International Journal of Engineering & Technology, 7 (4.35) (2018) 880-884

SPC

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper



One Dimensional Shallow Water Equation Streamflow Modeling using WASH123D Model

H. Basri*[1,2], L.M. Sidek[1,2], D.S. Shih[3], H.C. Lloyd[4], W.H. Azad[2,5], A.Z. Abdul Razad[6]

[1]*Civil Engineering Department, College of Engineering, Universiti Tenaga Nasional, Malaysia*
[2]*Sustainable Technology and Environment Group, Institute of Energy Infrastructure, Universiti Tenaga Nasional, Malaysia*
[3]*Department of Civil Engineering, National Chung Hsing University, Taiwan*
[4]*School of Engineering, Faculty of Science Engineering & Built Environment, Deakin University, Australia*
[5]*Department of Irrigation & Drainage, Malaysia*
[6]*TNB Research Sdn Bhd, Malaysia*
**Corresponding author E-mail:BHidayah@uniten.edu.my*

Abstract

Reservoir inflow forecasting assists dam operator in reservoir operation by providing advance information on lake level. This paper discusses on the application of the physical-based numerical model to simulate one-dimensional channel network using WASH123D Model. The model was developed to simulate streamflow at two locations namely Sg Kejar and Sg Tiang, located in the Temengor catchment. The WASH123D model performed channel routing using shallow water equation. The model input data includes rainfall from 5 rainfall stations, river cross sections and simulated runoff data using SCS Method. Due to unavailable observed data, results comparisons were performed using streamflow results obtained using InfoWorks RS Platform. The peak flow from simulation results at Sg Kejar & Sg Tiang Station is 152.6m^3/s and 36.6m^3/s. The analysis shows good agreement for both simulations with Nash-Sutcliffe Efficiency of 0.68 for Sg Kejar and 0.99 for Sg Tiang. It is suggested that model recalibration shall be made once there is enough water level data to enable more accurate representation of spatial heterogeneity in the catchment processes.

Keywords: *hydrological model, inflow forecasting, physical-based model, shallow water equation, WASH123D*

1. Introduction

In the last few decades water especially river management and restoration play an important role in the environment and thus, numerous works have been done in this applied research field [1]–[3]. The hydrological model often used for various water management functions such as flood and reservoir inflow forecasting and other real-time operational applications [4]–[6]. Apart from that, the hydrological model also applicable for off-line application for design & planning. There are many available hydrological models and can be categorized into three main types which are, the transfer function (empirical black box), lumped conceptual and physically based models [7]. Characterization of rainfall-runoff models is made by a different level of complexity and data requirement [8]. The model selection criteria can be made based on the type of application, the catchment characteristics and the data availability, different spatial and temporal scales, and different model conceptualizations and parameterizations [9].

Nowadays, one of the concerns for hydrologists is natural inflows to reservoirs [10]. Reservoir inflow forecasting enables appropriate planning of reservoir operation. Furthermore, the advance information can assist dam operators in ensuring the dam is operated at the optimized condition while maintaining the safety of the dams. This information is also pertinent to reduce the flood risk at the downstream area by proper planning of water release from the reservoir during a high rainfall event. The reservoir inflow forecasting incorporates catchment runoff process modeling, channel routing model, storage routing algorithm, and reservoir operating rules.

WASH123D is a finite element numerical model designed to simulate variably saturated, variable-density water flow, reactive chemical transport, and sediment transport in watershed systems [11].This model was used in this study due to its capability to simulate flow in various component systems or combinations of component systems of a watershed [12]–[15]. The model has been successfully used to perform flood routing analysis due to a typhoon in Yilan River, Taiwan [16].Additionally, the WASH123D model was also used to model overland flow simulations Huwei Science and Technology Park, Taiwan [17]. Another application of WASH123D model was a simulation of the typhoon-induced flood in Lanyang River basin, Taiwan [18]. However, currently, there is no literature found on the application of WASH123D for simulation of reservoir inflow forecasting in Malaysia.

This study uses the WASH123D model to simulate inflow into Temengor Reservoir. The objective of this research project is to develop an inflow forecasting system for the Temengor reservoir.According to [19], the inflow forecasting system provides most useful outputs including river elevations, reservoir water level and time of occurrence for peak discharges with lead times that are sufficient to initiate appropriate responses by dam owner and authorities.The information obtained from inflow forecasting also essential to minimize the risk of dam failure [20]. Furthermore, advanced warning on dam failure enables more efficient

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implementation of dam safety emergency response plan. This paper discusses on the application of the physical-based numerical model to simulate one-dimensional channel network using WASH123D Model.

2. 1D River Flow Model using WASH123D

WASH123D Model consists of 1-D Stream-River Network, 2-D Overland Regime, and 3-D Subsurface Media. As described in [12], the 1-D Stream-River Network component applied for governing equations of water flow in a one-dimensional river derived based on the conservation law of water mass and linear momentum. The law of mass conservation results in the Continuity Equation as follows:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = S_S + S_R - S_E + S_I + S_1 + S_2 \quad (1)$$

where;

t is time [T];

s is the axis along the curvilinear river/stream/

canal direction [L];

A is cross-sectional area of the river/stream [L^2];

Q is the flow rate of the river/stream/canal [L^3/T];

S_S is the man-induced source [$L^3/T/L$];

S_R is the source due to rainfall [$L^3/T/L$];

S_E is the sink due to evapotranspiration [$L^3/T/L$];

S_I is the source due to exfiltration from the subsurface media [$L^3/T/L$];

S_1 and S_2 are the source terms contributed to overland flow [$L^3/T/L$]

Subsequently, the law of conservation of linear momentum results in the Momentum Equation:

$$\frac{\partial Q}{\partial t} + \frac{\partial VQ}{\partial x} = -gA\frac{\partial (Z_o + h)}{\partial x} + (M_S + M_R - M_E + M_I + M_1 + M_2) + \frac{B\tau^s - P\tau^b}{\rho} \quad (2)$$

Where;

h is water depth [L];

V is river/stream/canal velocity [L/T];

g is gravity [L/T^2];

Z_o is bottom elevation [L];

M_S is the external momentum–impulse from artificial sources/sinks [L^3/T^2];

M_R is the momentum–impulse gained from rainfall [L^3/T^2];

M_E is the momentum–impulse lost to evapotranspiration [L^3/T^2];

M_I is the momentum–impulse gained from the subsurface due to exfiltration [L^3/T^2];

M_1 and M_2 are the momenta–impulse gained from the overland flow [L^3/T^2];

ρ is the water density [M/L^3];

B is the top width of the cross-section [L];

τ^s is the surface shear stress [$M/T^2/L$];

is the wet perimeter [L];

and τ^b is the bottom shear stress [$M/T^2/L$], which can be assumed proportional to the flow rate as $\tau^b / \rho = \kappa V^2$ where $\kappa = gn^2/R^{1/3}$ and *R* is the hydraulic radius [L] and *n* is the Manning's roughness.

3. Study Area & Model Setup

3.1. Study Area

The study area is located in Temengor catchment with an area of 3506 km^2. The catchment elevation ranges from 206m to 2156m, and mostly forest area. Temengor reservoir covers about 150 km^2 of the catchment area. Fig. 1 shows an overview of Temengor catchment and main rivers contributing to the reservoir. This paper presents the results of river flow simulation from two rivers located at the northern part of the catchment, Sg Kejar and Sg Tiang. These two rivers were selected because of the availability of newly installed water level stations. However, at the time when the model was developed, the water level stations are still under installation process, and the stage-discharge rating curve is yet to be established. Nevertheless, the site is selected to enable future model calibration when the water level stations data is sufficient.

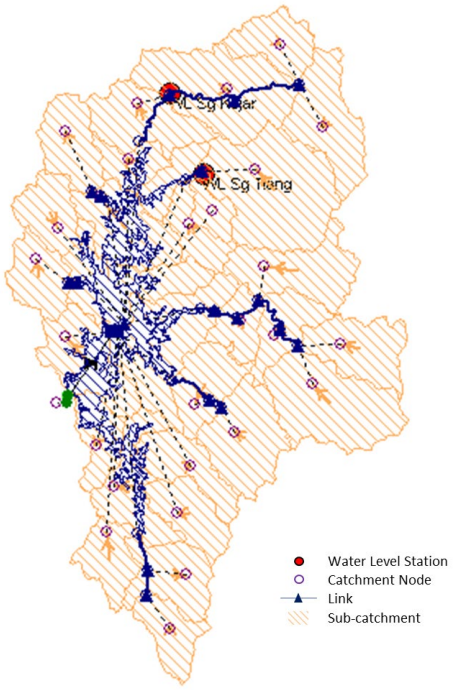


Fig.1: Overview of Temengor Catchment in InfoWorks RS Platform

3.2. Model Setup

The WASH123D Model setup requires catchment information such as rainfall data, river cross section & catchment runoff. Fig. 2 shows the overall methodology to conduct this study. In general, the river cross-section data were prepared in Groundwater Modeling System (GMS) platform. The second part is the preparation of catchment runoff as input data using a text file. Once all data preparation has been completed, the WASH123D simulation model was executed. Finally, once model simulation was completed, the simulation result consists of river depth and velocity were analyzed to obtain the river flow at selected locations.

3.2.2 River Cross Section

River cross-section data obtained from river survey was set up using GMS platform. There is 74 and 5 river cross-sections for Sg Kejar and Sg Tiang, respectively. Example of river cross-section setup for Sg Kejar in GMS platform is shown in Fig. 3. Once all cross-sections data has been keyed into the GMS system, the

model is mapped to WASH1D model to generate cross section file in WASH1D model, as shown in Fig. 4. Any correction to the river cross sections can be done in the generated .1dm file, using any text editor application.

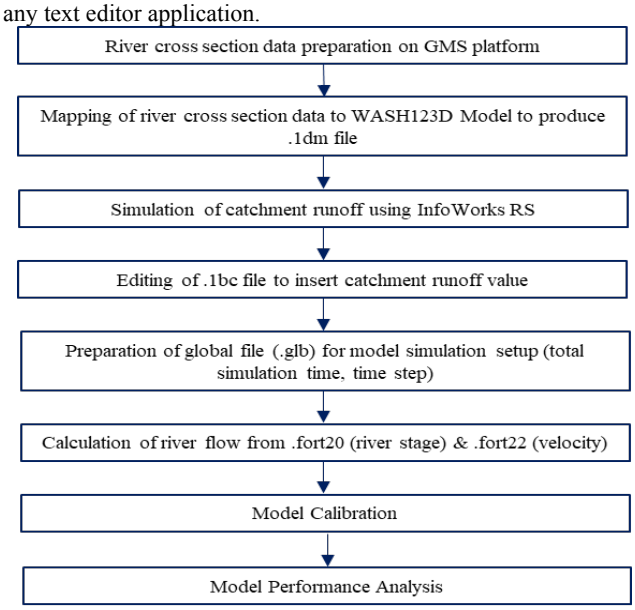


Fig.2: Overall WASH123D Model Simulation Methodology

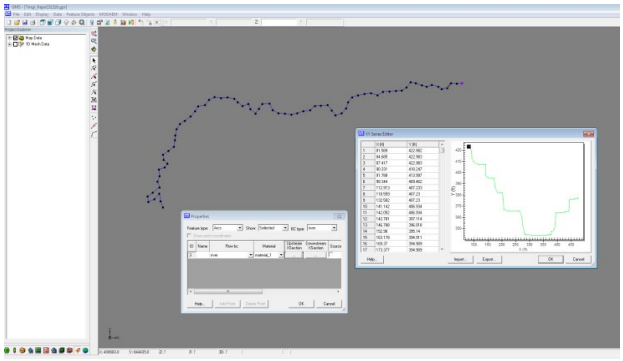


Fig.3: Cross section set up for Sg Kejar in GMS platform

```
WMS1DM
T1 SGKEJAR1
T2 HB
T3 08DEC2016
GE2 1 1 2 1
GE2 2 2 3 1
GE2 3 3 4 1
GE2 4 4 5 1
GE2 5 5 6 1
GE2 6 6 7 1
GE2 7 7 8 1
GE2 8 8 9 1
GN  1 401647.08 644588.22 346.982
GN  2 401272.86 644548.62 345.582
GN  3 400846.91 644584.76 344.082
GN  4 400542.09 644227.35 342.582
GN  5 400148.66 644129.81 341.082
```

Fig.4: Cross section generated from GMS in .1dm file

3.2.2 Rainfall Data

The input data used for model development are rainfall data from five rainfall stations located in the catchment. The rainfall data used for the model calibration is from 1 November 2009 to 1 January 2010, while model validation period starts from 1 July 2016 to 31 August 2016. This data was as input data to generate catchment runoff. The catchment runoff simulation was performed using the SCS Method in InfoWorks RS modeling platform.

3.2.3 Catchment Runoff

The catchment runoff was simulated using the SCS method for sub-catchments contributing to Sg Kejar and Sg Tiang respectively. Details of contributing sub-catchments are tabulated in Table 1.

Table 1: Sub-catchment Properties

River	Length (km)	Sub-catchment	Area (km^2)
Sg Kejar	36.8	SC1	86.76
		SC2	145.60
		SC3	229.14
		SC6	128.68
		Total	**590.19**
Sg Tiang	4.3	SC4	274.5
		Total	**274.5**

The catchment runoff result was used as an upstream boundary condition to WASH1D model to simulate flow routing in the river. Fig. 5 shows the example of input data from catchment runoff in .1bc file.

```
WMS1BC
T1
T2
T3
OP2 1 0 1 1 0
OP3 1.0
IP1 200 1.0e-6 1.0e-6 1.0e-6 1.0e-6 1.0e-6
PT1 1 1
ICH 0 0 0.05 'initial/base depth'
ICU 0 0.0
ICS 0
ICT 0.0
ICF 0
MP1 1 0.03 0.0 'mannings value setup'
OB1 1 2 0 10299 0 0
SS1 26 10300 'input from cathment'
SS1 50 10301 'input from cathment'
CS1 1 299 300 301
CS1 2 308 309 310
CS1 3 314 315 316
CS1 4 320 321 322
CS1 5 326 327 328
CS1 6 332 333 334
CS1 7 338 339 340
CS1 8 344 345 346
```

```
XYS 10249 1489 curve_10249 'input inflow'
0 0
3600  0.5
7200  0.5
10800 0.5
14400 0.5
18000 0.5
21600 0.5
25200 0.5
28800 0.5
32400 0.5
36000 0.5
39600 0.5
43200 0.5
46800 0.5
50400 0.5
54000 0.5
57600 0.5
61200 0.5
64800 0.5
68400 0.5
72000 0.5
75600 0.5
79200 0.5
82800 0.5
86400 0.5
```

Fig.5: Inflow Data in .1bc file

4. WASH1D Simulation Results

Once all input data has been prepared, the WASH1D simulation model was executed using the executable file. The completed WASH1D model simulation produced two files that consist of river depth and velocity at each cross section. Results in these two files will be used to compute the river flow using the following Equation (3). Sample output file from WASH1D simulation is shown in Fig.6.

$$Q = A \times V \quad (3)$$

Where;

Q is flow rate (m^3/s)

A is river cross-section area (m^2)

V is velocity (m/s)

```
DATASET
OBJTYPE MESH1D
BEGSCL
ND      75
NC      74
NAME NODAL_WD 'water stage'
TS   0  0.36000000E+04 'time step'
  0.34699319E+03 'Node 1'
  0.34559142E+03 'Node 2'
  0.34408866E+03 'Node 3'
  0.34258812E+03
  0.34109177E+03
  0.33959893E+03
  0.33811501E+03
  0.33662299E+03
```

```
DATASET
OBJTYPE MESH1D
BEGVEC
ND      75
NC      74
NAME NODAL_VEL 'velocity'
TS   0  0.36000000E+04 'time step'
  0.00000000E+00  0.00000000E+00 'Node 1'
  0.96833308E-01  0.85551560E-01 'Node 2'
  0.24867941E+00  0.15219462E+00 'Node 3'
  0.56875029E+00  0.33081940E+00
  0.55479313E+00  0.41353952E+00
  0.62520891E+00  0.76309300E+00
  0.30843658E+00  0.73269605E+00
  0.74355567E+00  0.20269052E+01
```

Fig.6: Sample Model Simulation Output File

4.1. Model Efficiency Analysis

Calculation of flow rate was made at selescted nodes, located at the water level stations in each river, shown in Fig. 6. As mention in 3.1, there is no river flow or water level data at the time the model was developed. However, to get the idea of a simulated river flow value, a reference was made with Infoworks RS simulation result. Infoworks RS simulation result was used as a reference to WASH1D simulated result as it also using the same

flow routing principal to perform flow routing in the river. In this study, the model interpretation and performance analysis were made to the referenced model, Infoworks RS. The first method is using a visual plot, by plotting the river flow results from both models.

Additionally, the following statistical and hydrology-specific indicators were also calculated. The analysis was made by adopting method applied for previous model simulation comparison between WASH123D model and other available commercial software as stated in [21].

(a) Mean Absolute Error (MAE) and peak clipping error

$$MAE = \frac{1}{n}|y-x| \quad (4)$$

MAE indicate mean differences between two models, and y is WASH123D simulated flow and x is Infoworks RS simulated flow, and n is a total number of data points. Peak clipping error calculates the differences at a flood peak.

$$E_p = \frac{x_{peak} - y_{peak}}{x_{peak}} \quad (5)$$

(b) Root Mean Square Error (RMSE)

$$RMSE = \sqrt{\frac{\Sigma(y-x)^2}{n}} \quad (6)$$

RMSE indicates a discrepancy between modelled and true values at an individual level.

(c) Pearson product-moment correlation coefficient (C^2)

$$C^2 = \left(\frac{\Sigma(y-y_m)(x-x_m)}{\sqrt{\Sigma(y-y_m)^2(x-x_m)^2}} \right)^2 \quad (7)$$

C^2 measures the degree of linear association between x and y represents variability in a single variable as correlated with another variable. x_m is the arithmetic mean of the references model, and y_m is the WASH123D model predicted means.

(d) Nash-Sutcliffe Efficiency (*NSE*)

$$NSE = 1 - \frac{\Sigma(y-x)^2}{\Sigma(x-x_m)^2} \quad (8)$$

NSE is an overall indication of the goodness-of-fit efficiency. Investigation using a visual plot in Fig. 7 to Fig. 10 shows simulation results predict the same trend of river flow at both locations. This result confirms the 1D simulation of both models, which employ a shallow water equation. Further analysis using the selected objective functions described earlier were made to confirm the visual analysis result. Table 2 tabulates the summary of model performance for both stations. Overall analysis shows that simulation for Sg Tiang provided better performance as compared to simulation for Sg Kejar. This is due to minor adjustment at the Sg Kejar cross section was made to ensure the simulation model is convergence.

Table 2: Summary of WASH123D Model Performance

Obj. Function	Sg Kejar		Sg Tiang	
	Calibration	Validation	Calibration	Validation
MAE	7.85	6.8	0.29	0.66
E_p	-0.42	0.07	-0.02	0.02
RMSE	14.4	9.43	0.50	0.92
C^2	0.99	0.99	0.99	0.99
NSE	0.68	0.98	0.99	0.99

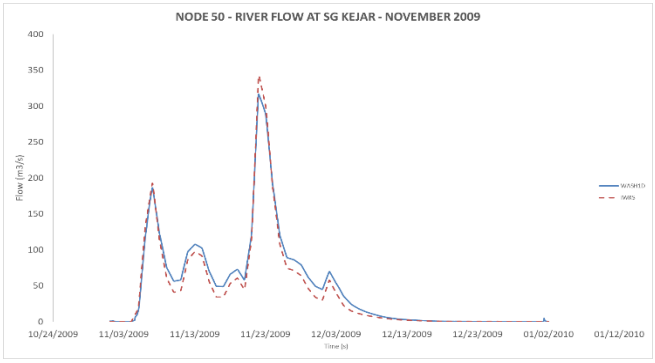


Fig.7: River Flow Simulation Result at Sg Kejar Station from 1 November 2009 to 1 January 2010

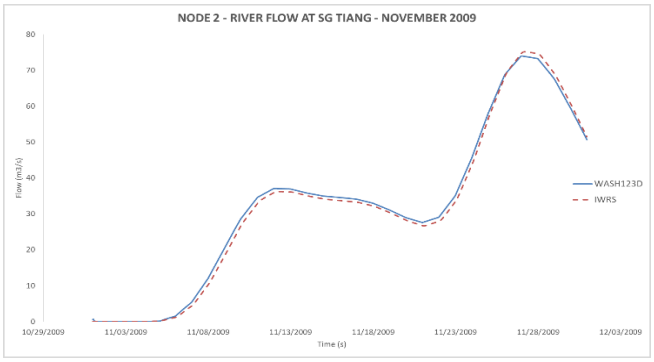


Fig.8: River Flow Simulation Result at Sg Tiang Station from 1 November 2009 to 1 January 2010

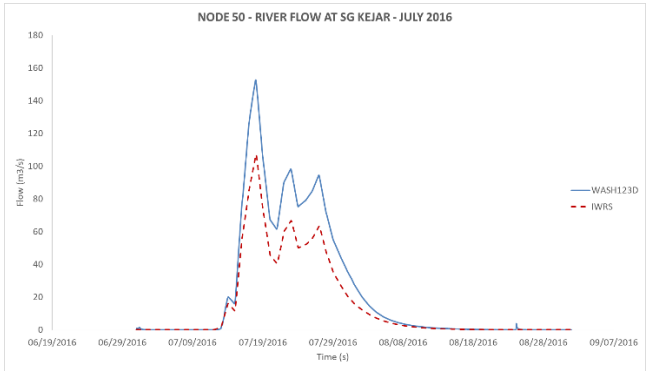


Fig.9: River Flow Simulation Result at Sg Kejar Station from 1 July 2016 to 31 August 2016

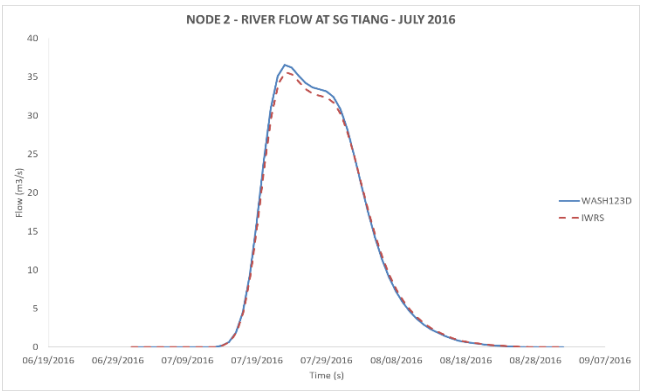


Fig.10: River Flow Simulation Result at Sg Tiang Station from 1 July 2016 to 31 August 2016

From the analysis, MAE for Sg Tiang is better as compared to Sg Kejar. According to [22], lower MAE values indicate better model prediction. During the calibration period, the peak error value is -0.42 and -0.02 for Sg Kejar and Sg Tiang, respectively. Nevertheless, the model performance improved during the validation period, which indicates a relatively good model performance [21].

On the other hand, RMSE for calibration result Sg Tiang Station is satisfactory, but analysis for Sg Kejar indicates opposite performance, as the simulated value using Infoworks RS is low throughout the time series. However, the model performance improved during the validation period for Sg Kejar. Overall, RMSE values obtained during both simulations period are satisfactory at all stations [23]. Both simulation results show a good correlation with the Pearson product-moment correlation coefficient with a value of 0.99. The NSE value of 0.68 at Sg Kejar station during the calibration period indicate good model performance, while very good model performance criteria were obtained during the validation period. Simulation result for both calibration & validation period of Sg Tiang with NSE value of 0.99 also indicates good model performance [23].

In summary, results obtained in this study indicates the capability of WASH123D model to reflect rainfall runoff phenomena as discussed in a previous study by [17] and [24]. Furthermore, this study also confirms the application of WASH123D model for flood forecasting as indicated in [25].

5. Conclusion & Recommendation

In this paper, simulation of the 1D model using shallow water equation was employed to obtain river flow at Sg Kejar and Sg Tiang water level station. Previously, there is no stream flow or water level station data at the selected location. Therefore, the simulation results comparison from WASH123D model were made with InfoWorks RS model. Comparisons of visual and statistical analysis for both models have shown a good agreement, which would be helpful to indicate the river flow at both locations.

The implications of no calibration at both water level stations is that the model only responds to the average processes occurring in the catchment and spatial heterogeneity in the catchment processes not be properly represented. This results in the poor reproduction of flows at internal flow stations. Therefore, it is suggested to recalibrate the model as a sufficient amount of data is available.

Acknowledgement

The research is supported by Tenaga Nasional Berhad, TNB Research Sdn Bhd, Department of Irrigation and Drainage Malaysia and Malaysian Ministry of Higher Education in the form of a Fundamental Research Grant Scheme 20170106FRGS.

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