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***Research paper***

# On R#-Closed and R#-Open Maps in Topological Spaces

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## Abstract

Defining and investigating properties of $R^{\#}$-closed maps, $R^{\#}$-open maps, $R^{\#}$*-closed maps and $R^{\#}$*-open maps in topological spaces.
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## 1. Introduction

S.R, Malghan [4] defined g closed map. N.Nagaveni [5] defined rwg closed maps in topological space. Recently, R S walli et all [6] defined $\alpha$rw-closed maps and $\alpha$rw-open maps in topological space. This paper aims to explain and inspect $R^{\#}$-closed maps, $R^{\#}$*-closed maps, $R^{\#}$-open maps and $R^{\#}$*-open maps..

## 2. Preliminaries

In the paper M or (M,$\tau$) and N or (N,$\sigma$) denote topological spaces and separation axioms are not assumed. For A⊆M, cl(A), int(A), M-A, represent closure of A, interior of A and complement of A in M respectively,CD denotes the closed, h denotes the function or map of the function.

**Definition 2.1:** B⊆ M is said to be $R^{\#}$-C(M) [2] if gcl(B) ⊆ W whenever B ⊆ W and W is R*O(M).

**Definition 2.2:** h is said to be $R^{\#}$-continuous [5] if $h^{-1}$(V) is $R^{\#}$-C(M) for every CD ⊆V of Y.

**Definition 2.3:** h is said to be $R^{\#}$-Irresolute [1] if $h^{-1}$(V) is $R^{\#}$C(M) for every $R^{\#}$C subset V of Y.

**Results 2.4: [1]**

i. Every CD (respectively r-CD, g-CD, w-CD, $\hat{g}$-CD) set is $R^{\#}$C(M).
ii. Every $R^{\#}$-C(M) is rg(respectively gpr-CD, rwg-CD, gspr-CD, r^g-CD, rg$\beta$-CD) set in M.

**Results 2.5: [1]** Assume B⊆ M then

i. If B is r-O(M) and rg-C(M), then B is $R^{\#}$C(M) in M.
ii. If B is g-O(M) and rg-C(M) then B is $R^{\#}$C(M) .
iii. If B is r-O(M) and rwg-C(M) then B is $R^{\#}$C(M).
iv. If B is r-O(M) and gpr-C(M) then B is $R^{\#}$C(M).
v. If B is r-O(M) and r^g-C(M) then B is $R^{\#}$C(M).
vi. If B is r-O(M) and $\beta wg^{**}$-C(M) then B is $R^{\#}$C(M).

**Definition 2.6:** B⊆X is said to be $T_R^{\#}$ space [1] if every $R^{\#}$-C(M) is CD

**Results 2.7: [3]**
1. If G and H ⊆ M then
   i. $R^{\#}$cl(G)=G
   ii. If H is any $R^{\#}$-O(X) set contained in G then H⊆$R^{\#}$int(G)
2. If N⊆ M is $R^{\#}$-O(M) then N is a $R^{\#}$-nbd of each of its points

## 3. $R^{\#}$- closed maps and $R^{\#}$-open maps

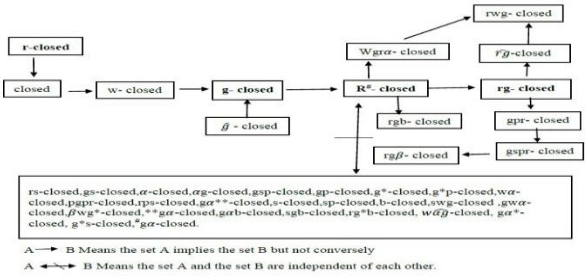
**Definition 3.1:** Assume g is known as $R^{\#}$-C(M) (resp-$R^{\#}$-O(M)), if the image of every CD (resp-open) in X is $R^{\#}$C(M) in N.

**Theorem 3.2:** If g is CD map then it is $R^{\#}$-CD map, converse is false.

Proof: Assume g be CD map, A∈C(M). Then g(A)∈C(N), hence f(M) is $R^{\#}$-C(N) set in Y, g is $R^{\#}$-C(M).

**Example 3.3:** Accept M=N={1, 2, 3}, $\tau$={∅, M, {1}, {2}, {1, 2}, {1, 3}}, $\sigma$={∅, N, {1}, {2}, {1, 2}}. Let g be an identical map then g is $R^{\#}$C(M) but not a CD map, G={2} in M is $g^{-1}${G}={2}∉C(Y).

**Theorem 3.4:** If g is g-CD map then it is $R^{\#}$-CD map, converse is false.



Proof: Based on every g -CD set is $R^{\#}C(X)$.

**Example 3.5:** Assume M=N={1, 2, 3, 4}, $\tau$={∅, M, {1}, {2}, {1, 2, 3}} and $\sigma$={∅, N, {1}, {2}, {1, 2}, {1, 2, 3}},g be map g(1)=4, g(2)=4, g(3)=4, g(4)=2 is $R^{\#}C$ but not a g-CD map because closed set {4} in M is {1, 2, 3}, which is not g-C(N).

**Theorem 3.6:** If g is w-CD map (resp. $\hat{g}$-CD map, r-CD map) then it is $R^{\#}$-CD map converse is false.

Proof: The proof is obvious every w-CD set is $R^{\#}$-CD.

**Example 3.7:** Assume M=N={1, 2, 3, 4}, $\tau$={∅, M, {1}, {2}, {1, 2}, {1, 2, 3}}, $\sigma$={∅, N, {1}, {1, 2}, {1, 2, 3}}, g be an identical map then it is $R^{\#}$-CD, not a g-CD map because CD {1, 2, 3} in M is {1, 2, 3}∉w-C(Y).

**Theorem 3.8:** If g be $R^{\#}$-CD map then it is rg-CD converse is false.

Proof: Accept g be $R^{\#}$-CD map and A∈C(M). Then g(A) is $R^{\#}$-C(N) and hence g(A) is rg-C(N).Hence g is rg-CD.

**Example 3.9:** In example 3.7 g(1)=4, g(2)=4, g(3)=4, g(4)=2 is rg-CD but not a $R^{\#}$-CD map, G={4}, $g^{-1}$(G)= {1, 2, 3} ∉ $R^{\#}$-C(N).

**Theorem 3.10:** Each $R^{\#}$-CD map is gspr-CD(resp. gpr,rgb, rg$\beta$**,**r^g,rwg, wgr$\alpha$-closed) convers is false.

Proof: The proof is obvious every $R^{\#}C(M)$ is gspr-C(M).

**Example 3.11:** In example 3.7, g(1)=4, g(2)=4, g(3)=3, g(4)=1 is gspr-CD but not a $R^{\#}$-CD map as $g^{-1}${4}is {1, 2} in Y∉ $R^{\#}$-C(N).

**Remark 3.12:** $R^{\#}$-CD map is independent with some existing CD maps in topological spaces as below.

**Example 3.13:** In example 3.11.Assume f be a map defined by g(1)=4, g(2)=4, g(3)=4, g(4)=2 is $R^{\#}$-CD map but not a rs, gs, $\alpha$g, gsp, gp,g*, g*p, w$\alpha$, pgpr, rps and g$\alpha$**-CD in N, as $g^{-1}(s) = \{1, 2, 3\}$ is not a rs, gs, $\alpha$g, gsp, gp, g*, g*p, w$\alpha$, pgpr, rps and g$\alpha$**- CD set in N.

**Example 3.14:** Assume M=N={1, 2, 3, 4}, $\tau$={$\phi$, M, {1, 2}, {3, 4}} and $\sigma$={∅, N, {1}, {1, 2}, {1, 2, 3}}, g be a map defined by g(1)=4, g(2)=4, g(3)=2, g(4)=4 is pre. Semi, sp, b, swg, gw$\alpha$, sgb,rg*b,w$\alpha\hat{g}$-,g$\alpha$*- g*s,#g$\alpha$, but not $R^{\#}$-CD in M as $g^{-1}(1,2) = \{3\}$ is pre, Semi pre, ,b, swg, gw$\alpha$, sgb, rg*b, w$\alpha\hat{g}$, g$\alpha$*, g*s and #g$\alpha$- CD set in M but not $R^{\#}$-C(N).

**Remark 3.15:** From the discussion and facts, the relation between $R^{\#}$-CD map and some existing CD maps in topological space is shown as follows.

**Theorem 3.16:** If g is contra r-CD and rg-CD map then g is $R^{\#}$-CD map.

Proof: Accept V ∈ C(X). Then g(A) is r-O and rg-C(N). By results 2.5, f(A) is $R^{\#}C(N)$. Therefore f is $R^{\#}$-CD map.

**Theorem 3.17:** If g is contra r-CD and rwg-CD map then g is $R^{\#}$-CD map.

Proof: Accept V ∈ C(M). Then g(A) is r-O(N) and rwg-C(N). By results 2.5, g(A) is $R^{\#}C(N)$. Therefore f is $R^{\#}$-CD map.

**Theorem 3.18:** If g is contra r-CD and gpr-CD map then g is $R^{\#}$-CD map.

Proof: Accept V ∈ C(M). Then g(A) is r-O(N) and gpr-C(N). By results 2.5, g(A) is $R^{\#}C(N)$. Therefore g is $R^{\#}$-CD map.

**Theorem 3.19**: If g is contra r-CD and r^g-CD map then g is $R^{\#}$-CD map.

Proof: Accept V ∈ C(M). Then f(A) is r-O(N) and r^g-C(N). By results 2.5,g(A) is $R^{\#}C(N)$. Therefore f is $R^{\#}$-CD map.

**Theorem 3.20:** If g is contra r-CD and $\beta$wg**-CD map then g is $R^{\#}$-CD map.

Proof: Accept V ∈ C(M). Then g(A) is r-O(N)and $\beta$wg**-C(N). By results 2.5, g(A) is $R^{\#}C(N)$. Therefore f is $R^{\#}$-CD map.

**Theorem 3.21:** If g is contra g-CD and rg-CD map then g is $R^{\#}$-CD map.

Proof: Accept V ∈ C(M). Then g(A) is g-O(N) and rg-C(N). By results 2.5, g(A) is $R^{\#}C(N)$. Therefore g is $R^{\#}$-CD map.

**Theorem 3.22:** If g is $R^{\#}$-CD map then $R^{\#}cl(g(K)) \subseteq g(cl(K))$ for every K ⊆ M.

Proof: Assume g be a $R^{\#}$-CD map and K⊆M. Then cl(H) is $R^{\#}$-C(M),f(cl(K)) is $R^{\#}$-C(N).(K) ⊆ g(cl(K)). By results 2.7, $R^{\#}cl(g(K)) \subseteq R^{\#}cl(g(cl(K)))$→ (1). Since g(cl(K)) is $R^{\#}C(N)$ then $R^{\#}cl(g(cl(K)))$=g(cl(K)) →(2) by results 2.7. From (1) and (2) we have $R^{\#}cl(g(K)) \subseteq g(cl(K))$ for every K ⊆M.

**Corollary 3.23:** If g is $R^{\#}$-CD map then g(K) is $\tau_{R^{\#}}$ CD in N for every K⊆C(M).

Proof: Assume K⊆C(M). By the fact, if g is $R^{\#}$-CD map then $R^{\#}cl(g(K)) \subseteq g(cl(K))$ → (1). Since K is CD, we have cl(K)=K and g(cl(K))=f(K) → (2). From (1) and (2), we have, $R^{\#}cl(g(K)) \subseteq g$ (K). We also know that, g(K)=$R^{\#}$cl(g(K)) and therefore $R^{\#}$cl(g(K)) =g(K). Therefore g(K) is $\tau_{R^{\#}}$-C(N).

**Theorem 3.24:** Accept M and N be any two topological spaces where "$R^{\#}$cl(K)=gcl(K) ∀ A ⊆N" and h be a map. Then these results are identical

(i) h is a $R^{\#}$-CD map
(ii) $R^{\#}cl(h(K))h(cl(K))$ ∀ A ⊆M.

Proof:
(i)→(ii)
The proof is as theorem 3.22.
(ii)→(i)
Let K be any CD set in M. Since K is CD we have cl(K)=K. Also h(K)=h(cl(K))⊇ $R^{\#}$cl(h(K)) by hypothesis. We know that h(K)⊆ $R^{\#}$cl(h(K)). Then h(K)= $R^{\#}$cl(h(K))=gcl(h(K)), by hypothesis. That is h(K) is g-CD in Y. Thus hg(K) is $R^{\#}$-CD in Y and hence f is $R^{\#}$-CD map.

**Remark 3.25:** The composition of two $R^{\#}$-CD maps need not be $R^{\#}$-CD.

**Example 3.26:** Assume M=N=Z={1, 2, 3, 4}.$\tau$={∅, M, {1}, {2}, {1, 2}, {1, 2, 3}},$\sigma$={∅,N, {1, 2}, {3, 4}} and $\eta$={ ∅, Z, {1}, {2}, {1, 2, 3}}. The identical map of f and g are $R^{\#}$-CD maps. g be another map defined by g(1)=4, g(2)=4, g(3)=4, g(4)=2. Then the composition gof:X→Z is not a $R^{\#}$-CD map as the image of the CD set {s} in M is {s} which is not a $R^{\#}$-CD set in (Z,$\eta$)That is (gof){3}=g{f(3)}=g(3)={1, 2, 3}.

**Theorem 3.27:** If g is CD map and h is $R^{\#}$-CD map then the composition goh is $R^{\#}$-CD map.

Proof: Accept F-C(M). Since g is CD then g(F)-C(N). As g is $R^{\#}$-CD map then g(f(F)) is $R^{\#}$-C(Z). therefore gof is $R^{\#}$-CD map.

**Remark 3.28:** If g is $R^{\#}$-CD map and h is CD map then the composition hog: M→Z is need not be a $R^{\#}$-CD map.

**Example 3.29:** Assume M=N=Z={1, 2, 3, 4}, $\tau$={∅, M, {1, 2}, {3, 4}}, $\sigma$={∅, N, {1}, {2}, {1, 2}, {1, 2, 3}} and $\eta$={ $\phi$, Z, {1}, {1, 2}, {1, 2, 3}}. Let a map f defined as f(1)=4, f(2)=2, f(3)=4, f(4)=2 and g as g(1)=1, g(2)=1, g(3)=4,g(4)=1. Then f and g are $R^{\#}$-CD maps and their composition gof is not a $R^{\#}$-CD map as the image of the CD set {3, 4}∈X is {1, 2)∉$R^{\#}$-C(Z) That is (gof){4}=g{f(4)}=g(1, 2, 3)={1, 2}.

**Theorem 3.30:** Assume g and h are $R^{\#}$-CD maps and N is $T_R^{\#}$ space then hog is $R^{\#}$-CD map. If N is $T_{1/2}$ space, then hog is $R^{\#}$-CD map.

Proof: Let H∈C(M). Since g is $R^{\#}$-CD then g(H) is $R^{\#}$-C(N). Since Y is $T_R^{\#}$space h(H) is CD. Since g is $R^{\#}$-C, h(g(H)) is $R^{\#}$-C(Z). Therefore gof is $R^{\#}$-CD map.

**Theorem 3.31:** If f is g-CD map, g is $R^{\#}$-CD map and N is $T_{1/2}$ space then gof is $R^{\#}$-closed map.

Proof: Let H∈C(M). As f is g-CD then f(H) is g-C(N). By hypothesis, f(A)∈C(M). Given g is $R^{\#}$-CD, then g(f(A)) is $R^{\#}$-C(Z). Therefore gof is $R^{\#}$-CD map.

**Definition 3.32:** h is known as $R^{\#}$-open map if g(A) is $R^{\#}$-O(Y) ∀ O(M).

By the known facts we have

**Theorem 3.33:**

i. Each open set (respectively r,g,w, $\hat{g}$-open) sets in M is $R^{\#}$-open map, converse is false.
ii. Each $R^{\#}$-O map is rg (respectively gpr,rwg, gspr,r^g,rg$\beta$-open) converse is not prove.

**Theorem 3.34:** For every bijection map g, the following results are identical

i) $g^{-1}$: Y→X is $R^{\#}$-continuous
ii) g is $R^{\#}$-O map
iii) g is $R^{\#}$-CD map

Proof:

(i)⇒(ii)
Let U∈O(X). As $g^{-1}$ is continuous then $(g^{-1})^{-1}$(U)=g(U)is $R^{\#}$-O(Y). Therefore f is $R^{\#}$-O.
(ii)⇒(iii)
Let F ∈C(X).Since g is $R^{\#}$-O then g($F^c$) is $R^{\#}$-O(Y). But g($F^c$)= $g(F)^c$. Therefore g(F) is $R^{\#}$-C(Y). Hence f is $R^{\#}$-CD.
(iii)⇒(i)
Let F∈C(X).Then g(F) is $R^{\#}$-C(Y). But g(F)=$(g^{-1})^{-1}$(F). Therefore $g^{-1}$ is continuous.

**Theorem 3.35:** Assume a map g is $R^{\#}$-O then g(int(H)) ⊆$R^{\#}$int(g(H)) ∀A⊆X

Proof: For a open map g,let H⊆M and int(H) be O(M). Then g(int(H)) is $R^{\#}$-O(N). We know that g(int(H))⊆ $g$(H). By results 2.7, g(int(H))⊆$R^{\#}$int(g(H))space. For the $R^{\#}$-closed set {q} with r∉ {q}, ∃ no two disjoint open sets U, V ∋ {q}⊆ U and r∈ V.

**Theorem 3.36:** If a map g is $R^{\#}$-O map then for each neighbourhood U of x in M, ∃ a $R^{\#}$-nbd W of f(x) in N such that W⊆f(U).

Proof: Given a $R^{\#}$-open map g. Let x$\epsilon$M and U be a nbd of x in M. By the definition of $R^{\#}$-nbd,∃ $R^{\#}$-O-G(M) such that x$\epsilon$G⊆U. Consider g(x)$\epsilon g$(G)⊆ $g$(U). Since g is $R^{\#}$-O then g(G) is $R^{\#}$-O(N). By results 2.7, F(G) is $R^{\#}$-nbd of each of its points. Take g(G)=W, W is $R^{\#}$-nbd of g(x) in N such that W=f(U).

## 4. $R^{\#}$*-closed maps and $R^{\#}$*-open maps

**Definition 4.1:** A map g is said to be a $R^{\#}$*-closed map if the image g(A) is $R^{\#}$-C(N) ∀$R^{\#}$-C(A) in M.

**Theorem 4.2:** Every $R^{\#}$*-closed map is a $R^{\#}$-CD converse is false.

Proof: The proof follows is obvious that every CD set is $R^{\#}$-CD.

**Example 4.3:** Assume M=N=Z={1, 2, 3, 4}, $\tau$={∅, M, {1, 2}, {3, 4}}, $\sigma$={∅,N, {1}, {2}, {1, 2}, {1, 2, 3}} and $\eta$={ $\phi$, Z, {1}, {1, 2}, {1, 2, 3}}.
Let f be a map, f(1)=4,f(2)=2,f(3)=2,f(4)=4 is $R^{\#}$-CD map but not a $R^{\#}$*-CD map as the image of the $R^{\#}$-C {1, 2} in M is {3} which is not a $R^{\#}$C(N).

**Theorem 4.4:** If the maps g and h are $R^{\#}$*-C maps then the composition hog: M→Z is $R^{\#}$*-C.

Proof: Accept F be any $R^{\#}$-C(M). Since g is $R^{\#}$*-C then g(F) is $R^{\#}$-C(N). Since h is $R^{\#}$*-C then h(g(F)) is $R^{\#}$-C(Z). Therefore hog is $R^{\#}$*-C map.

**Definition 4.5:** A map g is known as $R^{\#}$*-open map if g(H) is $R^{\#}$-O(N) ∀ $R^{\#}$-O-H(M).

**Theorem 4.6:** Each $R^{\#}$-O map is $R^{\#}$*-open map convers is false.

Proof: The proof is obvious.

**Example 4.7:** Assume M=N={1, 2, 3}, $\tau$={∅, M, {1}, {2}, {1, 2}} and $\sigma$={∅, N, {1}, {2}, {1, 2}, {1, 3}}. Let a map g defined by g(1)=3, g(2)=1, g(3)=2 is a $R^{\#}$-O map but not a $R^{\#}$*-O map as the image of the $R^{\#}$-O set {1, 2} in M is {2, 3} in N which is not a $R^{\#}$-O(N).

**Theorem 4.8:** Assume the maps g and h are $R^{\#}$*-O map then their composition hog is also $R^{\#}$*-open.

Proof: The proof is as theorem 4.6.

## 5. Conclusion

In this paper we defined and studied $R^{\#}$-closed maps, $R^{\#}$-open maps, $R^{\#}$*-closed maps and $R^{\#}$*-open maps.

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