5\beta^2+3\beta+3\alpha\beta-\alpha\beta^2-4)\ \frac{g_3(k-1)(\beta+3\alpha-4\alpha\beta)}{kg_2^2}] \tag{6}$$

Proof. If $f\in S_q^*(f,f',g)(\Phi)$, then there exists a Schawrz function w with $w(0)=0$ and $|w(z)|\le 1$ such that

$$(1-\alpha)\left(\frac{z(f*g)'(z)}{(f*g)f(z)}\right)^{\beta}+\alpha\left(\frac{(z(f*g)'(z))'}{(f*g)(z)}\right)^{1-\beta}=\psi(z)\left[\Phi(w(z))-1\right] \tag{7}$$

where the function $\Phi(z)$ is a modified sigmoid function defined as

$$\Phi(z)=1+\frac{z}{2}-\frac{z^3}{24}+\frac{z^5}{240}-\frac{z^6}{64}+\frac{779}{20160}z^7-............ \tag{8}$$

The analytic function, $\psi(z)$ in Δ, of the form

$$\psi(z)=c_0+c_1z+c_2z^2+c_3z^3+............. \tag{9}$$

Define a function $P(z)$ such that

$$w(z)=\frac{P(z)-1}{P(z)+1}\ \text{for some}\ P\in \mathrm{P} \tag{10}$$

From (8) and (10), by applying $w(z)$ on the right hand side of the (7) after simplification, we get

$$\psi(z)\left[\Phi(w(z))-1\right]=\frac{p_1c_0}{4}z+\left(\frac{p_1c_1}{4}+\frac{p_2c_0}{4}-\frac{p_1^2c_0}{4}\right)z^2+\left(p_3c_0-p_1p_2c_0+\frac{5p_1^3c_0}{24}+\frac{p_1c_2}{4}+\frac{c_1}{4}\left(p_2-\left[p_2-\frac{p_1^2}{2}\right]\right)\right)z^3+......... \tag{11}$$

By Simple computation on the left hand side of (7), we have

$$(1-\alpha)\left(\frac{z(f*g)'(z)}{(f*g)f(z)}\right)^{\beta}+\alpha\left(\frac{(z(f*g)'(z))'}{(f*g)(z)}\right)^{1-\beta}=(1-\alpha)[1+\beta a_2g_2z+\left(2\beta a_3g_3+\frac{\beta(\beta-1)}{2}a_2^2g_2^2\right)z^2.......]+\alpha[1+2(1-\beta)a_2z+\left(2(1-\beta)\left(3a_3g_3-(\beta+2)a_2^2g_2^2\right)\right)z^2...........] \tag{12}$$

By applying k[th] root transformation and comparing the coefficients of z and z^2 from (7), (11) and (12), we get

$$b_{k+1}=\frac{p_1c_0}{4kg_2\left(\beta+2\alpha-3\alpha\beta\right)} \quad (13)$$

$$b_{2k+1}=\frac{1}{2kg_3\left(\beta+3\alpha-4\alpha\beta\right)}\left\{\frac{p_1c_1}{4}+\frac{c_0}{4}[p_2-t_1p_1^2]\right\} \quad (14)$$

where t_1 is given by (6). Upon simplification on the right hand side of equation (13) by applying Lemma 3 one can obtain the result as in (4). Similarly by taking modulus and applying Lemma 4 one can obtain the result as in (5). The sharpness of the result can be obtained by the rotations of the functions $w(z)=z$ and $w(z)=z^2$.

4. Fekete-Szegö coefficient function for $f\in S_q^*(f,f'g)(\Phi)$

Theorem 8. If $f\in S_q^*(f,f'g)(\Phi)$, then

$$\left|b_{2k+1}-\mu\, b_{k+1}^2\right|\le\frac{1}{2kg_3\left(\beta+3\alpha-4\alpha\beta\right)}\max\ \{1,\ |t_2|\} \quad (15)$$

where

$$t_2=\frac{1}{2}+\frac{1}{4\left(\beta+2\alpha-3\alpha\beta\right)^2}[g_2^2(5\beta^2+3\beta+3\alpha\beta-\alpha\beta^2-4)\ \frac{g_3\left[(k-1)+2\mu\right]\left(\beta+3\alpha-4\alpha\beta\right)}{kg_2^2}] \quad (16)$$

The result is sharp.
Proof. From relations (13) and (14), consider

$$\left[b_{2k+1}-\mu b_{k+1}^2\right]=\frac{1}{2kg_3\left(\beta+3\alpha-4\alpha\beta\right)}\left\{\frac{p_1y}{4}+\frac{c_0}{4}\left[p_2-t_2p_1^2\right]\right\} \quad (17)$$

Since $\psi(z)$ given by (9) is analytic and bounded in Δ, therefore on using [12], we have for some $y(|y|\le1)$: such that $|c_0|\le1$ and $c_1=\left(1-c_0^2\right)y$. On substituting the value of c_1 in (17), we get

$$\left[b_{2k+1}-\mu b_{k+1}^2\right]=\frac{1}{2kg_3\left(\beta+3\alpha-4\alpha\beta\right)}\left\{\frac{p_1y}{4}+\frac{c_0}{4}\left[p_2-t_2p_1^2\right]-\frac{p_1c_0^2y}{4}\right\} \quad (18)$$

If $c_0=0$ in (18), one can be seen that

$$\left[b_{2k+1}-\mu b_{k+1}^2\right]=\frac{p_1c_1}{8kg_3\left(\beta+3\alpha-4\alpha\beta\right)} \quad (19)$$

Taking modulus on both sides and by applying Lemma 4 on the right hand side of (19), one can obtain the result

$$\left|b_{2k+1}-\mu b_{k+1}^2\right|\le\frac{1}{4kg_3\left(\beta+3\alpha-4\alpha\beta\right)} \quad (20)$$

If $c_0\ne0$ in (18), then suppose that

$$F(c_0):=\left\{\frac{p_1y}{4}+\frac{c_0}{4}\left[p_2-t_2p_1^2\right]-\frac{p_1c_0^2y}{4}\right\} \quad (21)$$

Which is a polynomial in c_0 and hence analytic in $|c_0|\le1$, and maximum value is attained at $c_0=e^{i\theta}$,

$$\max_{0\le\theta\le2\pi}\left|F\left(e^{i\theta}\right)\right|=\left|F(1)\right| \quad (22)$$

Here $\left|b_{2k+1}-\mu b_{k+1}^2\right|\le\frac{1}{2kg_3\left(\beta+3\alpha-4\alpha\beta\right)}\left|p_2-t_2p_1^2\right|$ (23)

By applying the Lemma 3 on the right hand side of (23), one can obtain the result as in (15).

If $g(z)=\frac{z}{1-z}$, then $(f*g)=f(z)$, the above result can be reduced to the following corollary.

Corollary 9. If $f\in S_q^*(f,f')(\Phi)$, then

$$\left|b_{2k+1}-\mu\, b_{k+1}^2\right|\le\frac{1}{2k\left(\beta+3\alpha-4\alpha\beta\right)}\max\ \{1,\ |t_3|\} \quad (24)$$

Where

$$t_3=\frac{1}{2}+\frac{1}{4\left(\beta+2\alpha-3\alpha\beta\right)^2}[(5\beta^2+3\beta+3\alpha\beta-\alpha\beta^2-4)\ \frac{\left[(k-1)+2\mu\right]\left(\beta+3\alpha-4\alpha\beta\right)}{k}] \quad (25)$$

The result is sharp.

5. Coefficient inequality for the function $\frac{z}{f(z)}$

Theorem 10. If $f\in S_q^*(f,f')(\Phi)$ and $G(z)=\frac{z}{f(z)}$**,** then for any real number μ, we have

$$\left|q_2-\mu\, q_1^2\right|\le\frac{1}{2\left(\beta+3\alpha-4\alpha\beta\right)}\max\{1,|t_4|\} \quad (26)$$

Where

$$t_4=\frac{1}{2}+\frac{1}{4\left(\beta+2\alpha-3\alpha\beta\right)^2}\left\{5\beta^2+5\beta-5\alpha\beta+6\alpha-\alpha\beta^2-4-2\mu\right\} \quad (27)$$

The result is sharp.

Proof.
As $f\in S_q^*(f,f')(\Phi)$ and

$$G(z)=\frac{z}{f(z)}=1+\sum_{n=1}^{\infty}q_nz^n \quad (28)$$

By a simple computation we get

$$\frac{z}{f(z)}=1-a_2z+\left(a_2^2-a_3\right)z^2-.......... \quad (29)$$

Upon equating the coefficients of z and z^2, from relations (27) and (28), we have

$$q_1=-a_2 \quad (30)$$

$$q_2 = -a_3 + a_2^2 \tag{31}$$

From equations (13), (14), (29) and (30), it can be obtained

$$q_1 = -\frac{p_1 c_0}{4(\beta + 2\alpha - 3\alpha\beta)} \tag{31}$$

$$q_2 = -\frac{1}{2(\beta+3\alpha-4\alpha\beta)}\{\frac{p_1c_1}{4} + \frac{c_0}{4}(p_2 - p_1^2[\frac{1}{2} + \frac{c_0}{4(\beta+2\alpha-3\alpha\beta)^2}\left(5\beta^2+5\beta-5\alpha\beta+6\alpha-\alpha\beta^2-4\right)\} \tag{32}$$

For any complex number μ, consider

$$q_2 - \mu\, q_1^2 = -\frac{1}{2(\beta+3\alpha-4\alpha\beta)}\{\frac{p_1c_1}{4} + \frac{c_0}{4}(p_2 - t_4 p_1^2)\} \tag{33}$$

Where t_4 is given by (27). By applying similar computation as in Theorem 8, one can obtain the result as in (24).

6. Coefficient Inequalities for the function f^{-1}

Theorem 11. If $f^{-1}(w) = w + \sum_{n=2}^{\infty} d_n w^n$ is the inverse of $f \in S_q^*(f, f')(\Phi)$, then for any complex parameter μ, we have

$$\left|d_3 - \mu d_2^2\right| \le \frac{1}{2(\beta+3\alpha-4\alpha\beta)} \max\ \left\{1, \left|t_5\right|\right\} \tag{34}$$

Where

$$t_3 = \frac{1}{2} + \frac{c_0}{4(\beta+2\alpha-3\alpha\beta)^2}\left\{(5\beta^2+5\beta-5\alpha\beta+6\alpha-\alpha\beta^2-4-2\mu)\right\} \tag{35}$$

Proof.

As $f^{-1}(w) = w + \sum_{n=2}^{\infty} d_n w^n$ (36)

is the inverse function of f, it can be seen that

$$f^{-1}\left(f(z)\right) = f\left(f^{-1}(z)\right) = z \tag{37}$$

From equations (36) and (37), it can be reduced to

$$z + (a_2 + d_2)z^2 + (a_3 + 2a_2d_2 + d_3)z^3 + ... = z \tag{38}$$

By comparing the coefficients of z and z^2, from relation (38), it can be seen that

$$d_2 = -\frac{p_1c_0}{4(\beta+2\alpha-3\alpha\beta)} \tag{39}$$

$$d_3 = -\frac{1}{2(\beta+3\alpha-4\alpha\beta)}\{\frac{p_1c_1}{4} + \frac{c_0}{4}(p_2 - p_1^2[\frac{1}{2} + \frac{c_0}{4(\beta+2\alpha-3\alpha\beta)^2}\left(5\beta^2+7\beta-13\alpha\beta+12\alpha-\alpha\beta^2-4\right)\} \tag{40}$$

For any complex number μ, we have

$$d_3 - \mu\, d_2^2 = -\frac{1}{2(\beta+3\alpha-4\alpha\beta)}\{\frac{p_1c_1}{4} + \frac{c_0}{4}(p_2 - t_5 p_1^2)\} \tag{41}$$

Where t_5 is given by (36). By applying similar computation as in Theorem 8 one can obtain the result as in equation (34).

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