International Journal of Engineering & Technology, 7 (4.10) (2018) 770-772

International Journal of Engineering & Technology

Website: www.sciencepubco.com/index.php/IJET

Research paper

On R#-Closed and R#-Open Maps in Topological Spaces

Raghavendra K[1]*, Basavaraj M. Ittanagi[2]

[1]*Department of Mathematics, ACS College of Engineering, Bangalore, Karnataka State, India*
[2]*Department of Mathematics, Siddaganga Institute of Technology, Tumakuru, Karnataka State, India*
**Corresponding author E-mail: raghumsc143@gmail.com*

Abstract

Defining and investigating properties of R#-closed maps, R#-open maps, R#*-closed maps and R#*-open maps in topological spaces. 2010 Mathematics Classification: 54A05, 54A10

Keywords: *R#-closed maps, R#-open maps, R#*-closed maps and R#*-open maps*

1. Introduction

S.R, Malghan [4] defined g closed map. N.Nagaveni [5] defined rwg closed maps in topological space. Recently, R S walli et all [6] defined αrw-closed maps and αrw-open maps in topological space. This paper aims to explain and inspect R#-closed maps, R#*-closed maps, R#-open maps and R#*-open maps..

2. Preliminaries

In the paper M or (M,τ) and N or (N,σ) denote topological spaces and separation axioms are not assumed. For A⊆M, cl(A), int(A), M-A, represent closure of A, interior of A and complement of A in M respectively,CD denotes the closed, h denotes the function or map of the function.

Definition 2.1: B⊆ M is said to be R#-C(M) [2] if gcl(B) ⊆ W whenever B ⊆ W and W is R*O(M).

Definition 2.2: h is said to be R#-continuous [5] if h^{-1}(V) is R#-C(M) for every CD ⊆V of Y.

Definition 2.3: h is said to be R#-Irresolute [1] if h^{-1}(V) is R#C(M) for every R#C subset V of Y.

Results 2.4: [1]

i. Every CD (respectively r-CD, g-CD, w-CD, $\hat{g}$-CD) set is R#C(M).
ii. Every R#-C(M) is rg(respectively gpr-CD, rwg-CD, gspr-CD, r^g-CD, rgβ-CD) set in M.

Results 2.5: [1] Assume B⊆ M then

i. If B is r-O(M) and rg-C(M), then B is R#C(M) in M.
ii. If B is g-O(M) and rg-C(M) then B is R#C(M) .
iii. If B is r-O(M) and rwg-C(M) then B is R#C(M).
iv. If B is r-O(M) and gpr-C(M) then B is R#C(M).
v. If B is r-O(M) and r^g-C(M) then B is R#C(M).
vi. If B is r-O(M) and βwg^{**}-C(M) then B is R#C(M).

Definition 2.6: B⊆X is said to be $T_{R^{\#}}$ space [1] if every R#-C(M) is CD

Results 2.7: [3]

1. If G and H ⊆ M then
   i. R#cl(G)=G
   ii. If H is any R#-O(X) set contained in G then H⊆R#int(G)
2. If N⊆ M is R#-O(M) then N is a R#-nbd of each of its points

3. R#- closed maps and R#-open maps

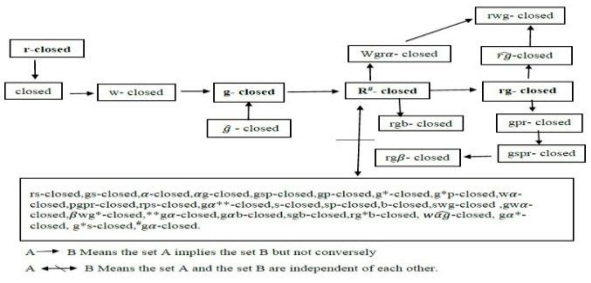
Definition 3.1: Assume g is known as R#-C(M) (resp-R#-O(M)), if the image of every CD (resp-open) in X is R#C(M) in N.

Theorem 3.2: If g is CD map then it is R#-CD map, converse is false.

Proof: Assume g be CD map, A∈C(M). Then g(A)∈C(N), hence f(M) is R#-C(N) set in Y, g is R#-C(M).

Example 3.3: Accept M=N={1, 2, 3}, τ={∅, M, {1}, {2}, {1, 2}, {1, 3}}, σ={∅, N, {1}, {2}, {1, 2}}. Let g be an identical map then g is R#C(M) but not a CD map, G={2} in M is g^{-1}{G}={2}∉C(Y).

Theorem 3.4: If g is g-CD map then it is R#-CD map, converse is false.



Copyright © 2018 Authors. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Proof: Based on every g -CD set is $R^{\#}C(X)$.

Example 3.5: Assume M=N={1, 2, 3, 4}, τ={∅, M, {1}, {2}, {1, 2, 3}} and σ={∅, N, {1}, {2}, {1, 2}, {1, 2, 3}},g be map g(1)=4, g(2)=4, g(3)=4, g(4)=2 is $R^{\#}C$ but not a g-CD map because closed set {4} in M is {1, 2, 3}, which is not g-C(N).

Theorem 3.6: If g is w-CD map (resp. $\hat{g}$-CD map, r-CD map) then it is $R^{\#}$-CD map converse is false.

Proof: The proof is obvious every w-CD set is $R^{\#}$-CD.

Example 3.7: Assume M=N={1, 2, 3, 4}, τ={∅, M, {1}, {2}, {1, 2}, {1, 2, 3}}, σ={∅, N, {1}, {1, 2}, {1, 2, 3}}, g be an identical map then it is $R^{\#}$-CD, not a g-CD map because CD {1, 2, 3} in M is {1, 2, 3}∉w-C(Y).

Theorem 3.8: If g be $R^{\#}$-CD map then it is rg-CD converse is false.

Proof: Accept g be $R^{\#}$-CD map and A∈C(M). Then g(A) is $R^{\#}$-C(N) and hence g(A) is rg-C(N).Hence g is rg-CD.

Example 3.9: In example 3.7 g(1)=4, g(2)=4, g(3)=4, g(4)=2 is rg-CD but not a $R^{\#}$-CD map, G={4}, $g^{-1}(G)$= {1, 2, 3} ∉ $R^{\#}$-C(N).

Theorem 3.10: Each $R^{\#}$-CD map is gspr-CD(resp. gpr,rgb, rgβ,r^g,rwg, wgrα-closed) convers is false.

Proof: The proof is obvious every $R^{\#}C(M)$ is gspr-C(M).

Example 3.11: In example 3.7, g(1)=4, g(2)=4, g(3)=3, g(4)=1 is gspr-CD but not a $R^{\#}$-CD map as g^{-1}{4}is {1, 2} in Y∉ $R^{\#}$-C(N).

Remark 3.12: $R^{\#}$-CD map is independent with some existing CD maps in topological spaces as below.

Example 3.13: In example 3.11.Assume f be a map defined by g(1)=4, g(2)=4, g(3)=4, g(4)=2 is $R^{\#}$-CD map but not a rs, gs, αg, gsp, gp,g*, g*p, wα, pgpr, rps and gα**-CD in N, as $g^{-1}(s) = \{1, 2, 3\}$ is not a rs, gs, αg, gsp, gp, g*, g*p, wα, pgpr, rps and gα**- CD set in N.

Example 3.14: Assume M=N={1, 2, 3, 4}, τ={ϕ, M, {1, 2}, {3, 4}} and σ={∅, N, {1}, {1, 2}, {1, 2, 3}}, g be a map defined by g(1)=4, g(2)=4, g(3)=2, g(4)=4 is pre. Semi, sp, b, swg, gwα, sgb,rg*b,w$\alpha\hat{g}$-,gα*- g*s,#gα, but not $R^{\#}$-CD in M as $g^{-1}(1, 2) = \{3\}$ is pre, Semi pre, ,b, swg, gwα, sgb, rg*b, w$\alpha\hat{g}$, gα*, g*s and #gα- CD set in M but not $R^{\#}$-C(N).

Remark 3.15: From the discussion and facts, the relation between $R^{\#}$-CD map and some existing CD maps in topological space is shown as follows.

Theorem 3.16: If g is contra r-CD and rg-CD map then g is $R^{\#}$-CD map.

Proof: Accept V ∈ C(X). Then g(A) is r-O and rg-C(N). By results 2.5, f(A) is $R^{\#}C(N)$. Therefore f is $R^{\#}$-CD map.

Theorem 3.17: If g is contra r-CD and rwg-CD map then g is $R^{\#}$-CD map.

Proof: Accept V ∈ C(M). Then g(A) is r-O(N) and rwg-C(N). By results 2.5, g(A) is $R^{\#}C(N)$. Therefore f is $R^{\#}$-CD map.

Theorem 3.18: If g is contra r-CD and gpr-CD map then g is $R^{\#}$-CD map.

Proof: Accept V ∈ C(M). Then g(A) is r-O(N) and gpr-C(N). By results 2.5, g(A) is $R^{\#}C(N)$. Therefore g is $R^{\#}$-CD map.

Theorem 3.19: If g is contra r-CD and r^g-CD map then g is $R^{\#}$-CD map.

Proof: Accept V ∈ C(M). Then f(A) is r-O(N) and r^g-C(N). By results 2.5,g(A) is $R^{\#}C(N)$. Therefore f is $R^{\#}$-CD map.

Theorem 3.20: If g is contra r-CD and βwg**-CD map then g is $R^{\#}$-CD map.

Proof: Accept V ∈ C(M). Then g(A) is r-O(N)and βwg**-C(N). By results 2.5, g(A) is $R^{\#}C(N)$. Therefore f is $R^{\#}$-CD map.

Theorem 3.21: If g is contra g-CD and rg-CD map then g is $R^{\#}$-CD map.

Proof: Accept V ∈ C(M). Then g(A) is g-O(N) and rg-C(N). By results 2.5, g(A) is $R^{\#}C(N)$. Therefore g is $R^{\#}$-CD map.

Theorem 3.22: If g is $R^{\#}$-CD map then $R^{\#}$cl(g(K))⊆ g(cl(K)) for every K ⊆ M.

Proof: Assume g be a $R^{\#}$-CD map and K⊆M. Then cl(H) is $R^{\#}$-C(M),f(cl(K)) is $R^{\#}$-C(N).(K) ⊆ g(cl(K)). By results 2.7, $R^{\#}$cl(g(K))⊆$R^{\#}$cl(g(cl(K)))→ (1). Since g(cl(K)) is $R^{\#}C(N)$ then $R^{\#}$cl(g(cl(K)))=g(cl(K)) →(2) by results 2.7. From (1) and (2) we have $R^{\#}$cl(g(K))⊆g(cl(K)) for every K ⊆M.

Corollary 3.23: If g is $R^{\#}$-CD map then g(K) is $\tau_{R^{\#}}$ CD in N for every K⊆C(M).

Proof: Assume K⊆C(M). By the fact, if g is $R^{\#}$-CD map then $R^{\#}$cl(g(K))⊆ g(cl(K)) → (1). Since K is CD, we have cl(K)=K and g(cl(K))=f(K) → (2). From (1) and (2), we have, $R^{\#}$cl(g(K))⊆ g (K). We also know that, g(K)=$R^{\#}$cl(g(K)) and therefore $R^{\#}$cl(g(K)) =g(K). Therefore g(K) is $\tau_{R^{\#}}$-C(N).

Theorem 3.24: Accept M and N be any two topological spaces where "$R^{\#}$cl(K)=gcl(K) ∀ A ⊆N" and h be a map. Then these results are identical

(i) h is a $R^{\#}$-CD map
(ii) $R^{\#}$cl(h(K))h(cl(K)) ∀ A ⊆M.

Proof:
(i)→(ii)
The proof is as theorem 3.22.
(ii)→(i)
Let K be any CD set in M. Since K is CD we have cl(K)=K. Also h(K)=h(cl(K))⊇ $R^{\#}$cl(h(K)) by hypothesis. We know that h(K)⊆ $R^{\#}$cl(h(K)). Then h(K)= $R^{\#}$cl(h(K))=gcl(h(K)), by hypothesis. That is h(K) is g-CD in Y. Thus hg(K) is $R^{\#}$-CD in Y and hence f is $R^{\#}$-CD map.

Remark 3.25: The composition of two $R^{\#}$-CD maps need not be $R^{\#}$-CD.

Example 3.26: Assume M=N=Z={1, 2, 3, 4}.τ={∅, M, {1}, {2}, {1, 2}, {1, 2, 3}},σ={∅,N, {1, 2}, {3, 4}} and η={ ∅, Z, {1}, {2}, {1, 2, 3}}. The identical map of f and g are $R^{\#}$-CD maps. g be another map defined by g(1)=4, g(2)=4, g(3)=4, g(4)=2. Then the composition gof:X→Z is not a $R^{\#}$-CD map as the image of the CD set {s} in M is {s} which is not a $R^{\#}$-CD set in (Z,η)That is (gof){3}=g{f(3)}=g(3)={1, 2, 3}.

Theorem 3.27: If g is CD map and h is $R^{\#}$-CD map then the composition goh is $R^{\#}$-CD map.

Proof: Accept F-C(M). Since g is CD then g(F)-C(N). As g is $R^{\#}$-CD map then g(f(F)) is $R^{\#}$-C(Z). therefore gof is $R^{\#}$-CD map.

Remark 3.28: If g is $R^{\#}$-CD map and h is CD map then the composition hog: M→Z is need not be a $R^{\#}$-CD map.

Example 3.29: Assume M=N=Z={1, 2, 3, 4}, τ={∅, M, {1, 2}, {3, 4}}, σ={∅, N, {1}, {2}, {1, 2}, {1, 2, 3}} and η={ ϕ, Z, {1}, {1, 2}, {1, 2, 3}}. Let a map f defined as f(1)=4, f(2)=2, f(3)=4, f(4)=2 and g as g(1)=1, g(2)=1, g(3)=4,g(4)=1. Then f and g are $R^{\#}$-CD maps and their composition gof is not a $R^{\#}$-CD map as the image of the CD set {3, 4}∈X is {1, 2)∉$R^{\#}$-C(Z) That is (gof){4}=g{f(4)}=g(1, 2, 3)={1, 2}.

Theorem 3.30: Assume g and h are $R^{\#}$-CD maps and N is $T_R{}^{\#}$ space then hog is $R^{\#}$-CD map. If N is $T_{1/2}$ space, then hog is $R^{\#}$-CD map.

Proof: Let H∈C(M). Since g is $R^{\#}$-CD then g(H) is $R^{\#}$-C(N). Since Y is $T_R{}^{\#}$space h(H) is CD. Since g is $R^{\#}$-C, h(g(H)) is $R^{\#}$-C(Z). Therefore gof is $R^{\#}$-CD map.

Theorem 3.31: If f is g-CD map, g is $R^{\#}$-CD map and N is $T_{1/2}$ space then gof is $R^{\#}$-closed map.

Proof: Let H ∈C(M). As f is g-CD then f(H) is g-C(N). By hypothesis, f(A)∈C(M). Given g is $R^{\#}$-CD, then g(f(A)) is $R^{\#}$-C(Z). Therefore gof is $R^{\#}$-CD map.

Definition 3.32: h is known as $R^{\#}$-open map if g(A) is $R^{\#}$-O(Y) ∀ O(M).

By the known facts we have

Theorem 3.33:

i. Each open set (respectively r,g,w, $\hat{g}$-open) sets in M is $R^{\#}$-open map, converse is false.
ii. Each $R^{\#}$-O map is rg (respectively gpr,rwg, gspr,r^g,rgβ-open) converse is not prove.

Theorem 3.34: For every bijection map g, the following results are identical

i) g^{-1}: Y→X is $R^{\#}$-continuous
ii) g is $R^{\#}$-O map
iii) g is $R^{\#}$-CD map

Proof:

(i)⇒(ii)
Let U∈O(X). As g^{-1} is continuous then $(g^{-1})^{-1}$(U)=g(U)is $R^{\#}$-O(Y). Therefore f is $R^{\#}$-O.
(ii)⇒(iii)
Let F ∈C(X).Since g is $R^{\#}$-O then g(F^c) is $R^{\#}$-O(Y). But g(F^c)= g(F)^c. Therefore g(F) is $R^{\#}$-C(Y). Hence f is $R^{\#}$-CD.
(iii)⇒(i)
Let F∈C(X).Then g(F) is $R^{\#}$-C(Y). But g(F)=$(g^{-1})^{-1}$(F). Therefore g^{-1} is continuous.

Theorem 3.35: Assume a map g is $R^{\#}$-O then g(int(H)) ⊆$R^{\#}$int(g(H)) ∀A⊆X

Proof: For a open map g,let H⊆M and int(H) be O(M). Then g(int(H)) is $R^{\#}$-O(N). We know that g(int(H))⊆ g(H). By results 2.7, g(int(H)) ⊆$R^{\#}$int(g(H))space. For the $R^{\#}$-closed set {q} with r∉ {q}, ∃ no two disjoint open sets U, V ∋ {q}⊆ U and r∈ V.

Theorem 3.36: If a map g is $R^{\#}$-O map then for each neighbourhood U of x in M, ∃ a $R^{\#}$-nbd W of f(x) in N such that W⊆f(U).

Proof: Given a $R^{\#}$-open map g. Let xϵM and U be a nbd of x in M. By the definition of $R^{\#}$-nbd,∃ $R^{\#}$-O-G(M) such that xϵG⊆U. Consider g(x)ϵ g(G)⊆ g(U). Since g is $R^{\#}$-O then g(G) is $R^{\#}$-O(N). By results 2.7, F(G) is $R^{\#}$-nbd of each of its points. Take g(G)=W, W is $R^{\#}$-nbd of g(x) in N such that W=f(U).

4. $R^{\#}$*-closed maps and $R^{\#}$*-open maps

Definition 4.1: A map g is said to be a $R^{\#}$*-closed map if the image g(A) is $R^{\#}$-C(N) ∀$R^{\#}$-C(A) in M.

Theorem 4.2: Every $R^{\#}$*-closed map is a $R^{\#}$-CD converse is false.

Proof: The proof follows is obvious that every CD set is $R^{\#}$-CD.

Example 4.3: Assume M=N=Z={1, 2, 3, 4}, τ={∅, M, {1, 2}, {3, 4}}, σ={∅,N, {1}, {2}, {1, 2}, {1, 2, 3}} and η={ ϕ, Z, {1}, {1, 2}, {1, 2, 3}}.
Let f be a map, f(1)=4,f(2)=2,f(3)=2,f(4)=4 is $R^{\#}$-CD map but not a $R^{\#}$*-CD map as the image of the $R^{\#}$-C {1, 2} in M is {3} which is not a $R^{\#}$C(N).

Theorem 4.4: If the maps g and h are $R^{\#}$*-C maps then the composition hog: M→Z is $R^{\#}$*-C.

Proof: Accept F be any $R^{\#}$-C(M). Since g is $R^{\#}$*-C then g(F) is $R^{\#}$-C(N). Since h is $R^{\#}$*-C then h(g(F)) is $R^{\#}$-C(Z). Therefore hog is $R^{\#}$*-C map.

Definition 4.5: A map g is known as $R^{\#}$*-open map if g(H) is $R^{\#}$-O(N) ∀ $R^{\#}$-O-H(M).

Theorem 4.6: Each $R^{\#}$-O map is $R^{\#}$*-open map convers is false.

Proof: The proof is obvious.

Example 4.7: Assume M=N={1, 2, 3}, τ={∅, M, {1}, {2}, {1, 2}} and σ={∅, N, {1}, {2}, {1, 2}, {1, 3}}. Let a map g defined by g(1)=3, g(2)=1, g(3)=2 is a $R^{\#}$-O map but not a $R^{\#}$*-O map as the image of the $R^{\#}$-O set {1, 2} in M is {2, 3} in N which is not a $R^{\#}$-O(N).

Theorem 4.8: Assume the maps g and h are $R^{\#}$*-O map then their composition hog is also $R^{\#}$*-open.

Proof: The proof is as theorem 4.6.

5. Conclusion

In this paper we defined and studied $R^{\#}$-closed maps, $R^{\#}$-open maps, $R^{\#}$*-closed maps and $R^{\#}$*-open maps.

References

[1] Basavaraj M Ittanagi and Raghavendra K, On $R^{\#}$-Continuous and $R^{\#}$-Irresolute maps in Topological Spaces Int. J. Adv. Res. Mar-2018-6(2), 461-470
[2] Basavaraj M Ittanagi and Raghavendra K On $R^{\#}$-closed sets in Topological spaces,IJMA- 8(8),2017,134-141
[3] Basavaraj M Ittanagi and Raghavendra K On $R^{\#}$-open sets in Topological spaces,JCMS- 8(11),2017,614-620
[4] Malghan.S.R,Generalized closed maps, J. Karnatak Univ. Sci., 27(1982), 82-88.
[5] Nagaveni.N, Studies on Generalizations of Homeomorphisms in Topological Spaces, Ph. D. Thesies, Bharathiar University, Coimbatore, 2000
[6] Wali R S and Prabhavati S. Mandalgeri, On αrw-Closed and αrw-Open maps Maps in Topological Spaces, International Journal of Applied Research, 1(11) 2015, 511-518.