International Journal of Engineering & Technology, 7 (4.10) (2018) 940-941

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper



Mathematical Model for Traffic Flow

Keerthika, C[1]., Narahari Greeshma[2], Priya Vyshnavi[3], Keerthan Kumar Reddy[4], K. Indhira[5], V.M. Chandrasekaran[6]*

School of Advanced Science, Vellore Institute of Technology
Vellore 632 104, India
**Corresponding author E-mail: vmcsn@vit.ac.in*

Abstract

Every year countless hours are lost in traffic jams. When the density of traffic is sufficiently high small disturbances in vehicle's accelerations can cause phantom traffic jams. We can relate the traffic flow to mathematics and physics like that of liquids and gases. This paper presents mathematical model for phantom jams and Gauss Jordan elimination for traffic flow.

Keywords: *Gauss-Jordan elimination; Jamiton; Phantom jam; Traffic jam.*

1. Introduction

Countless hours are lost every year in traffic jams. The most frustrating are all those traffic jams without a clear cause, without accidents, without stationary vehicles, without closed lanes for construction. Such phantom jams can be formed when there are a large number of cars on the road. In this high traffic density, minor faults (a driver who presses too hard on the brake or who gets too close to another car) can quickly become a traffic jam that is self-sufficient and complete. If the vehicles are moving in a normal speed, then any change in vehicle acceleration will reflect immediately and a heavy traffic jam will happen. This type of jams follow a particular rule that we call as "jamitons", similar to the behaviour of the wave called soliton, in the same way as a jamiton. A solution that can even reduce traffic congestion by 1 percent will have a huge economic impact. Therefore, several researchers have tried to make more sophisticated models that contain jamitons and phantom traffic jams.

2. Concept

Mathematical theory of traffic flow was introduced by Frank Knite in 1920's and later several researchers have tried to create different mathematical models on traffic flow. Mathematicians from the University of Exeter have developed a model to show how major delays occur on our roads, with no apparent cause. Like liquids and gases that we study in mathematical physics, we can study the flow of traffic in real life. Seibold who is passionate about traffic says that "we often call them 'stop-and-go waves' of traffic, where you drive, and suddenly you have to brake because the person in front of you brakes, and then as a consequence you force the person behind you to brake, and this then triggers a wave that goes backwards on the road, a wave of braking vehicles". Flynn et al. observed the relation between traffic waves and detonation waves. Detonation wave is a particular type of wave produced by combusting gas. They have described the Phantom Jam using a mathematical model similar to that of fluid flow. "Phantom jams" can't be fixed but can be slowed down considerably.

Let us consider the following situation. In a heavy traffic road with full of vehicles, suppose a car brakes, then the person next to them will also do this, reducing the speed a little more. Very quickly we will not go anywhere fast.
Consider the following equation which represents how and when we have to ready for frustration (this equation is considered and used at MIT and University of Alberta)-

$$\rho_L = \frac{\rho M}{2}\left[1-\sqrt{1-\frac{4\beta}{u^2}}\right]$$

where

ρ_L denotes least traffic density at which jam will happen. If atleast this number of cars are on the road, then watch out.

ρM denotes highest traffic density in that road that we have taken.

β denotes the driver behavior and the road conditions. The steeper the road or the more uncertain the climate, the longer the deceleration will take. That gives a small number and permits us to minimize the number of vehicles required to create a jam pop up.
Denotes the negative sign, creates the equation to get the smallest density for which phantom jam will happen. Similarly, a greatest density will happen. Maximum can be obtained by replacing the minus with a plus.
If more number of vehicles are on the road, they are all instructed to go slowly to avoid any traffic jam.

u denotes the speed if we are not in heavy traffic.

3. Gauss-Jordan elimination in traffic flow

The flow of the traffic between the four intersections can be represented using a system of linear equations obtained by equating the number of vehicles entering and leaving at each intersection. This system can be solved by using Gauss-Jordan elimination technique. This predicts the number of vehicles passing through each road. Let A be an $(m\times n)$ matrix obtained from the system of linear equations. Let $A = [a_{ij}]_{m\times n}$ be a given matrix. Using Gauss-Jordan elimination method, we can reduce the given matrix to the row-reduced echelon form as shown below:

Copyright © 2018 Authors. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

In Step 1, we take the first column of the given matrix *A*.
If all the elements in the first column are zero, then we can go to the second column. Otherwise, choose a row of the matrix A, for example k^{th} row, so that in the first column it has a non-zero element. Next, we interchange the 1^{st} and k^{th} rows. If the element $a_{11} \neq 0$ and not one, then we divide the complete row by a_{11} so that the first element in that row becomes one. Using this row, make all the elements under a_{11} column as zero.
In Step 2, suppose all the elements in our new matrix under a_{11} are zero, we can consider the submatrix, say $B = [b_{ij}]_{(m-1)\times(n-1)}$. Similar to the first step, we make all the elements under b_{11} after dividing the row by b_{11}, as zero.
In Step 3, we repeat the Step 1 untill we get a matrix whose elements under leading main diagonal are zero. Let us denote the matrix by *C*. Note that the new matrix *C* has the following properties:

I. All the elements in the leading diagonal of C becomes ones. So the leading elements of *C* are 1's and the columns having these leading elements should be leading columns.
II. Also the elements below the leading diagonal elements are zero.

In Step 4 we use the leading term in the s^{th} row to make all entries in the s^{th} leading column equal to 0.
In Step 5, we use the leading term in the $(s\text{-}1)^{th}$ row to convert all entries in the $(s\text{-}1)^{th}$ leading column equal to 0 and continue untill we reach to the first leading element or column.
Finally, we get the row-reduced echelon form of the given matrix *A*. By solving the row reduced echelon form of matrix, number of vehicles passing through a street can be identified.

4. Conclusion

By using Gauss-Jordan elimination technique, we can identify the number of vehicles passing through a street. If the number of vehicles on the road is known, traffic can be controlled by using automatic speed limit system which displays the speed limit. Now the vehicles will flow with the speed limit and hence traffic can be controlled.

References

[1] I.K. Adu, D.K. Boah & V. Tulasi(2014), Application of system of linear equations to traffic flow for a network of four one-way streets in Kumasi, Ghana, *International Journal of Contemporary Mathematical Sciences*,9 (14), 653-660.
[2] M.R. Flynn, A.R. Kasimov, J.C. Nave, R.R. Rosales & B. Seibold (2009), Self-sustained nonlinear waves in traffic flow, *Physical Review E*, 79(5), 056113-1-056113-13.
[3] Gauss - Jordan elimination, available online: http://www.nptel.ac.in /courses/122104018/node20.html
[4] Mathematicians take aim at 'Phantom' traffic jams, available online: https://www.usnews.com/science/articles/2009/06/15/mathematicians-take-aim-at-phantom-traffic-jams
[5] Professor uses math to solve traffic jams, available online: https://temple-news.com/professor-uses-math-solve-traffic-jams/
[6] Traffic jam mystery solved by mathematicians, available online: https://phys.org/news/2007-12-traffic-mystery-mathematicians.html
[7] Sakthivel, A and Kavitha, T. N.(2016), A trial to solve the puzzles by modeling linear equations and using Gauss- method. *Int J Recent Sci Res*. 7(10), 13777-13781.